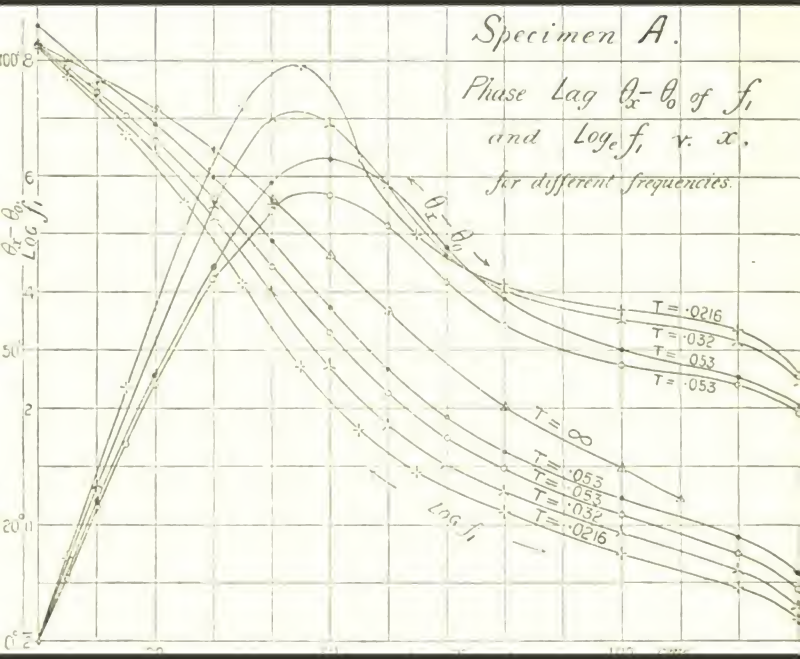

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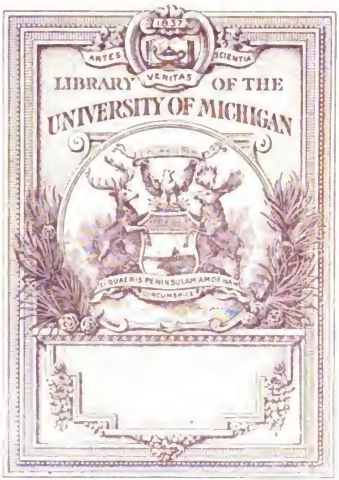

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JOHN JOLY, M.A. D.Sc. F.R.S. F.G.S.

AND

WILLIAM FRANCIS, F.L.S.

"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster ilior quia ex alienis libamus ut apes." JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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"Meditationis est perscrutari occulta; contemplationis est admirari perspicua .... Admiratio generat quæstionem, quæstio investigationem, investigatio inventionem."—*Hugo de S. Victore.*

——"Cur spirent venti, cur terra dehiscat,
Cur mare turgescat, pelago cur tantus amaror,
Cur caput obscura Phœbus ferrugine condat,
Quid toties diros cogat flagrare cometas,
Quid pariat nubes, veniant cur fulmina cœlo,
Quo micet igne Iris, superos quis conciat orbes
Tam vario motu."

J. B. Pinelli ad Mazonium.

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CONTENTS OF VOL. XII.

(SIXTH SERIES).

NUMBER LXVII.—JULY 1906.

	Page
Mr. W. Sutherland on the Molecular Constitution of Aqueous Solutions	1
Mr. G. A. Schott on the Electron Theory of Matter and the Explanation of Fine Spectrum Lines and of Gravitation	21
Mr. G. F. H. Smith on the General Determination of the Optical Constants of a Crystal by means of Refraction through a Prism	29
Mr. R. Hargreaves on Ellipsoidal Harmonics, Æolotropic and Isotropic	36
Dr. J. G. Leathem on a Simplification of the Mathematical Discussion of the Effect of a Long Straight Wire on Electric Waves	46
Prof. J. H. Jeans on the Thermodynamical Theory of Radiation	57
Prof. J. H. Jeans on the H-Theorem and the Dynamical Theory of Gases	60
Letter from Mr. Barber Starkey describing remarkable results of a Discharge of Lightning upon an Oak-tree: with Note by Lord Kelvin	62
Mr. H. Davies on the Solution of Problems in Diffraction by the Aid of Contour Integration	63
Prof. R. W. Wood on Interference Colours of Chlorate of Potash Crystals, and a New Method of Isolating Heat Waves. (Plate I.)	67
Mr. A. A. Campbell Swinton on the Effect of Radium in Facilitating the Visible Electric Discharge in Vacuo	70
Dr. Howard L. Bronson on the Periods of Transformation of Radium A, B, and C	73
Dr. O. Hahn on some Properties of the α Rays of Radiothorium (II.)	82
Notices respecting New Books:—	
Dr. G. Jäger's Die Fortschritte der kinetischen Gastheorie.	93
Baumert's Lehrbuch der Gerichtlichen Chemie. Zweiter Band	94

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Digitized by Google

Page
Dr. B. Schmid's Philosophisches Lesebuch 94
G. Zahikjanz's Die Theorie, Berechnung und Konstruktion der Dampfturbinen 95
Proceedings of the Geological Society :—
The President's Anniversary Address 95

NUMBER LXVIII.—AUGUST.

Lord Rayleigh on the Experimental Determination of the Ratio of the Electrical Units 97
Dr. W. H. Eccles on the Effect of Electrical Oscillations on Iron in a Magnetic Field 109
Mr. J. S. Dow on Colour Phenomena in Photometry 120
Prof. E. Rutherford on Retardation of the α particle from Radium in passing through Matter. (Plate II. fig. 3).... 134
Prof. R. W. Wood on Abnormal Polarization and Colour of Light scattered by Small Absorbing Particles 147
Mr. P. Blackman on Atomic Conductivities of the Ions 150
Prof. E. Rutherford on the Distribution of the Intensity of the Radiation from Radioactive Sources. (Plate II.) 152
Prof. R. W. Wood on Fish-Eye Views, and Vision under Water. (Plate III.) 159
Notices respecting New Books :—
Prof. R. W. Wood's Physical Optics 162
Prof. Carl Barus's The Nucleation of the Uncontaminated Atmosphere 164
Prof. E. Guarini's Derniers Progrès du Telphérage Électrique 164
Proceedings of the Geological Society :—
Mr. R. D. Oldham on the Constitution of the Interior of the Earth, as revealed by Earthquakes 165
Miss E. M. R. Wood on the Tarannon Series of Tarannon 166
Mr. G. M. Cockin on the Occurrence of Limestone of the Lower Carboniferous Series in the Cannock-Chase Portion of the South-Staffordshire Coalfield 168
Rev. E. Hill on the Chalk and Drift in Möen 168
Prof. T. G. Bonney on the Relations of the Chalk and Boulder-Clay near Royston 169
Prof. G. A. L. Lebour & Dr. J. A. Smythe on a Case of Unconformity and Thrust in the Coal-Measures of Northumberland 170
Messrs. H. J. O. White & Ll. Treacher on the Phosphatic Chalks of Winterbourne and Boxford 171
Prof. G. de Lorenzo on the Eruption of Vesuvius in April 1906 173
Mr. D. C. Evans on the Ordovician Rocks of Western Caermarthenshire 174

Digitized by Google

Page
Prof. W. J. Sollas on Recumbent Folds produced as a Result of Flow 175
Dr. H. Pjetursson on the Crag of Iceland—an Intercalation in the Basalt-Formation 176

NUMBER LXIX.—SEPTEMBER.

Dr. M. Levin on the Origin of the β Rays emitted by Thorium and Actinium 177
Mr. A. S. Eve on the Radioactive Matter in the Earth and the Atmosphere 189
Messrs. W. Hawthorne and W. B. Morton on the Deflexions caused by a Break in an Overhead Wire carried on Poles . 200
Mr. A. Russell on the Dead Points of a Galvanometer Needle for Transient Currents 202
Mr. J. A. Brown: An Investigation of the Potential required to maintain a Current between Parallel Plates in a Gas at Low Pressures 210
Dr. J. Morrow on the Lateral Vibration of Bars subjected to Forces in the Direction of their Axes 233
Dr. O. Hahn on the Ionization Ranges of the α Rays of Actinium 244
Dr. J. A. Ewing on the Molecular Structure of Metals 254
Prof. L. T. More on the Dielectric Strain along the Lines of Force 268
Prof. H. A. Wilson on the Theory of "Moving Coil" and other kinds of Ballistic Galvanometers 269

NUMBER LXX.—OCTOBER.

Mr. R. D. Kleeman on the Recombination of Ions made by α, β, γ, and X Rays 273
Dr. S. R. Milner on the Use of the Secohmmeter for the Measurement of Combined Resistances and Capacities.... 297
Dr. P. E. Shaw on the Disruptive Voltage of Thin Liquid Films between Iridio-Platinum Electrodes.—Part I. Voltages 25–400 317
Prof. R. W. Wood on the Fluorescence, Magnetic Rotation, and Temperature Emission Spectra of Iodine Vapour. (Plate IV.) 329
Prof. F. T. Trouton and Mr. C. Searle on Leakage Currents in the Moisture condensed on Glass Surfaces 336

Digitized by Google

Page

Prof. E. Rutherford on the Mass and Velocity of the α particles expelled from Radium and Actinium. (Plate V.) 348

Prof. E. Rutherford and Dr. O. Hahn on the Mass of the α particles from Thorium. (Plate V.) 371

Mr. J. A. Crowther on the Coefficient of Absorption of the β Rays from Uranium 379

Profs. R. B. Moore and H. Schlundt on some New Methods for Separating Uranium X from Uranium 393

Sir W. Ramsay and Dr. J. F. Spencer on Chemical and Electrical Changes induced by Ultraviolet Light. (Plates VI. & VII.) 397

Prof. E. L. Hancock on the Effect of Combined Stresses on the Elastic Properties of Iron and Steel. (Plate VIII.) .. 418

Prof. E. L. Hancock on Tests of Metals in Reverse Torsion. (Plates IX. & X.) 426

Notices respecting New Books:—

Dr. T. Muir's The Theory of Determinants in the Historical Order of Development 431

Mr. T. B. Strong's Lectures on the Method of Science . 431

Dr. F. Niethammer's Moteurs à Collecteur à Courants Alternatifs 432

NUMBER LXXI.—NOVEMBER.

Prof. T. R. Lyle and Mr. J. M. Baldwin: Experiments on the Propagation of Longitudinal Waves of Magnetic Flux along Iron Wires and Rods 433

Mr. A. Campbell on the Use of Chilled Cast Iron for Permanent Magnets 468

Mr. E. F. Burton on the Action of Electrolytes on Colloidal Solutions 472

Prof. R. A. Lehfeldt on the Acceleration of Gravity at Johannesburg 479

Prof. C. A. Skinner: Comparative Observations on the Evolution of Gas from the Cathode in Helium and Argon . 481

Lord Rayleigh on the Interference-Rings, described by Haidinger, observable by means of Plates whose Surfaces are absolutely Parallel. (Plate XVI.) 489

Mr. C. V. Raman on Unsymmetrical Diffraction-bands due to a Rectangular Aperture 494

Mr. L. B. Tuckerman's Remarks on Prof. Jeans's Article "On the Thermodynamical Theory of Radiation" 498

Prof. R. W. Wood on the Fluorescence and Magnetic Rotation Spectra of Sodium Vapour, and their Analysis. (Plates XI.–XV.) 499

Notices respecting New Books:—

Prof. J. A. Fleming's The Principles of Electric Wave Telegraphy 525

Digitized by Google

Page

Dr. G. von Neumayer's Anleitung zu Wissenschaftlichen Beobachtungen auf Reisen 526
S. Vogl's Die Physik Roger Bacos 527
Jamin's Cours de Physique de l'École Polytechnique, Troisième Supplément par Prof. Bouty 527
Drs. Millikan and Gale's First Course in Physics 528
G. de Metz's La Double Réfraction accidentelle dans les Liquides 528
Proceedings of the Geological Society :—
Dr. Maria Ogilvie Gordon on Interference-Phenomena in the Alps 529
Mr. W. R. Baldwin-Wiseman on the Influence of Pressure and Porosity on the Motion of Sub-Surface Water 531
Intelligence and Miscellaneous Articles :—
Atomic Conductivities of the Ions, by P. Blackman 532

NUMBER LXXII.—DECEMBER.

Mr. W. A. Scoble on the Strength and Behaviour of Ductile Materials under Combined Stress 533
Dr. H. W. Malcolm on Double Refraction in Colloids produced by Electric Endosmose 548
Dr. G. Bakker on the Theory of Surface Forces 557
Dr. S. Tolver Preston on Certain Questions connected with Astronomical Physics 570
Prof. E. H. Barton and Mr. J. Penzer on Simultaneous Vibration-Curves of String and Air photographically obtained from a Monochord. (Plate XIX.)..... 576
Mr. G. A. Schott : A Kinematical Explanation of Groups of Spectrum Lines with Constant Frequency-Difference 579
Dr. W. Marshall Watts and Mr. H. M. Wilkinson on the "Swan" Spectrum 581
Prof. R. W. Wood on the Intensification of Glass Diffraction Gratings, and the Diffraction Process of Colour Photography. 585
Prof. J. A. Fleming on the Electric Radiation from Bent Antennæ. (Plates XVII. & XVIII.) 588
Dr. R. S. Willows on the Electrical Resistance of Alloys .. 604
Notices respecting New Books :—
W. H. Wheeler's Practical Manual of Tides and Waves. 610
Dr. C. Doelter's Petrogenesis 610

Index 611

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PLATES.

I. Illustrative of Prof. R. W. Wood's Paper on Interference Colours of Chlorate of Potash Crystals and a New Method of Isolating Heat Waves.

II. Illustrative of Prof. E. Rutherford's Papers on Retardation of the α particle from Radium in passing through Matter, and on the Distribution of the Intensity of the Radiation from Radioactive Sources.

III. Illustrative of Prof. R. W. Wood's Paper on Fish-Eye Views and Vision under Water.

IV. Illustrative of Prof. R. W. Wood's Paper on the Fluorescence and other Spectra of Iodine Vapour.

V. Illustrative of Prof. E. Rutherford's Paper on the Mass and Velocity of the α particles expelled from Radium and Actinium; and of Prof. E. Rutherford and Dr. O. Hahn's Paper on the Mass of the α particles from Thorium.

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IX. & X. Illustrative of Prof. E. L. Hancock's Paper on Tests of Metals in Reverse Torsion.

XI.–XV. Illustrative of Prof. R. W. Wood's Paper on the Fluorescence and Magnetic Rotation Spectra of Sodium Vapour, and their Analysis.

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[SIXTH SERIES.]

JULY 1906.

I. *The Molecular Constitution of Aqueous Solutions.* *By* WILLIAM SUTHERLAND *.

IN the recent study of aqueous solutions most attention has been given to the solute, little being devoted to the equally important changes in the solvent. From the data of "The Molecular Constitution of Water" (Phil. Mag. [5] vol. l. 1900) it is possible to calculate the effect of solute on solvent, and so to arrive at a tolerably complete conception of the molecular constitution of aqueous solutions. In that paper, it was shown by the development of ideas of Röntgen and others that water consists of a mixture of trihydrol $(H_2O)_3$ and dihydrol $(H_2O)_2$, ice being pure trihydrol, and water at its critical temperature nearly pure dihydrol, while aqueous vapour is hydrol, H_2O. At about 15° C. water is one-third trihydrol and two-thirds dihydrol, the percentages at other temperatures being given in Table I. of that paper. Various physical constants of both constituents are collected in Table XIII., supplying necessary data for the following inquiry, in which it will be shown that a solute causes the dissociation of trihydrol into dihydrol, whence the abnormal behaviour of aqueous solutions. Electrolytic solutes will be taken as the most general type, the results of which can be applied to non-electrolytes by making the electrolytic dissociation of these vanish. The laws of the dissociation of trihydrol by solutes will be investigated by considering for

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solutions: 1. Density; 2. Theory of Density; 3. Special case of Acids and Alkalies; 4. Specific Heat; 5. Surface Tension: 6. Summary.

1. *Density.*

As a typical instance of the variation of density with concentration the data of Bousfield and Lowry for NaOH (Phil. Trans. A. 204. 1904) will be studied first. Suppose 1 gramme of solution to be made of p_2 grm. of water and p_4 of solute. Bousfield and Lowry show that for values of p_4 from 0·05 to 0·18 the density ρ at 18° C. is given by

$$\rho = 1{\cdot}05454 + 1{\cdot}105(p_4 - 0{\cdot}05). \quad . \quad . \quad . \quad . \quad (1)$$

This reproduces the experimental densities within 2 parts in 10^5. For values of p_4 between 0 and 0·05 their formula is

$$\rho = 0{\cdot}99866 + 1{\cdot}141p_4 - 0{\cdot}52p_4^2. \quad . \quad . \quad . \quad (2)$$

Similarly the data of Kohlrausch for NH_4Cl at 18° C. from $p_4 = 0{\cdot}05$ to 0·25 can be expressed by

$$\rho = 1 + 0{\cdot}2856p_4, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

with a maximum error of 4 in 10^4. For values of p_4 less than 0·05 a term in p_4^2 would have to be introduced also, as in (2), with modification of the coefficient of p_4. But we shall confine our attention to those solutions in which a linear relation between ρ and p_4 suffices. Now to facilitate the study of solutions in general, we can use the principle of modular properties discovered by Valson (*Comptes Rendus*, lxiii. 1871) and extended by Bender (Wied. *Ann.* xx.). Valson studied solutions made by dissolving gramme equivalents in 1 litre of water, and found that if such a solution of a salt MR has its density compared with that for another QR, the difference of the two is independent of R; so that if NH_4 is made a fixed standard of reference M, $\rho_{QCl} - \rho_{MCl}$ when added to ρ_{MR}, the density of NH_4R solution, gives that of QR solution, whatever R may be. This difference is Valson's modulus for the positive ion Q. Similarly a modulus exists for the negative ion, and was specified by Valson with reference to Cl as the fixed standard. He gave moduluses for 13 positive and 6 negative ions. Bender, in extending the principle to other strengths of solutions, found it better to compare solutions having equal numbers of gramme equivalents in a litre of solution, and gave the following formula for the density of a solution containing n equivalents per litre,

$$\rho_n = R_n + n(P + N), \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where R_n is the density of a solution of NH_4Cl containing n gramme equivalents per litre, P being the modulus for the

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positive ion and N for the negative. Hence by means of (3) and a list of values for P and N we can obtain the density ot a solution of any compound containing the ions in the list. Bender's values of P and N at 18° C. are given in Table I. below.

We must now consider how these densities are connected with the changes occurring in water during solution. Let 1 gramme of water contain p_1 dihydrol and p_2 trihydrol. Let n gramme equivalents e of solute in dissolving dissociate $n\tau$ gramme-molecules of $(H_2O)_3$ into $3n\tau/2$ of $(H_2O)_2$ in a litre of solution, which contains ne grammes of solute and $10^3\rho_n - ne$ of water. Hence

$$p_4 = ne/10^3\rho_n, \quad p_3 = 1 - ne/10^3\rho_n.$$

Before solution p_3 consists of $p_1 p_2 (H_2O)_2$ and $p_2 p_3 (H_2O)_3$. After solution the water of a litre consists of $p_1(10^3\rho_n - ne) + 54n\tau$ grammes of $(H_2O)_2$ and $p_2(10^3\rho_n - ne) - 54n\tau$ grammes of $(H_2O)_3$. If p_1' and p_2' denote the fractions of the water in a solution which are $(H_2O)_2$ and (H_2O_3), we have

$$\left.\begin{aligned} p_1' = p_1 + 54n\tau/(10^3\rho_n - ne) = p_1 + 54n\tau/10^2 \text{ approx.} = p_1 + 54\tau p_4/e \\ p_2' = p_2 - 54\tau p_4/e \end{aligned}\right\}. \quad (5)$$

Let v_1, v_2, and v_4 be the volumes of a gramme of $(H_2O)_2$, $(H_2O)_3$, and solute at 18°; then, on the assumption that no other changes of volume occur during solution than that due to dissociation of $(H_2O)_3$, we have for the volume v_n of a gramme of solution

$$v_n = 1/\rho_n = p_2(p_1'v_1 + p_2'v_2) + p_4 v_4 = (1 - p_4)\{p_1'(v_1 - v_2) + v_2\} + p_4 v_4. \quad (6)$$

Hence for the change of volume on solution

$$p_2(p_1 v_1 + p_2 v_2) + p_4 v_4 - v_n$$

we have

$$54 p_4 (1 - p_4)(v_2 - v_1)\tau/e. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In $p_4(1 - p_4)$ this reminds us of mass action at work. It was shown from experiments by A. Geritsch to be the law of contraction (Nature, xxxix. p. 334; Journ. Russ. Chem. Phys. Soc. xx.). Recently Happart has shown contraction to be proportional to $p_4(1 - p_4)$ for a large number of solutions (Landolt and Börnstein, *Tabellen*, 3rd edition).

Substituting, making the approximations

$$p_1 v_1 + p_2 v_2 = 1 = p_1' v_1 + p_2' v_2 \text{ and } \rho_n = 1,$$

after expanding as far as the first power of p_4 we get

$$v_n = 1 + p_4\{v_4 - 1 + 54\tau(v_1 - v_2)/e\}. \quad . \quad . \quad . \quad (8)$$

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But from (3) and (4),

$$v_n = 1/\{1 + 0{\cdot}2856 p_4 \mathrm{E}/e + n(\mathrm{P}+\mathrm{N})\},$$

where E is the equivalent mass of NH_4Cl.

$$\therefore \quad v_n = 1 - p_4\{0{\cdot}2856\mathrm{E}/e + 10^3(\mathrm{P}+\mathrm{N})/e\} \text{ approx.} \quad . \quad (9)$$

Comparing (8) and (9) we have

$$\tau = \{(1-v_4)e - 0{\cdot}2856\mathrm{E} - 10^3(\mathrm{P}+\mathrm{N})\}/54(v_1 - v_2). \quad . \quad (10)$$

In this expression $v_4 e$ is the volume of a gramme equivalent of the solute, which for most electrolytes is the sum of parts due to each ion, e is the sum of such parts, as is also P + N. Therefore (10) contains the rather important result that the number of molecules of $(H_2O)_3$ dissociated by an equivalent of solute is the algebraic sum of parts due to its positive and negative ions and a part $0{\cdot}2856\mathrm{E}/54(v_1 - v_2)$, whose apportionment to positive and negative ions will be discussed immediately. Its value is $-1{\cdot}29$, of which we suppose $-1{\cdot}29 + x$ and $-1{\cdot}29 + y$ to belong to the positive and negative ions.

We shall now tabulate the data requisite for the calculation of τ by (10) and the parts of τ due to each ion together with $-1{\cdot}29 + x$ or $-1{\cdot}29 + y$. The parts of $v_4 e$ due to each ion are obtainable from "Further Studies on Molecular Force" (Phil. Mag. [5] xxxix. 1895, pp. 15, 24, 26, 31), where they are denoted by B for the whole gramme atom; P and N from Bender (*loc. cit.*). As e is the usual equivalent weight, it is not tabulated.

$$\mathrm{E} = 18 + 35{\cdot}4, \; v_1 = 0{\cdot}9179, \; v_2 = 1{\cdot}1364.$$

These are the values of v_1 and v_2 at 0°, the latter not being accurate enough to make it worth while correcting for 18°.

TABLE I.

	Li.	Na.	K.	NH_4.	Mg.	Ca.	Sr.	Ba.	Zn.	Cd.	Pb.	Ag.
$v_4 e$	2·0	7·4	18·6	18·0	2·8	4·3	5·3	8·3	5·3	6·25	4·9	6·8
10^3P	7·7	23·5	29·6	0	22·1	28·2	52·2	73·9	41·0	60·6	1·09	1·07
$\tau - 1{\cdot}29 + x$	0·23	0·67	0·78	0	1·09	1·06	1·16	1·14	1·16	0·91	0·88	0·49
τ	1·52	1·96	2·07	1·29	2·38	2·35	2·45	2·43	2·45	2·20	2·17	1·78

	Cl.	Br.	I.	NO_3.	SO_4.	CO_3.	CH_3COO.
$v_4 e$	19	26	36	31	17	13	42
10^3N	0	37	73·3	16	20	14	−1·5
$\tau - 1{\cdot}29 + y$	−1·39	−1·44	−1·50	−1·27	−0·93	−0·25	−1·57
τ	−1·39	−1·44	−1·50	−1·27	−0·93	−0·25	−1·57

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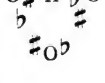
The striking result in this Table is that $\tau - 1{\cdot}29 + x$ for all the positive ions has a positive sign and $\tau - 1{\cdot}29 + y$ for all the negative. Most probably then the same phenomenon holds in regard to τ. If so, then, so far as I know, this is the first case in molecular physics where, apart from purely electrical considerations, the opposite charges of the two sorts of ions are known to manifest themselves by opposite actions. Positive ions decompose $(H_2O)_3$ into $(H_2O)_2$, while negative ions form (H_2O_3) out of $(H_2O)_2$.

As regards the $-1{\cdot}29 + x$ and $-1{\cdot}29 + y$ which associate with τ, the problem is to find x and y. I have been able only to guess at a solution by the following train of thought. Since $\tau - 1{\cdot}29 + y$ has for Cl a negative value numerically much greater than the positive values for Li, Na, or K, it appears probable that 1·29 belongs almost wholly to the positive ions, and hardly at all to the negative. Tentatively then I have added 1·29 to $\tau - 1{\cdot}29 + x$ as given for the positive ions, and 0 to $\tau - 1{\cdot}29 + y$ as given for the negative: and I have tabulated the results in Table I. as τ; that is to say, I have arbitrarily put $x = 0$, $y = 1{\cdot}29$, a procedure to be justified by the results to which it leads in the next section. But altogether apart from this, the table supplies independent confirmation of the hypothesis of electrolytic dissociation in aqueous solutions.

2. *Theory of Density.*

From the values of τ in Table I. it appears that, with the exception of CO_2, an equivalent of an ion unmakes or makes from 1 to 2·5 molecules of $(H_2O)_3$. This seems to indicate that an ion controls the structure of only those molecules of water which are in its immediate neighbourhood. So we are constrained to form a theory to account for the novel type of dissociation here presented with its manifest bearings on catalytic actions and the coagulation of colloids. The electron formula proposed for $(H_2O)_3$ in "Ionization, Ionic Velocities, and Atomic Sizes" (Phil. Mag. [6] iii. p. 162) assumes tetrad valency for the O atom due to 3 negative electrons ♭ and one positive ♯. The binding of the three O atoms in $(H_2O)_3$ may be represented thus, with the letter A to denote one of the three effective bonds ♯♭:—



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Suppose an external ♯ to approach A from the left. It will repel the ♯ of A and attract ♭, thus increasing the chemical cohesion at A by forcing the two oxygen atoms closer together. But ♯ on the right of A will lessen the chemical cohesion by repelling the ♯ and attracting the ♭ of A. Now I have shown in "The Dielectric Capacity of Atoms" (Phil. Mag. [6] vii.; Austr. Assoc. Adv. Sci. x. 1904) that all the ordinary ions have a much smaller dielectric capacity than water; so the electric force between electrons in an ion and electrons in adjacent water molecules will be greater than the forces between electrons in the water molecules at the same distance apart. This shows that an electron can break or strengthen the bonds of $(H_2O)_3$ and $(H_2O)_2$ according to the relative position of the ion and the bonds. The ions must be continually making and unmaking $(H_2O)_3$ and $(H_2O)_2$. The reaction accompanying this action is the process by which water ionizes the solute.

This theory of Ionization has already been advanced in "Ionization &c." (*loc. cit.*). Hence the formulæ for the density and ionization of solutions are practically equations for the chemical equilibrium of the complex mixture $(H_2O)_3$, $(H_2O)_2$, solutes and ions. Our present concern is with the rates at which the processes of making and unmaking $(H_2O)_3$ go on. The chief cause is the electric force at the surface of each ion. The average position for the electron of an ion is at the centre of the ion, although actually the electron may never be there. In the case of a divalent ion we have to assume that each electron acts on the average as though it were at the centre, although the mutual repulsion of the electrons will not allow them both to be at the centre at the same time. If a denote the radius of an ion and K its dielectric capacity, then $1/Ka^2$ is the chief cause of making and unmaking $(H_2O)_3$. The simplest presentation of the complex reactions is to assume that a positive ion keeps a certain number of $(H_2O)_3$ molecules split into three separate groups ♯H_2O♭ which differ from the H_2O of steam by the large electric moment of ♯ and ♭ before they have coalesced as in H_2O. This form of charged molecule or atom I have proposed to call a stion. A positive ion produces from trihydrol a certain number of stions, each of which has a very short separate existence. By their recombination three at a time these form $(H_2O)_3$ again. Equilibrium ensues when the number formed in unit time equals the number broken. For simplification we have merged the action of a positive ion on $(H_2O)_2$ in that on $(H_2O)_3$. Let a positive ion produce x stions, then the number of $(H_2O)_3$ molecules formed from these in

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unit time will be proportional to x^3. Moreover, the chance of the ions bringing ♯ of one stion and ♭ of another into such a position as favours their forming a bond may be taken proportional to $(1/Ka^2)^2$, the square being taken because there are two independent electrons to be controlled by the electric force at the surface of the ion. In $(H_2O)_3$ there are 3 bonds to be considered; but probably the formation of one causes the other two to form almost automatically, because $(H_2O)_3$ forms spontaneously with such ease. Thus we are going to measure the rate of formation of $(H_2O)_3$ from stions by $x^3/(Ka^2)^2$. The rate at which a positive ion breaks up $(H_2O)_3$ must be proportional to the rate at which the ion hemmed in amongst $(H_2O)_3$ molecules moves so as to get near to a bond in such a way as to break it. The ion is kept on the move by alternate attractions and repulsions proportional to $1/Ka^2$ generating velocity proportional to λ/Ka^2, where λ is the ionic velocity under standard external electric force. Hence for equilibrium

$$x^3/(Ka^2)^2 = c\lambda/Ka^2, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where in dilute solutions c may be treated as a constant, though for concentrated ones it becomes appreciably a function of x. Now, though x has been taken to represent a number of H_2O stions, $x/3$ really represents the number of $(H_2O)_3$ molecules destroyed by the presence of the positive ion, the stions ♯H_2O♭ existing mostly as $(H_2O)_2$ in process of formation from $(H_2O)_3$, or into $(H_2O)_3$. Hence x is proportianal to τ, and (11) gives us the result that for a positive ion with $a \propto B^{\frac{1}{2}}$,

$$\tau/(\lambda K B^{\frac{3}{2}})^{\frac{1}{2}} \text{ is constant}, \quad . \quad . \quad . \quad . \quad (12)$$

In a similar way we can make the problem of the action of the negative ions tractable by treating it as a process of building up $(H_2O)_2$ in excess of its amount in pure water through the agency of an imaginary supply of stions representing the actual increase of flux and reflux between $(H_2O)_2$ and $(H_2O)_3$ caused by the presence of the negative ion. On account of the 2 in the formula of $(H_2O)_2$ we obtain for the negative ion instead of (12)

$$\tau/(\lambda K B^{\frac{3}{2}})^{\frac{1}{4}} \text{ is constant}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In the next table these relations (12) and (13) are tested by means of the values of $\tau/(\lambda K B^{\frac{3}{2}})^{\frac{1}{2}}$ and $\tau/(\lambda K B^{\frac{3}{2}})^{\frac{1}{4}}$, given in the last row from the data, namely τ from Table I., λ from Kohlrausch (Wied. *Ann.* lxvi.), B from Table I. in which $\tau_1 e = B$ for monovalent ions and $= B/2$ for divalent, and K from "The Dielectric Capacity of Atoms" (*loc. cit.*), or from the formula there given.

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TABLE II.

	Li.	Na.	K.	NH_4.	Mg.	Ca.	Sr.	Ba.	Zn.	Cd.	Pb.	Ag.
λ	35·5	44·4	65·3	64·2	48	53	54	57·3	47·5	(52·5	(57)	55·7
K	6·27	3·24	1·62	1·67	6·58	5·16	4·72	3·83	5·36	4·60	4·59	2·66
$10^3\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$	215	240	229	143	238	224	229	216	229	201	204	229

	Cl.	Br.	I.	NO_3.	SO_4.	CO_3.	CH_3COOH.
λ	65·9	67·5	66·7	60·8	69·7	(81)	35·4
K	1·59	1·40	1·27	1·47	2·48	2·33	2·28
$10^3\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$	51	50	49	43	22	6	50

For the positive ions the values of $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$ are nearly constant except in the case of NH_4, the mean value being 0·222. For the negative ions, except the divalent SO_4 and CO_3, $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$ is nearly constant, say 0·05.

This sketch of the theory of the dissociation of $(H_2O)_3$ in solution does not show why there should be a specific difference in the action of the two sorts of electricity; this difference having been assumed as a matter of fact established by Table I. In electrical theory as hitherto developed there is no reason why one sort of electricity should have a different effect from the other. It seems as though positive and negative electricity on the molecular scale have different energy relations to dielectric capacity in the surrounding medium. Concerning frictional electricity, we have the valuable generalization discovered by A. Coehn (Wied. *Ann.* lxiv.), that if two insulators are rubbed together, the one with the larger dielectric capacity is positively charged. Here, then, we have direct experimental evidence of the difference of relation of the two kinds of electricity to dielectric capacity; and the similar case just made out for atomic electric charges confirms it in a way which shows that this difference is fundamental in the relation of electricity to matter and needs to be fully investigated.

Probably the effect of ions in dissociating and associating $(H_2O)_3$ and $(H_2O)_2$ is a simple typical instance of their action in coagulating and dissolving colloids and in stimulating and depressing muscle and nerve.

3. *Special Case of Acids and Alkalies.*

The H and OH ions of acids and alkalies occupy a special position on account of the high ionic velocities usually ascribed to them, namely 318 for H and 174 for OH, while SO_4 comes next with only 69·7 and Cs with 67·8. Moreover H and OH are quite exceptional in regard to the relation

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$\lambda B^{\frac{1}{2}}K/\nu = 280$ shown in "Ionization &c." (*loc. cit.*) to hold for the majority of ions. For H the product, even with the too small value of B used in that paper, is 980 instead of 280, while for OH it is 765. It will now be shown that the ionic velocities hitherto assigned to H and OH are not the real velocities of these ions, but the real velocities increased by velocities belonging to ions derived from water. The exceptional properties of H and OH ions can be traced back to the single property, that of being able to ionize H_2O.

From the data for NaOH solutions with 2·13 for the density of NaOH, the contraction in forming a gramme of solution can be written $0{\cdot}49\ p_4(1-p_4)$, whence from (7) τ for NaOH is 1·66. But for Na τ is 1·96, so for OH τ is $-0{\cdot}30$. From the data for KOH the value $-0{\cdot}28$ is obtained, the mean being $-0{\cdot}29$.

To determine τ for the H ion of acids we may take HNO_3 as a convenient typical acid, free from the exceptional relations of H_2SO_4 towards water, and having a definite liquid density at ordinary temperature and pressure unlike HCl, HBr, and HI. The data of Lunge and Rey show a contraction $0{\cdot}26p_4(1-p_4)$ on mixing $1-p_4$ grm. of water with p_4 grm. of HNO_3 for all values of p_4. From this $\tau = 1{\cdot}39$ for HNO_3. But for NO_3 τ is $-1{\cdot}27$; so for H the value of τ is 2·66. If we treat the data for H_2SO_4 in the same way, they yield a value 1·88 for τ for H. With this acid some other action accompanies that we are now investigating.

The data for the halogen acids show a remarkable departure from (7) which makes the contraction proportional to $p_4(1-p_4)$. With these the factor $1-p_4$ falls out over a considerable range of p_4. The difference of the volume of a gramme of solution and of the original volume of the water in it for HCl at 15° is very nearly $0{\cdot}5p_4$ up to $p_4 = 0{\cdot}1$. For HF the same difference is nearly $0{\cdot}6p_4$ up to $p_4 = 0{\cdot}2$; for HBr $0{\cdot}3p_4$ up to $p_4 = 0{\cdot}2$, and for HI $0{\cdot}27p_4$ up to $p_4 = 0{\cdot}67$. It appears then that in the dissolving of halogen acids in water there is an additional contraction of amount $p_4/(1-p_4)$ times the normal one which we are investigating. As this fraction vanishes with p_4 we can obtain the contraction for HCl for instance in the form $(b-0{\cdot}5)p_4$ for small values of p_4, where b is the volume of the molecules in a gramme of HCl at 15°. The difficulty is to determine this rather fictitious volume. A gramme of liquid HCl under its vapour-pressure at 15°·85 occupies 1·2 c.c. This would probably be too large a value to take. The best way to proceed then is to use the limiting volume of the molecules in a gramme as corresponding to the volumes used for solid salts going into solution. This

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limiting volume will be too small and will give the value of τ too small. According to the principles of "The Laws of Molecular Force" (Phil. Mag. [5] xxxv.) the limiting volume of a gramme-molecule of HCl is found to be 27. As the value of B for Cl is 19, that for H is 8. This is double the limiting volume of a gramme-atom of H in the CH_2 of organic compounds. But the refraction equivalent of H in the halogen acids is 3·5, while that in organic compounds is only 1·3. The limiting volume of a gramme of HCl is thus 0·741. Taking B for Br as 26 and for I as 36, the limiting volume of a gramme of HBr is 0·42, and of HI is 0·343. Hence for the coefficient of $p_4(1-p_4)$ in the limiting contraction for HCl, HBr, and HI we have (0·741−0·5), or 0·241, and 0·120 and 0·073. For τ these give 0·74 for HCl, 0·82 for HBr, and 0·79 for HI. Using the values of τ for the halogens, these give for τ for H the values 2·13, 2·22, and 2·34. These are necessarily too small, and are not inconsistent with the value 2·66 obtained from HNO_3, which will be adopted as the correct apparent one. To find $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$ for H for comparison with the average 0·222 for positive ions we need K. Now the refraction equivalent $(N-1)B$ for H in the halogen acids is 3·5, and B is 8, so $N = 1·44$ and $K = N^2 = 2·07$. With $\tau = 2·66$ we get the product desired, 0·19. This is not seriously different from the value for other positive ions, but we shall see soon that this approximate agreement is only accidental. When we evaluate $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}}$ for the OH ion with $\tau = 0·29$, $\lambda = 174$, $K = 2·07$, and $B = 10$, we get 0·007 in place of the 0·050 characteristic of monovalent negative ions. In this respect OH is strongly exceptional.

It will now be shown that all these seemingly exceptional results for H and OH become quite regular, if it is assumed that these ions ionize H_2O into H and OH. The simplest way to demonstrate this is to calculate first what the regular ionic velocities of H and OH ought to be by the equation $\lambda B^{\frac{1}{3}}K/\nu = 280$. These are 67·5 for H and 62·7 for OH, which are much nearer to the values for ordinary ions than the experimental 318 and 174. The sum of the velocities of the ions of H_2O is 130·2, so that if H ionizes (318−67·5)/130·2 or 1·92 times H_2O, the apparently high ionic velocity assigned to H would be accounted for as being nearly three times the velocity of H with nearly twice that of OH. In the same way OH ionizes (174–62·7)/130·2 or 0·86 times H_2O, its apparent ionic velocity being nearly twice the true amount with that of H added on. Now we can readily test these deductions by applying them to the values of τ. Again,

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the simplest way to proceed is to find τ for H by $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}} = 0{\cdot}222$ using for λ 67·5, and to find τ for OH by $\tau/(\lambda KB^{\frac{2}{3}})^{\frac{1}{2}} = 0{\cdot}05$ with $\lambda = 62{\cdot}7$. The results are 1·83 and 1·23. Each H ion keeps 1·83 molecules of $(H_2O)_3$ out of existence, and each OH ion keeps 1·23 in existence. But as regards the H ion of an acid, we have seen that it also breaks up $1{\cdot}92H_2O$ into H and OH, and we ought to consider this H_2O derived from the $(H_2O)_2$ into which $(H_2O)_2$ is broken down by the ion. So there is an extra change of volume due to this ionization. The limiting volume of the H ion being 8 and of OH being 10, the mean volume of a gramme of ionized H_2O is 18/18 or 1. Thus $\frac{1}{2}(H_2O)_2$ in changing from dihydrol into the ions H and OH undergoes expansion $18(1-0{\cdot}9179)$, so the ionization of 1·92 times $\frac{1}{2}(H_2O)_2$ causes expansion 2·84. But the change of $(H_2O)_3$ into $\frac{3}{2}(H_2O)_2$ involves a contraction 11·8. Hence the contraction due to the ionization of H_2O caused by the H ion is equivalent to the forming of 2·84/11·8 or 0·24 molecules of $(H_2O)_3$ from $(H_2O)_2$. Hence for the apparent value of τ for the H ion we have 1·83 due to itself, $1{\cdot}92 \times 1{\cdot}83$ due to the 1·92 ions of H it liberates from H_2O diminished by $1{\cdot}92 \times 1{\cdot}23$ for the contrary effect of the 1·92 ions of OH formed by it, and then further diminished by the 0·24 just found. This gives for the apparent value of τ for H the value 2·74 to be compared with the 2·66 obtained from the HNO_3 data. A similar calculation for the OH ion gives for τ the apparent value

$$-1{\cdot}86 \times 1{\cdot}23 + 0{\cdot}86 \times 1{\cdot}83 + 0{\cdot}86 \times 0{\cdot}1364 \times 18/11{\cdot}8$$

or $-0{\cdot}53$ to be compared with $-0{\cdot}29$ obtained from the NaOH and KOH data. In both cases the agreement is sufficient to confirm the conclusion reached from the apparent ionic velocities of H and OH.

From this investigation of the characteristic ions of acids and alkalies we have obtained the following results:—That the ionic velocities ascribed to H and OH are not the real velocities of these ions, but the sum of the true velocities and the velocities of the ions which these produce from H_2O. In "Ionization &c." (*loc. cit.*) I suggested that the large exceptional velocities of H and OH might be due to the fact that these being the ions of H_2O might enter $(H_2O)_3$ and $(H_2O)_2$ at one end and liberate the corresponding ion at the other by electric action, these ions thus appearing to have larger velocities than the true ones. Here we have reached the more definite conclusion that the exceptionally large velocities are due to the ionization of H_2O by H and OH, whereby the

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number of carriers of electricity is increased, though all travel with velocities which are not exceptional. Indirectly we have confirmed equation (16) of "Ionization &c.," connecting ionic velocities with the properties of ion and solvent. We have also confirmed the relation established in Table II. of the present paper by bringing H into line with the other positive ions and OH into line with the other monovalent negative ions.

4. *Specific Heat.*

It has long been known that if the specific heat of an electrolytic solution is c', that of water being c, and of solute c_4, the difference $p_3c+p_4c_4-c'$ is mostly positive; there is a diminution of specific heat on solution. In "The Molecular Constitution of Water" (*loc. cit.*) it was shown that the specific heat of $(H_2O)_3$ is of the order 0·6, and that of $(H_2O)_2$ about 0·8. The specific heat of water is largely absorbed in changing $(H_2O)_3$ into $(H_2O)_2$. The direct action of solutes in changing $(H_2O)_3$ into $(H_2O)_2$ is to increase the specific heat of the solution above $p_3c+p_4c_4$, because of the increase in the amount of $(H_2O)_2$ having a higher specific heat than the $(H_2O)_3$ which it replaces. If this were the sole action of the solute, $p_3c+p_4c_4-c'$ would be negative instead of generally positive according to experimental evidence. Hence the chief effect of the solute in making $p_3c+p_4c_4-c'$ positive is an alteration of the rate of change of $(H_2O)_3$ into $(H_2O)_2$ with rise of temperature. For the specific heat of water we have formula (25) of "The Mol. Const. of Water,"

$$c=p_1c_1+p_2c_2+(\mathrm{D}+dh/dp_2)dp_2/dt, \quad . \quad . \quad . \quad (14)$$

where D is the heat absorbed in dissociating a gramme of $(H_2O)_3$ into $(H_2O)_2$, and h is the heat evolved when p_1 of $(H_2O)_2$ is mixed with p_2 of $(H_2O)_3$, and t denotes temperature. It is shown that

$$\mathrm{D}+dh/dp_2=-78-3290(0{\cdot}425-p_2)^2. \quad . \quad . \quad (15)$$

For a solution then

$$c'=p_3\{p_1'c_1+p_2'c_2+(\mathrm{D}+dh/dp_2')dp_2'/dt\}+p_4c_4. \quad . \quad (16)$$

Remembering that $p_1+p_2=1=p_1'+p_2'$, we get

$$p_3c+p_4c_4-c'=p_3\{(c_1-c_2)(p_2'-p_2)-(\mathrm{D}+dh/dp_2')dp_2'/dt + (\mathrm{D}+dh/dp_2)dp_2/dt\}. \quad . \quad . \quad . \quad . \quad (17)$$

To fix the probable relative importance of the terms on the right, let us consider the case of a solution of NaCl at 18°

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containing a gramme-molecule per litre. $c_1-c_2=0{\cdot}2$, and $p_2'-p_2=-54\tau/10^3$; but

$$\tau=1{\cdot}96-1{\cdot}39=0{\cdot}57, \text{ so } (c_1-c_2)(p_2'-p_2)=-0\ 0062.$$

At 18° $p_2=0{\cdot}326$; so from (16) $D+dh/dp_2=-110$. Now dp_2/dt is of the order $-0{\cdot}006$; and we see that the term $(D+dh/dp_2)dp_2/dt$ is about 100 times $(c_1-c_2)(p_2'-p_2)$. Hence there is a strong *a priori* probability that the variation caused in this term by the presence of the solute is large compared with $(c_1-c_2)(p_2'-p_2)$. Moreover, experiment shows that this $(c_1-c_2)(p_2'-p_2)$ is of opposite sign to $p_3c+p_4c_4-c'$. Hence, as pointed out above, the chief phenomenon in the specific heat of solutions is connected with $(D+dh/dp_2')dp_2'/dt$. The corresponding term for pure water having a value about 0·66, it follows that with $(c_1-c_2)(p_2'-p_2)$ neglected $(D+dh/dp_2')dp_2'/dt$ must have a value less than 0·66 by sufficient to leave a difference equal to the experimental $(p_3c+p_4c_4-c')/p_3$, which for our particular solution of NaCl is about 0·02. It appears then that a rise of temperature in a solution generally produces a smaller total absorption of heat due to the change of $(H_2O)_3$ into $(H_2O)_2$ than occurs in the water of the solution when separate from the solute.

The simplest assumption we can make about the somewhat complicated difference

$$(D+dh/dp_2)dp_2/dt-(D+dh/dp_2')dp_2'/dt$$

is that it is proportional to the amount of trihydrol dissociated by p_4 of solute, namely $\tau p_4/e$, in which we can merge $(c_1-c_2)(p_2'-p_2)$, as it also is proportional to $\tau p_4/e$. This assumption transforms (17) into $p_3c+p_4c_4-c'=ap_3p_4\tau/e$, in which a is an absolute constant. An appeal to the experimental data shows that $(p_3c+p_4c_4-c')/p_3p_4$ is not constant, but a linear function of p_4. Thus we are led to adopt

$$(p_3c+p_3c_4-c')/p_3p_4=a\tau/e-bp_4, \quad . \quad . \quad . \quad (18)$$

in which b is a parameter characteristic of each solute. From the data for NaCl given by Thomsen (*Thermochemische Untersuchungen*, Bd. i. p. 40), $a\tau/e=0{\cdot}49$ and $b=1{\cdot}76$, furnishing the following comparison:—

$1000p_4$....	245	139	97	61	31	16
$1000c'$ exp....	791	863	895	931	962	978
" calc....	794	861	896	930	963	980
$1000(p_3c+p_4c_4-c')$...	16	28	29	21	14	9
$100(p_3c+p_4c_4-c')/p_3p_4$ exp....	9	23	33	37	46	58
" " " calc....	7	25	32	39	44	46

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Solutions of Na_2SO_4 may be taken as typical of those in which $p_3c+p_4c_4-c'$ becomes a negative at a certain concentration. The experimental data are taken from the *Tabellen* of Landolt and Börnstein, 3rd edit., as are all the subsequent data for specific heats. I find $a\tau/e=0{\cdot}49$ and $b=1{\cdot}7$, with the following comparison:—

$1000p_4$...	303	193	108	19
$1000c'$ exp. ...	781	813	892	977
,, calc....	784	835	892	977
$1000(p_3c+p_4c_4-c')$...	−14	+9	25	8
$1000(p_3c+p_4c_4-c')/p_3p_4$ exp. ...	−7	+6	26	43
,, ,, ,, calc. ...	−8	+11	26	42

The next table contains the values of $a\tau$ and be for those electrolytes for which the necessary data are available.

TABLE III.

	NaCl.	NaBr.	NaI.	$NaNO_3$.	Na_2SO_4.	NaOH.	KCl.	KBr.	KI.
$a\tau$.....	29	32	31	32	32	51	41	39	41
be.....	99	56	51	124	125	122	91	36	43

	K_2SO_4.	NH_4Cl.	$MgCl_2$.	$CaCl_2$.	$SrCl_2$.	$BaCl_2$.	$MgSO_4$.	$Pb(NO_3)_2$.
$a\tau$.....	39	34	41	40	44	44	42	21
be.....	104	57	55	64	79	112	105	43

The first point to arrest attention in this table is that $a\tau$ for NH_4Cl has a positive value 34, although τ is $1{\cdot}29-1{\cdot}39$ or $-0{\cdot}1$. The second is that $a\tau$ for all the Na compounds except NaOH has a value near to 31, and for all the K compounds a value near to 40. For $MgCl_2$ and $MgSO_4$ the values of $a\tau$ are nearly the same. From these facts it follows that the ordinary negative ions contribute no part to τ in $a\tau$. The reason for this is fairly obvious. A rise of temperature makes trihydrol more unstable, causing more of it to become dihydrol in pure water, whereas a rise of temperature produces no appreciable effect on dihydrol, the predominant ingredient of water. Now the negative ions attack dihydrol to make trihydrol out of it; but as a rise of temperature does not make $(H_2O)_2$ more unstable, the negative ions contribute a negligible part to $(D+dh/dp_2')dp_2'/dt$, and therefore also a negligible part to $a\tau$. We can test this reasoning by applying it to the case of NaOH. The OH ion dissociates $0{\cdot}86$ H_2O into $0{\cdot}86$ OH and $0{\cdot}86$ H; but the OH contributes no part to $a\tau$; so that the value 51 for NaOH is the sum of 31 for Na and $0{\cdot}86$ times the part of $a\tau$ due to H, which is $(51-31)/0{\cdot}86$ or 23. We have then the result that the

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values of $a\tau$ from specific heats, and of τ from densities, stand as follow for the chief positive ions:—

	H.	Na.	K.	NH_4.	Mg.	Ca.	Sr.	Ba.	Pb.
$a\tau$.....	23	31	40	34	41	40	44	44	21
100τ.....	183	196	207	129	238	235	245	243	217

Although the values of $a\tau$ are not in a strictly constant ratio to those of τ, there is a close enough correspondence between the two sets to show that the theory advanced for the complicated specific heat of solutions is in substantial agreement with the facts and theory of the contraction of volume on solution.

Concerning the coefficient *be* for different compounds I have noticed that it shows a remarkable rough proportionality to the square of the heat of formation of the compound from its elements. If H is the heat of formation of a gramme equivalent in kilocalories,

$$be = 0{\cdot}01\mathrm{H}^2, \text{ approximately,} \quad . \quad . \quad . \quad (19)$$

as the following table shows.

TABLE IV.

	NaCl.	NaBr.	NaI.	$NaNO_3$.	Na_2SO_4.	NaOH.	KCl.	KBr.	KI.
H.....	98	86	69	111	164	102	104	95	80
be calc....	96	74	48	123	269	104	108	90	64
be.....	99	56	51	124	125	122	91	36	43

	K_2SO_4.	NH_4Cl.	$MgCl_2$.	CaCl.	SrCl.	BaCl.	$MgSO_4$.	$Pb(NO_3)_2$.
H.....	172	76	75	85	92	97	151	53
be calc....	295	58	56	72	85	94	228	28
be.....	104	57	55	64	79	112	105	43

The calculated values of *be* for the three sulphates are about double the actual values. The bromides are exceptional, and the calculated values for K compounds are larger than the calculated for the Na compounds, while the relation of the actual values is the reverse. Nevertheless, although other physical properties of the ions seem to influence the values of *be*, the chief phenomenon is the connexion of *be* with the square of the heat of formation. The following is a brief theoretical account of the reason for such a relation. In Section 2 we treated the positive and negative ions as too far apart to affect one another's action in unmaking and making $(H_2O)_3$. We considered only the field of electric force at the surface of an ion as an isolated unit. But there is another type of electrical action, namely that of a positive and a

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negative ion, when they come so near one another as to cause a specially intense field lying between them. By the principle of energy, $(H_2O)_3$ having a larger dielectric capacity than $(H_2O)_2$ will tend to move into such a field from the weaker field around more quickly than $(H_2O)_2$. For the same reason such a field will be a place favouring the change of $(H_2O)_2$ into $(H_2O)_3$. The number of such fields will be proportional to the number of each sort of ion, therefore to p_4^2. Also the force drawing a mobile dielectric into a field of electric force of intensity R is proportional to R^2. Hence, since a number of fields of force proportional to p_4^2 and of average effect proportional to R^2 act on an amount of dihydrol p_3p_1', we have an amount of trihydrol formed proportional to $p_3p_4^2R^2p_1'$. The rate of variation of this will be proportional to dp_1'/dt chiefly. Hence we see how the term $-bp_4$ comes into (18) and how *be* is proportional to R^2. But as the ions of different compounds are similarly distributed through their solutions, R for different solutes must be proportional to the difference of electric potential between positive and negative ions, say V. But the heat of dissociation of the solute molecule or equivalent into ions is ϵV, where ϵ is the electron charge. If this heat of dissociation is the principal part of the heat of formation H of the solid compound from its elements, then V is approximately proportional to H. So it comes about that R^2 is roughly proportional to H^2. The fuller investigation of this relation can hardly be undertaken in the present connexion.

5. *Surface Tension.*

In the case of surface tension we have a still more complicated set of conditions. It has been shown in "The Molec. Const. of Water" (*loc. cit.* p. 473) that the surface film of water between 0° and 40° consists of pure trihydrol, but that at higher temperatures the surface tension is sufficiently reduced to allow some dihydrol to form in the surface film. In a theory of the surface tension of solutions we are confronted with the additional difficulty of having to find the solubility of the solute in the surface film. Moreover, the dissociation of $(H_2O)_3$ in the surface takes place under different physical conditions, which affect the value of τ to an unknown extent. Accordingly the present investigation will be devoted chiefly to establishing the broad principle that the anomalies in the surface tensions of aqueous solutions are caused by the action of the solute in changing $(H_2O)_3$ into $(H_2O)_2$ to about the same extent as has been demonstrated already in the previous sections. In "Molecular Force and the Surface Tension of Solutions" (Phil. Mag. [5] vol. l. p. 477)

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it is shown that while for normal mixtures the surface tension α and the density ρ are connected with α_3, α_4, ρ_3, ρ_4 for the pure ingredients by the formula

$$\alpha/\rho^2 = (p_3\alpha_3^{\frac{1}{2}}/\rho_3 + p_4\alpha_4^{\frac{1}{2}}/\rho_4)^2, \quad . \quad . \quad . \quad . \quad (20)$$

this does not apply to aqueous solutions, because it contains no provision for taking account of the change produced in the water by the solute. It is also shown that α for aqueous solutions is given by the empirical formula

$$\begin{aligned}\alpha/\rho^2 &= p_3^2\alpha_3/\rho_3^2 + p_4^2 b + 2p_3p_4c \\ &= \alpha_3/\rho_3^2 + p_4^2(\alpha_3/\rho_3^2 + b - 2c) + 2p_4(c - \alpha_3/\rho_3^2), \quad . \quad (21)\end{aligned}$$

in which b and c are parameters characteristic of the solute. The parameter b in this has not the value α_4/ρ_4^2, as in the formula (20) for normal mixtures. This then is the chief abnormality in the surface tension of solutions. In the paper just mentioned values of b and c were found for a large number of solutes. We shall be able to use these by establishing the connexion between (21) and a theoretical equation, taking account of the change in the amount of $(H_2O)_3$ caused by the solute. Let all symbols in this section refer to the surface-film: for instance, if there are p_1 of $(H_2O)_2$ and p_2 of $(H_2O)_3$ in a gramme of surface water, the values of p_1 and p_2 are very different from those in the bulk; but no special symbol will be provided to distinguish surface from bulk. So also, when a solution is made of p_3 water and p_4 solute in a gramme, p_4 in the surface is different from p_4 in the bulk. For the surface tension of water we have

$$\alpha_3/\rho_3^2 = (p_1\alpha_1^{\frac{1}{2}}/\rho_1 + p_2\alpha_2^{\frac{1}{2}}/\rho_2)^2. \quad . \quad . \quad . \quad . \quad (22)$$

This equation and (20) summarize the theory of the surface tension of solutions. As most of the experiments have been made below 40°, we shall avail ourselves of the simplification that in the surface of pure water at these temperatures $p_2 = 1$. Hence by (5) in a solution

$$p_2' = 1 - p_1' = 1 - 54\tau p_4/e,$$

and

$$\frac{\alpha}{\rho^2} = \left[p_3 \left\{ \frac{54\tau}{e}\, p_4 \left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1} - \frac{\alpha_2^{\frac{1}{2}}}{\rho_2} \right) + \frac{\alpha_2^{\frac{1}{2}}}{\rho_2} \right\} + p_4 \frac{\alpha_4^{\frac{1}{2}}}{\rho_4} \right]^2.$$

With $p_3 + p_4 = 1$ and expanding in powers of p_4 as far as

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the second,

$$\frac{\alpha}{\rho^2}=\frac{\alpha_2}{\rho_2^2}+\frac{2\alpha_2^{\frac{1}{2}}}{\rho_2}\left\{\frac{54\tau}{e}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}+\frac{\alpha_4^{\frac{1}{2}}}{\rho_4}\right\}p_4+$$

$$+\left[\left\{\frac{54\tau}{e}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}+\frac{\alpha_4^{\frac{1}{2}}}{\rho_4}\right\}^2-\frac{2\alpha_2^{\frac{1}{2}}}{\rho_2}.\frac{54\tau}{e}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)\right]p_4^2+\ldots\ldots(23)$$

By comparing this and (21) we obtain the desired relations, remembering that in (21) p_4 is the amount of solute in a gramme of the bulk solution. Let p_4 in the surface equal k times p_4 in the bulk, then we have

$$c-\frac{\alpha_3}{\rho_3^2}=\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\left\{\frac{54\tau}{e}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}+\frac{\alpha_4^{\frac{1}{2}}}{\rho_4}\right\}k,\quad\ldots\ldots(24)$$

$$\frac{\alpha_3}{\rho_3^2}+b-2c=\left[\left\{\frac{54\tau}{e}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}+\frac{\alpha_4^{\frac{1}{2}}}{\rho_4}\right\}^2-\frac{2\alpha_2^{\frac{1}{2}}}{\rho_2}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)\frac{54\tau}{e}\right]k^2.\quad(25)$$

In these equations α_3, ρ_3 refer to pure water, and for temperatures below 40° C. may be identified with α_2, ρ_2. So

$$\left(\frac{c-\alpha_2/\rho_2^2}{\alpha_2^{\frac{1}{2}}/\rho_2}\right)^2-\frac{\alpha_2}{\rho_2^2}-b+2c=\frac{2\alpha_2^{\frac{1}{2}}}{\rho_2}\left(\frac{\alpha_1^{\frac{1}{2}}}{\rho_1}-\frac{\alpha_2^{\frac{1}{2}}}{\rho_2}\right)\frac{54\tau}{e}k^2,\quad,\quad(26)$$

With the values of b and c from Table III. of "Mol. Force and Surf. Tension of Solutions" (*loc. cit.*), identifying surface ρ with bulk ρ and taking α_1, α_2, ρ_1, ρ_2 from "The Mol. Const. of Water," we can use the last equations for calculating τk^2 (surface) for various solutions, and then (24) gives α_4/ρ_4^2 for the solute. The surface tensions are expressed in milligrammes weight per millimetre. In this unit $\alpha_1=7{\cdot}73$ and $\alpha_2=7{\cdot}44$ at 20°, while $\rho_1=1{\cdot}07$, $\rho_2=0{\cdot}877$.

TABLE V.

	NaCl.	KCl.	$NaNO_3$.	KNO_3.	$MgCl_2$.	$CaCl_2$.	$SrCl_2$.	$BaCl_2$.
$100c$	361	380	270	333	290	271	170	180
$100b$	647	380	460	313	650	667	416	120
$100\tau k^2$ (surf.)..	170	100	110	130	165	200	240	105
100τ (bulk) ...	57	68	69	80	99	96	106	104
$\tau k^2/\tau$	3·0	1·4	1·6	1·6	1·7	2·1	2·3	1·0

	Na_2CO_3.	K_2CO_3.	Na_2SO_4.	K_2SO_4	$MgSO_4$.	$ZnSO_4$.	$CuSO_4$.
$100c$	80	180	180	160	80	20	20
$100b$	910	668	530	1820	790	520	730
$100\tau k^2$(surf.)..	320	290	250	950	320	320	410
100τ (bulk) ...	171	182	103	114	145	152	—
$\tau k^2/\tau$	1·9	1·6	2·4	8·4	2·2	2·1	—

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If we exclude the exceptional ratio for K_2SO_4, the mean value of the ratio of τk^2 in the surface to τ in the bulk is 1·9, with a range from 1 to 3. Experimental uncertainty affects the value of the ratio much. The main result, however, is clear, that dissociation produced by the solute in the surface layer is the cause of the exceptional behaviour of the surface tension of solutions. If we can assume that an equivalent of solute dissociates as many molecules of $(H_2O)_3$ in the surface as in the bulk, then $k^2 = 1·9$ and $k = 1·4$; that is to say, that on the average the concentration of a salt solution is 1·4 times as great in the surface film as in the bulk. The values of α_1 obtainable from (24) have no direct bearing on the present subject. They have been already considered in the paper cited. All the other properties of solutions, such as refraction and viscosity, can be treated in the same way as those selected for discussion in this paper.

6. *Summary.*

The contraction occurring when electrolytes dissolve in water is due to their changing some of the trihydrol of the water into dihydrol. The number τ of gramme molecules of $(H_2O)_3$ changed into $(H_2O)_2$ by a gramme equivalent of solute is the algebraic sum of parts τ due to the positive and negative ions. For positive ions τ is positive, and for negative negative; that is to say, a positive ion changes $(H_2O)_3$ into $(H_2O)_2$, and a negative changes $(H_2O)_2$ into $(H_2O)_3$. This supplies a strong confirmation of the theory of the ionic dissociation of dissolved electrolytes. It indicates that the energy relations of positive and negative electricity to the surrounding dielectric are different; a fact to be taken account of in a complete theory of electricity.

If λ is ionic velocity, K dielectric capacity of the stuff of an atom, and B the limiting volume of the gramme atom, then for positive ions $\tau/(\lambda K B^{\frac{2}{3}})^{\frac{1}{3}}$ is constant, and for negative $\tau/(\lambda K B^{\frac{2}{3}})^{\frac{1}{3}}$ is constant. These formulæ are obtained by a theory of the electric action of ions on $(H_2O)_3$ and $(H_2O)_2$ molecules. This theory tends to confirm the existence in water of what has been called a stion, an atom or a molecule like H_2O with both a positive and a negative electron which have not coalesced as in the H_2O of water vapour. This stion is an intermediate product in the ceaseless making and unmaking of $(H_2O)_3$ in water, and plays an important part in ionization.

From a consideration of the special properties of the H ion

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in acids and of the OH ion in alkalies, it is shown that the large exceptional ionic velocities assigned to these ions are not their true velocities. Both of these ions cause H_2O to dissociate into H and OH. At ordinary temperature each H dissociates 1·92 H_2O, and each OH 0·86 H_2O. The ionic velocity hitherto assigned to H is the sum of 2·92 times the true velocity of H and 1·92 times that of OH. Similarly the velocity hitherto assigned to OH is 1·86 times the true velocity of OH and 0·86 times that of H. The true velocities of H and OH conform to the law $\lambda B^{\frac{1}{3}}K/\nu$=constant (where ν is valency), to which their hitherto accepted velocities make them decided exceptions, and thus they confirm it.

The departure of the specific heats of solutions from the simple law of mixtures is traced to the change of $(H_2O)_3$ into $(H_2O)_2$ by the solute. The thermal effect of the dissociation of $(H_2O)_3$ by rising temperature is modified by the presence of the solute in such a way as to make the specific heat of most solutions less than that given by the simple law of mixtures. It is shown that $(p_3c+p_4c_4-c')/p_3p_4$ is of the form $a\tau/e-bp_4$. But here τ of the negative ion does not appear in $a\tau/e$, because $(H_2O)_3$ is stable to rising temperature, while $(H_2O)_3$ is unstable. This is the fundamental fact in the molecular constitution of water, and is confirmed by this striking failure of the negative ion to produce an effect in connexion with the term $a\tau/e$. The values of $a\tau$ for the positive ions obtained from the specific heats of solutions are in a general way proportional to the values of τ obtained from the contraction of volume on solution. It is shown that be (e is equivalent) for a compound is proportional to the square of its heat of formation, a theory of this being briefly sketched. The chief point in this theory is the consideration of the intense electric field between a positive and a negative ion which are near to one another without being united.

In the surface tensions of solutions the failure of the normal law for mixtures is traced to the change of $(H_2O)_3$ into $(H_2O)_2$ by the solute. The amount of this change deduced from surface tensions is comparable with that previously found; but the comparison is complicated by the entrance of unknowns such as the solubility of solute in the surface film.

Melbourne, April 1906.

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II. *On the Electron Theory of Matter and the Explanation of Fine Spectrum Lines and of Gravitation.* By G. A. SCHOTT, *B.A., B.Sc., University College of Wales, Aberystwyth**.

§ 1. LORD RAYLEIGH † and Mr. Jeans ‡ have recently discussed, in the pages of the Philosophical Magazine, the question of the explanation of the fineness of the spectrum lines. Jeans finds it necessary to postulate forces of unknown origin between electrons in addition to the electric and magnetic forces required by Maxwell's theory of the electromagnetic field, and makes the periods of vibration of systems of electrons depend on the action of these two types of forces; on this view spectrum lines are of dynamical origin. Lord Rayleigh, on the other hand, maintains that the relations between the spectrum lines studied by Balmer, Rydberg, and Kayser and Runge, point to a kinematical origin, but finds difficulty in explaining the existence of fine lines at all. The explanation developed in the present paper overcomes this difficulty, and at the same time leaves us free to adopt a kinematical or dynamical origin for the spectrum series. Moreover it offers some prospect of explaining gravitation on the lines of the hydrodynamic theories of Bjerknes and Pearson.

§ 2. We shall consider the case of n equidistant electrons moving in a circle with uniform velocity. Let the radius of the circle be ρ, the velocity v, the velocity of light C, $\frac{v}{C}=\beta$, the mass of an electron m, its charge e.

Further let T, P be the components, along the tangent and towards the centre, of the mechanical force on one of the electrons. The equations of motion are

$$T = \frac{d(mv)}{dt} + \frac{2e^2\beta^3}{3\rho^2(1-\beta^2)^2},$$

$$P = \frac{mv^2}{\rho}.$$

The second term in the first equation represents the reaction of the æther on the electron due to radiation.

The first term vanishes in steady motion; it is retained here for the sake of generality, so as to include the possibility of m varying while v is constant. This is essential for what follows.

The general equations of motion of an electron, of which those just written constitute a special case, can be obtained

* Communicated by the Author.

† Phil. Mag. ser. 5, xliv. p. 356; ser. 6, xi. p. 123.

‡ Phil. Mag. ser. 6, ii. p. 421; xi. p. 604.

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from Maxwell's equations of the electromagnetic field by direct integration. They are identical with those obtained by Abraham * by an indirect process.

The mass m is of electromagnetic origin, at any rate in part. It can be shown by the method of dimensions that the electromagnetic part of the mass is of the form $\frac{e^2}{2a}\psi(\beta)$, where a is a length, which we shall take to be the mean radius of the electron, and $\psi(\beta)$ depends on β and on the shape of the electron. If a be supposed invariable, as is usually done, m is constant when β is so; but this restriction is unnecessary. The variation allowable in a is very small; for $\frac{\dot{m}}{m}=-\frac{\dot{a}}{a}$, and m cannot possibly vary at a greater rate than, let us say, $\frac{1}{3}$ per cent. per annum; that is, $\frac{\dot{a}}{a}$ is certainly less, and probably very much less, than 10^{-10}.

§ 3. The force (T, P) consists of two parts:

(1) (T_1, P_1) due to electrons outside the ring;

(2) (T_2, P_2) due to the remaining electrons of the ring.

In the first place, T_1 vanishes for a system of electrons in steady motion.

For T_1 is due partly to a steady electric force, partly to a steady magnetic force. The electric part vanishes, because the lines of force of a steady electric field cannot form closed curves. The magnetic part vanishes, because the mechanical action of a magnetic field on a moving electron is always perpendicular to its velocity.

Secondly, I find for (T_2, P_2) the expressions

$$T_2=\frac{2e^2\beta^3}{3\rho^2(1-\beta^2)^2}-\frac{e^2}{\rho^2}n\mathrm{U},$$

$$P_2=-\frac{e^2}{\rho^2}(1+\beta^2)\mathrm{K}+\frac{e^2}{\rho^2}\mathrm{V},$$

where

$$\mathrm{K}=\sum_{i=1}^{i=n-1}\frac{1}{4}\operatorname{cosec}\frac{\pi i}{n},$$

$$\mathrm{U}=2\sum_{s=1}^{s=\infty}\Big[sn\beta^2\mathrm{J}'_{2sn}(2sn\beta)-s^2n^2(1-\beta^2)\int_0^\beta \mathrm{J}_{2sn}(2snx)dx\Big],$$

$$\mathrm{V}=\sum_{s=0}^{s=\infty}(s+\tfrac{1}{2})\cot\frac{(2s+1)\pi}{2n}\Big[\beta(1-\beta^2)\mathrm{J}_{2s+1}\{(2s+1)\beta\}$$
$$+(1+\beta^2)\int_0^\beta \mathrm{J}_{2s+1}\{(2s+1)x\}dx\Big].$$

* *Theorie der Elektrizität*, vol. ii. p. 123 (85).

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The notation is the usual notation of Bessel functions.

The function U is of special importance for our purpose. The series converges when $\beta < 1$, very rapidly for large values of n and small values of β. In this case it is sufficient to use the first term only. Whatever the value of β, U is positive. The equations of motion become

$$nU = -\frac{\rho^2}{e^2}\frac{d(mv)}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_1 - \frac{e^2}{\rho^2}(1+\beta^2)K + \frac{e^2}{\rho^2}V = \frac{mv^2}{\rho} \quad . \quad . \quad . \quad . \quad (2)$$

§ 4. Equation (1) shows that a strictly steady motion is impossible when we restrict ourselves to the case of an invariable electron and exclude all forces not of electromagnetic origin. For given values of n, β, ρ it determines $\dot{\beta}$. Equation (2) determines ρ when β is given, but β itself remains arbitrary. This is the difficulty first pointed out by Jeans (§ 1). There is nothing to fix the constitution of the atom, if it be supposed built up of invariable electrons in orbital motion, with only electromagnetic forces between them.

The difficulty can be overcome by supposing the electron to vary slowly in radius.

In this case,

$$\frac{dm}{dt} = -m\frac{\dot{a}}{a} = -\frac{e^2}{C^2}\psi(\beta)\frac{\dot{a}}{a^2} \quad (\S\ 2)$$

Equation (1) becomes

$$\frac{nU}{\beta} = \frac{\rho^2\psi(\beta)}{C}\frac{\dot{a}}{a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation together with (2) determines β and ρ when n and $\frac{\dot{a}}{a^2}$ are given. Since U is essentially positive $\dot{a}$ must be positive.

We easily find on examination that the right-hand member is of order 10^{-24} at most for the negative electron, if $\frac{\dot{a}}{a} = 10^{-10}$; probably it is very much less. The factor $\rho^2\psi(\beta)$ depends on β, but does not vary very rapidly with it; the function U on the other hand, for fairly large values of n, varies very rapidly with β; in consequence an approximate knowledge of $\rho^2\psi(\beta)$ is quite sufficient to allow us to calculate β accurately when n and $\frac{\dot{a}}{a^2}$ are given. When β is known ρ is calculated by (2).

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The following table gives approximate values of β calculated for $\frac{\dot{a}}{a} = 10^{-16}$ and $\frac{\dot{a}}{a} = 10^{-46}$.

	n ...	10	20	50	100	1000
β	$\frac{\dot{a}}{a} = 10^{-16}$	·023	·131	·398	·566	·894
	$\frac{\dot{a}}{a} = 10^{-46}$	·00074	·023	·187	·398	·833

It is noteworthy how little difference even the order of $\frac{\dot{a}}{a}$ makes in the value of β when n is at all large.

§ 5. We shall now consider in what way we must modify the common view as to the constitution of the electron and of the æther, so as to allow of the expansion of the electron, which we have found necessary to account for the definiteness in the structure of the atom on the electron theory.

When the electron is invariable the energy equation may be written in the form.

$$v \,.\, \mathrm{T} = \frac{d\mathrm{E}}{dt} + \mathrm{R}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $v\mathrm{T}$ is the rate of working of the mechanical force acting on the electron, E is a function of its velocity, mass, and acceleration, and R is the rate at which the electron loses energy by radiation. E includes the electromagnetic energy of the electron; its characteristic is that $d\mathrm{E}$ is a perfect differential while $\mathrm{R}\,dt$ is not.

When the electron varies in radius (4) is no longer true, but must be completed by the addition of a term to the right-hand member of the form $\frac{e^2\dot{a}}{a^2} f(\beta)$. This additional term represents the rate at which work is done by the internal electromotive forces of the electron on account of the expansion. The form of $f(\beta)$, like that of $\psi(\beta)$ in § 2, depends on the assumptions made as to the structure of the electron.

We must assume the existence of internal stresses in the electron which resist its expansion.

In calculating the internal electromotive force we find that the principal term is derived from a potential Φ, the "convection potential" of Searle and Lorentz; the value of Φ is the same as it would be if the electron were moving

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uniformly and without relative motion of its parts with the velocity it actually possesses. The remaining terms in the electromotive force, due to acceleration and to relative motion, are only of order $\frac{a}{\rho}$ relative to the principal term, and may be neglected.

Let (P, Q, R, S, T, U) be the stress which balances the internal electromotive force. It is given by the equations:

$$\frac{\partial P}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial T}{\partial z} = \rho' \frac{\partial \Phi}{\partial x},$$

$$\frac{\partial U}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial S}{\partial z} = \rho' \frac{\partial \Phi}{\partial y},$$

$$\frac{\partial T}{\partial x} + \frac{\partial S}{\partial y} + \frac{\partial R}{\partial z} = \rho' \frac{\partial \Phi}{\partial z},$$

where ρ' is the electric density.

When ρ' is a function of Φ alone the stress reduces to a hydrostatic pressure

$$p = -\int \rho' d\Phi.$$

This occurs when ρ' is constant.

When the electricity is distributed over the surface of the electron there is a discontinuity of pressure at the surface.

When the electron is an oblate spheroid of eccentricity β, with its minor axis in the line of motion, the pressure in either case is constant at the surface. This is the case with the forms of electron proposed by Lorentz and by Bucherer.

§ 6. The stress just considered may be accounted for in two ways:—

(1) We may suppose it due to attractions between the elementary parts of the electron. In that case the attractions cannot follow the Newtonian law; otherwise two negative electrons would have no electrostatic action on each other, since the new hypothetical force would balance the electrostatic repulsion between the electrons when at rest. Hence the new force cannot be used to account for gravitation, and we require to postulate two types of actions at a distance in addition to the electromagnetic actions. This explanation is equivalent to Jeans's assumption of additional forces between electric charges. These forces not only require explanation themselves, but are hardly consonant with an electron theory.

(2) We may suppose the stress due to interaction between the electron and the æther.

The simplest assumption is that the electron is an oblate

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spheroid (§ 5), that the stress is a pressure, and that the surface of the electron is a surface of constant pressure.

This assumption has the advantage that the resultant action of the æther on the electron vanishes, so that the electron in moving through the æther experiences no resistance on account of its expansion. Nevertheless the æther resists the expansion of the electron with a pressure, which, as we have seen, depends on the size of the electron and on its velocity of translation.

The electron in expanding exerts an equal pressure on the surrounding æther and must be supposed to push it outwards. We can only form a mechanical conception of this action in two ways :—

(1) The æther fills the space occupied by the electron ; in this case it must be expansible.

(2) The electron is an empty space surrounded by æther ; in this case the æther need not be expansible.

§ 7. In the first case an expanding electron produces a pressure at a distant point varying approximately inversely as the distance, and in consequence acts on a second distant electron with a force inversely as the square of the distance, but the force is a repulsion. This is in accordance with the view expressed by Maxwell that gravitational attraction cannot be transmitted by means of stress in a medium. If the æther be expansible, its bulk-modulus must be so great that inequalities of pressure due to expansion are carried away by waves almost instantaneously. The first case becomes practically identical with the second for our purpose.

§ 8. Let us assume for the sake of definiteness that the æther is a continuous isotropic medium, which possesses inertia and rotational elasticity, and may, or may not, possess volume elasticity. This is the type assumed by Larmor.

In such a medium permanent rotational motions are impossible ; any permanent motion is irrotational.

The expanding electron in this æther virtually constitutes a hydrodynamic source ; two sources attract each other with a force proportional directly to their strengths and to the density of the medium, and inversely to the square of the distance between them. Therefore two expanding electrons attract each other according to the same law.

If a, a' be their radii, and μ the density of the medium, the hydrodynamic attraction is $\dfrac{4\pi\mu a^2 a'^2 \ddot{a}\dot{a}'}{r^2}$.

When we consider two systems of n, n' electrons respectively, at a distance large compared with the dimensions of either

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system, the hydrodynamic attraction between the two systems is

$$nn' \frac{4\pi\mu a^2 a'^2 \dot{a}\dot{a}'}{r^2}.$$

The electromagnetic masses of the systems are given by

$$M = nm,\ M' = nm',$$

where m, m' are the masses of the electrons.

Hence the attraction is $\frac{MM'}{r^2} \frac{4\pi\mu a^2 \dot{a} a'^2 \dot{a}'}{mm'}$.

This is the law of gravitation. The last factor is the gravitation constant. In order to account for gravitation quantitatively it must have the value $6{\cdot}7 \times 10^{-8}$.

§ 9. When the electrons are in motion the values of m, m' and those of $\dot{a}$, $\dot{a}'$ depend on the velocity. This makes the gravitation constant depend on the velocity. To avoid the difficulty several ways can be suggested:

(1) The mass m of the electron, and the pressure at its surface, differ from their values when the electron is at rest by quantities of the order β^2; when the velocity of the electron is not a large fraction of the velocity of light, the difference is very small.

(2) An atom, if built up out of electrons, probably contains electrons moving with very different velocities; the gravitational attraction between two atoms is therefore an average taken for electrons with very different velocities. The table of values of β for rings of electrons given in § 4 shows that for rings of considerable numbers of electrons the value of β does not alter much with the number of electrons in the ring, and for these rings the values of β are large; the gravitational attraction diminishes as β approaches the value unity, because the pressure at the surface of the electron diminishes and because the mass of the electron increases. Hence rings of few electrons, which move slowly, produce an effect which is practically the same as that for electrons at rest; rings of many electrons, which move quickly, produce a smaller effect, but the difference is very slight for a considerable difference in the number of electrons in the ring. The mean gravitation constant for the whole atom need not differ very much for atoms containing very different numbers of electrons, and therefore of very different atomic weights.

Whether this be the reason why the gravitation constant is the same for all kinds of atoms or not, the same difficulty arises in Lorentz's theory of gravitation. Lorentz supposes

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electrons of opposite sign to attract each other more than those of like sign repel each other. The gravitation between atoms is proportional to the number of electrons in each, while their masses depend on the masses of the electrons, and therefore on their velocities as well.

§ 10. While we may admit that the hypothesis of the expanding electron affords a qualitative explanation of the law of gravitation, yet we can hardly rest satisfied unless it gives for the gravitation constant a value of the right order. After many trials I have not succeeded in devising a mechanical model of the electron which shall do this in a satisfactory manner. When we treat the problem as that of a source in an incompressible liquid with the pressure at infinity equal to a given constant, we find that a is an elliptic function of the time which varies between two definite limits. Since $\frac{\dot{a}}{a}$ must be small, a must be very close to one of the two limits; thus there are possible two electrons of different sizes, each varying very slowly in size; their radii being different, so also are their electromagnetic masses; by identifying them with the negative and positive electrons we can account for the difference in mass of the two kinds of electrons. In order to account for the smallness of $\frac{\dot{a}}{a}$ we must suppose the density μ of the æther to be very large. In order to get the proper value of the gravitation constant, we must suppose μ to be of the order 10^{100}, whether we assume gravitation to be due to one or both kinds of electrons.

In itself a large value of μ is not an objection. But it leads to a difficulty, which however is not peculiar to the present view, but is shared by every theory which assumes a mechanical action between the electron and the æther. A sphere moving through a liquid has an effective mass, greater than its true mass, by one half the mass of liquid displaced. If this were true of the electron, its effective mass would be enormously greater than its electromagnetic mass. For the negative electron this is not the case; it moves through the æther without experiencing any appreciable resistance beyond that due to electromagnetic radiation. Yet we require it to force back the æther while it expands.

§ 11. The conclusions of this investigation are as follows:—

(1) In order to account for the definiteness of atomic structure, as evidenced by the fineness of the spectrum lines, on an electron theory without the assumption of forces between electrons other than electromagnetic, it is necessary

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and sufficient to assume that the electron expands at a certain slow rate (§ 4).

(2) In order to satisfy the principle of the conservation of energy it is necessary to assume that the expansion is resisted by an internal stress, which in the simplest case is a hydrostatic pressure (§ 5).

(3) In order to explain this pressure in accordance with the spirit of the electron theory, that is, without action at a distance, we must assume that it is due to the action of the æther on the electron (§§ 6, 7).

(4) The reaction of the electron on the æther produces a pressure in the æther at the surface of the electron; if this is not exactly balanced by the pressure at infinity the electron expands, presses back the æther and acts like a source in a liquid. Hence electrons gravitate towards each other, as do sources in a liquid (§ 8).

III. *The General Determination of the Optical Constants of a Crystal by means of Refraction through a Prism.* By G. F. Herbert Smith, *M.A., Assistant in the Mineral Department of the British Museum**.

By far the simplest method of obtaining the three principal indices of refraction of a crystalline substance is that of total-reflexion, since observations need be made on only one face or section of the crystal; but this method is necessarily restricted to substances of lower refractive power than the constant medium (usually a hemisphere of dense glass) or the liquid used to obviate the intervention of a film of air. In other cases, however, recourse must be had to refraction through a prism formed by a pair of natural or artificial faces on the crystal, inclined to one another at not too large an angle. If possible, it is arranged that the plane bisecting the angle between these faces is one of symmetry in the crystal, so that the method of minimum deviation and the corresponding simple formula are available. But it is often the case in natural crystals that advantage cannot be taken of this simple method: the crystal may not have suitable natural faces inclined at a small enough angle, and may be so small itself that artificial faces cannot be ground on to it. We propose to show that, even in such cases, it is possible by taking observations sufficiently far round a zone to determine the optical constants.

* Communicated by the Author. The substance of the paper was discussed by him at meetings of the Mineralogical Society, June 19, 1900, and November 17, 1903.

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In his classical report on double refraction which was presented to the British Association at the Cambridge meeting of 1862, Stokes pointed out * that the velocity of a wave, the normal of which lies in a plane perpendicular to the faces of a prism, and its direction with respect to the faces and, therefore, to the crystallographic axes, can be accurately determined from observations of the angle of incidence and the corresponding angle of deviation, and he gave the formulæ required. Cornu † has more thoroughly discussed the same question, especially in regard to the remarkable tilt of the refracted image with respect to the edge of the prism which is observable in such strongly birefringent substances as calcite and sodium nitrate. He, indeed, measured this tilt in the case of the image given by the extraordinary ray and made use of it for the determination of the extraordinary indices of these substances. In most substances the tilt is not sufficiently perceptible to be used for purposes of measurement.

The formulæ required in the case of refraction through a prism are as follows:—

$$\sin i = n \sin r, \quad . \quad . \quad (1) \qquad\qquad r + r' = \mathrm{A}, \quad . \quad . \quad . \quad (3)$$

$$\sin i' = n \sin r', \quad . \quad . \quad (2) \qquad\qquad i + i' = \mathrm{A} + \mathrm{D}, \quad . \quad . \quad (4)$$

where i and i' are the angles which the incident and emergent waves make with the faces of incidence and emergence respectively,

r and r' the angles which the refracted wave makes with the same faces,

n is the index of refraction of the wave in the crystalline medium, that of the surrounding medium being assumed to be unity,

A the angle of the prism, and D the angle of deviation.

The above equations reduce to

$$\tan\left(r - \frac{\mathrm{A}}{2}\right) = \tan\frac{\mathrm{A}}{2}\tan\left(i - \frac{\mathrm{A}+\mathrm{D}}{2}\right)\cot\frac{\mathrm{A}+\mathrm{D}}{2}, \quad . \quad (5)$$

and we further have

$$\phi = \frac{r - r'}{2} = r - \frac{\mathrm{A}}{2} = \frac{\mathrm{A}}{2} - r', \quad . \quad . \quad . \quad . \quad (6)$$

* Report, p. 273.

† "De la réfraction à travers un prisme suivant une loi quelconque," *Annales scientifiques de l'Ecole Normale Supérieure*, ser. 2, 1872, vol. i. pp. 231-272; 1874, vol. iii. pp. 1-46; *cf.* also Liebisch, *Physikalische Krystallographie*, Leipzig, 1891, pp. 295-8, 376-8, and Walker, 'The Analytical Theory of Light,' Cambridge, 1904, pp. 8-15.

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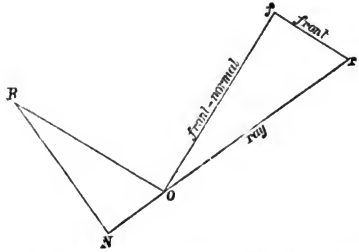
where ϕ is the angle which the wave-front in the crystal makes with the plane bisecting the acute angle of the prism.

The angles i and D can be found by observation with a goniometer, and the values of n and ϕ may be calculated from equations (5) and (1) or (2). Thus we can completely determine the velocity (*i. e.* the inverse of the refractive index, n) of any plane-wave within the crystal and its orientation with regard to the crystallographic axes.

Suppose we are able to take a series of observations round a zone, *i. e.* through pairs of faces with parallel edges, we should obtain a series of values for the velocity of propagation and the corresponding direction within the crystal, and can plot out curves with the former as ordinates and the latter as abscissæ. In the case of a doubly-refracting substance there will be two curves, the nature of which we will now proceed to determine.

Let us take the most general case in which the indicatrix* is an ellipsoid. Let R be any point on the surface,



RN the normal at that point, and O the centre of the ellipsoid. Through O draw ON at right angles to RN and produce it backwards to r so that $Or = \frac{1}{RN}$. In the plane ROr, Of is drawn at right angles to OR to meet rf, drawn parallel to OR, in f. Then $Of = \frac{1}{OR}$. We know that Of is the normal corresponding to a ray Or, and the plane through fr at right angles to Of is the wave-front, which

* The notation is the same as that employed by Mr. L. Fletcher, F.R.S., "The Optical Indicatrix and the Transmission of Light in Crystals," Mineralogical Magazine, 1891, vol. ix. pp. 278–388; published separately, London, Henry Frowde, 1892. The diagram above is his fig. 12, the block of which was courteously lent to the Author.

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touches the wave-surface at r; and further, that for rays with the same direction of wave-normal, the corresponding values of the wave-normal, $p\left(=\frac{1}{n}\right)$, are the inverses of the semi-axes of the perpendicular central-section. Whenever the wave-normal is at right angles to a principal axis, one of the two values obtained is half the inverse of that axis. Hence, if the axes are α, β, γ, supposed to be in ascending order of magnitude, it is obvious from geometrical considerations that the maximum and minimum values in a complete semi-revolution will give us γ and α. Since each curve has a maximum and a minimum value, the question remains what do the lower maximum and the higher minimum values give; one must clearly be β.

Let $a^2x^2+b^2y^2+c^2z^2=1$ be the equation to the indicatrix, where $a=\frac{1}{\alpha}$, $b=\frac{1}{\beta}$, $c=\frac{1}{\gamma}$; and let $\lambda\mu\nu$ be the direction cosines of the central line parallel to the edge of the prism. Since the wave-normal is always perpendicular to this line, the central section turns round it. Let the equation of this section be $lz+my+nz=0$.

Then $$l\lambda+m\mu+n\nu=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now the semi-axes of the section are given by

$$\frac{l^2}{a^2-p^2}+\frac{m^2}{b^2-p^2}+\frac{n^2}{c^2-p^2}=0, \quad . \quad . \quad . \quad (8)$$

where p is the inverse of the semi-axis and therefore the value of the corresponding wave-normal.

Further we have $$l^2+m^2+n^2=1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For critical values of p, $dp=0$. Hence we differentiate the above three equations in the usual way with regard to l, m, and n and eliminate the differentials, and arrive at the determinant

$$\begin{vmatrix} \frac{l}{a^2-p^2} & \frac{m}{b^2-p^2} & \frac{n}{c^2-p^2} \\ \lambda & \mu & \nu \\ l & m & n \end{vmatrix} = 0 \quad . \quad . \quad . \quad (10)$$

which reduces to

$$\frac{\lambda}{l}(b^2-c^2)(a^2-p^2)+\frac{\mu}{m}(c^2-a^2)(b^2-p^2)+\frac{\nu}{n}(a^2-b^2)(c^2-p^2)=0. \quad (11)$$

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Substituting this value for p^2 in (8), and remembering the relation in (7), we deduce

$$lmn\left\{\frac{l}{\lambda}(b^2-c^2)+\frac{m}{\mu}(c^2-a^2)+\frac{n}{\nu}(a^2-b^2)\right\}=0,$$

whence we see that the critical values are

$$l=0,\ m=0,\ n=0, \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{l}{\lambda}(b^2-c^2)+\frac{m}{\mu}(c^2-a^2)+\frac{n}{\nu}(a^2-b^2)=0. \quad . \quad (13)$$

If $l=0$, we have from (11) $p=a$; and similarly, if m or $n=0$, we obtain $p=b$ or c.

Combining (13) and (7), we get

$$\frac{l}{\lambda\{\mu^2(a^2-b^2)-\nu^2(c^2-a^2)\}}=\frac{m}{\mu\{\nu^2(b^2-c^2)-\lambda^2(a^2-b^2)\}}$$

$$=\frac{n}{\nu\{\lambda^2(c^2-a^2)-\mu^2(b^2-c^2)\}},$$

and substituting in (11) after some reduction we finally obtain

$$p^2=a^2\lambda^2+b^2\mu^2+c^2\nu^2.$$

It will be noticed that this value of p is the inverse of the radius of the ellipsoid which is parallel to the edge of the prism.

We might arrive at the same result in another way. The directions of the axes of any central-section, whose equation is $lx+my+nz=0$, is its intersection with the cone

$$\frac{l}{x}(b^2-c^2)+\frac{m}{y}(c^2-a^2)+\frac{n}{z}(a^2-b^2)=0.$$

Comparing these equations with (13) and (7) we see at once that the fourth critical value is the inverse of the semi-diameter parallel to the edge of the prism.

Let us consider which of the critical values are maxima or minima. We must differentiate equations (7), (8), and (9) twice, regarding one of the variables, let us say m, as independent. We have

$$\frac{dl}{dm}=\frac{m\nu-n\mu}{n\lambda-l\nu}, \qquad \frac{dn}{dm}=\frac{l\mu-m\lambda}{n\lambda-l\nu},$$

$$\frac{d^2l}{dm^2}=\frac{\nu}{(n\lambda-l\nu)^3}, \qquad \frac{d^2n}{dm^2}=\frac{-\lambda}{(n\lambda-l\nu)^3}.$$

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Arrange (11) thus

$$p^4 - p^2\{l^2(b^2+c^2)+m^2(c^2+a^2)+n^2(a^2+b^2)\}+l^2b^2c^2+m^2c^2a^2+n^2a^2b^2=0$$

and differentiate twice, remembering that $\frac{dp}{dm}$ vanishes for the critical values.

$$p\frac{d^2p}{dm^2}\left\{2p^2-l^2(b^2+c^2)-m^2(c^2+a^2)-n^2(a^2+b^2)\right\}-p^2\Big[(b^2+c^2)\left\{l\frac{d^2l}{dm^2}\right.$$

$$\left.+\left(\frac{dl}{dm}\right)^2\right\}+(c^2+a^2)+(a^2+b^2)\left\{n\frac{d^2n}{dm^2}+\left(\frac{dn}{dm}\right)^2\right\}\Big]$$

$$+b^2c^2\left\{l\frac{d^2l}{dm^2}+\left(\frac{dl}{dm}\right)^2\right\}+c^2a^2+a^2b^2\left\{n\frac{d^2n}{dm^2}+\left(\frac{dn}{dm}\right)^2\right\}=0.$$

Putting $p=b$ and $m=0$, we have

$$b\frac{d^2p}{dm^2}.\{2b^2-l^2(b^2+c^2)-n^2(a^2+b^2)\}=(a^2-b^2)(b^2-c^2).$$

Suppose b is greater than a and c. The term on the right is negative, and the expression within the bracket on the left, which can be arranged thus $l^2(b^2-c^2)+n^2(b^2-a^2)$, is positive. Hence $\frac{d^2p}{dm^2}$ is negative, and, as indeed was otherwise obvious, the greatest value of p is a maximum value of the corresponding curve. Similarly the least value is a minimum value.

If now b is the mean value, the term on the right is positive. Again, since $m=0$, $l\lambda+n\nu=0$, and, therefore,

$$l^2=\frac{\nu^2}{\nu^2+\lambda^2},\quad n^2=\frac{\lambda^2}{\nu^2+\lambda^2}.$$

Substituting these values in the expression within the bracket we have $\{b^2-(a^2\lambda^2+b^2\mu^2+c^2\nu^2)\}/(\nu^2+\lambda^2)$. But $a^2\lambda^2+b^2\mu^2+c^2\nu^2$ is the square of the semi-diameter parallel to the edge of the prism. Hence, if the diameter be greater than the mean axis of the ellipsoid, the value corresponding to the latter is a minimum value; if less, a maximum value. Now at any point lying on the circular sections, the semi-diameter is equal to the mean semi-axis. If, therefore, the direction parallel to the edge of the prism lies towards the greatest axis of the ellipsoid, the corresponding value is a maximum; but, if towards the least, it is a minimum; if it lies on either of the circular sections, the two curves touch.

The two intermediate values may be discriminated by determining the angles of extinction with regard to the edge

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of the prism. The plane of polarization of any wave is at right angles to the normal at the corresponding point on the indicatrix. The angles of extinction will be in the case of the principal axes $\cos^{-1}\lambda$, $\cos^{-1}\mu$, $\cos^{-1}\nu$, and for the fourth value $\cos^{-1}\dfrac{a^2\lambda^2+b^2\mu^2+c^2\nu^2}{\sqrt{a^4\lambda^2+b^4\mu^2+c^4\nu^2}}$.

The values for the three principal axes must satisfy the condition $$\lambda^2+\mu^2+\nu^2=1.$$

It may be noted that the planes of polarization for the two rays with parallel wave-fronts are in general not strictly at right angles, since the normals at the extremities of the axes of the central-section to which they are perpendicular are not themselves in general at right angles.

Another point that is worthy of notice is that the two curves are in general not symmetrical about the critical positions. The abscissæ, as we have remarked above, represent the angles which the wave-normal makes with some fixed direction; or, in other words, the angles which the central section revolving round the diameter parallel to the edge of the prism makes with some fixed plane. Supposing the upper curve contains the greatest and the mean of the principal indices, the angle between the corresponding positions of the central section is

$$\cos^{-1}\frac{\lambda\mu}{\sqrt{(\mu^2+\nu^2)(\nu^2+\lambda^2)}},$$

which in general is not a right angle; on the lower curve the angle is

$$\cos^{-1}\frac{\lambda\mu(a^2-b^2)}{\sqrt{(\lambda^2+\mu^2)\{(b^2-c^2)^2\mu^2\nu^2+(c^2-a^2)^2\nu^2\lambda^2+(a^2-b^2)^2\lambda^2\mu^2\}}}.$$

If the crystal be uniaxial the upper curve becomes a straight line, and the lower curve, which is still undulating, is now symmetrical, since $a=b$ and the angle between the critical positions is a right angle.

Although the tilt of the refracted image, as was mentioned above, is in general not sufficiently perceptible for purposes of measurement, it may be indirectly of service, because its existence signifies that the corresponding direction in the crystal is not near one of the critical positions and, therefore, no observations need be taken. The range inside the crystal corresponding to a particular prism may be increased if the observations are made on an inverted goniometer in some liquid medium. In an instrument of this kind which is in the British Museum light passes into and out of the liquid by means of double-ended glass-prisms. As the angle of incidence

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diminishes, it is clear that a point will be reached beyond which it will not be possible to measure it directly because the telescope would come into contact with the collimator. Indirect, though perfectly accurate, determinations may be made by clamping the crystal-holder rigidly to the graduated circle and noting the reading in any position when the telescope is directly opposite to the collimator. If a certain direct reading θ has been found to correspond to an angle of incidence i, then a reading $\theta \pm \omega$ (depending on whether the angle of incidence diminishes or increases with the reading) corresponds to an angle of incidence $i-\omega$. This is, indeed, the most convenient way of determining the angles of incidence.

Obviously much labour may be avoided if previously the orientation of the indicatrix has been determined either from the crystallographical symmetry where possible, or from observations under the polariscope.

IV. *Ellipsoidal Harmonics, Æolotropic and Isotropic.* *By* R. Hargreaves, *M.A.**

It is the first object of the present paper to show that Lamé's functions are applicable to the æolotropic form of the equation of Laplace, with some modification in the meaning of the constants used, and practically no change in the form of the results. It is also shown that in æolotropic and in isotropic work, the parametral equations required to determine Lamé's functions in every case involve a single constant of the ellipsoid. This point is obscured in Heine's treatment of the functions by the use of the unsymmetrical notation in which squares of semi-axes are represented by λ^2, λ^2-a^2, λ^2-b^2. If $a^2+\lambda$, $b^2+\lambda$, $c^2+\lambda$ are used there is exact correspondence between the classification of curvilinear and Cartesian forms, *i. e.* distinctions depend on the presence or absence of the radicals $\sqrt{\lambda+a^2}, \ldots$ in the one case, of the linear factors $x\,y\,z$ in the other. This follows from the connexion between curvilinear parameters $\lambda\,\mu\,\nu$ and Cartesians, which for isotropy is given by

$$f = u_\theta - 1 \equiv \frac{x^2}{a^2+\theta} + \frac{y^2}{b^2+\theta} + \frac{z}{c^2+\theta} - 1 \equiv \frac{\overline{\lambda-\theta}\;\overline{\mu-\theta}\;\overline{\nu-\theta}}{\overline{a^2+\theta}\;\overline{b^2+\theta}\;\overline{c^2+\theta}}.$$

If R_n is used for a product $f_1 \ldots f_n$ in which the subscripts correspond to different values $\theta_1, \ldots \theta_n$, there are normal

* Communicated by the Author.

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solutions of $\nabla^2 H = 0$ of the types

$$\text{(i.)}\quad H_{2n} = R_n,$$

$$\text{(ii.)}\quad H_{2n+1} = (x, y, \text{ or } z) R_n,$$

$$\text{(iii.)}\quad H_{2n+2} = (xy, yz, \text{ or } zx) \times R_n,$$

and

$$\text{(iv.)}\quad H_{2n+3} = xyz \times R_n,$$

the subscript giving the degree of the harmonic in xyz. The polynomials in $\lambda\mu\nu$ involved in R_n are distinct in the four cases, but their coefficients depend on a parameter satisfying an equation of degree $n+1$ in each case, so that there are $n+1$ values of each R_n. This statement gives the symmetrical classification according to the degree of H as follows:—

(*a*) degree m even $= 2n$: there are $4n+1$, *i.e.* $3n+(n+1)$ solutions; the $n+1$ solutions have the form R_n, and there are n solutions of each of the types $(xy, yz, \text{ or } zx) \times R_{n-1}$; (*b*) degree m odd $= 2n+1$: there are $4n+3$, *i.e.* $n+3(n+1)$ solutions; the n solutions have the form $xyz \times R_{n-1}$, and there are $n+1$ solutions of each of the types $(x, y, \text{ or } z) \times R_n$.

For æolotropy a more general cubic takes the place of

$$(a^2+\lambda)(b^2+\lambda)(c^2+\lambda),$$

and the radicals of its factors appear in place of $\sqrt{\lambda+a^2}, \ldots$; but the linear factors in xyz cease to have the simple form varying as x, y, or z.

It is the second object of the paper to consider the Cartesian form of harmonics, with special reference to æolotropy in the equation of Laplace; and to give proofs of a few fundamental points in a manner independent of the curvilinear treatment.

§ 1. The notation of the paper on æolotropic potential * is followed, but u_λ or u_θ is used (instead of u_α there) for the quadric $\Sigma(\alpha x^2 - 2\alpha' yz)$, when it is necessary to distinguish the variable λ or θ underlying the coefficients $\alpha \ldots$. We found that when ϕ depends only on λ,

$$-\nabla_e^2 \phi = \frac{1}{\dot{u}_\lambda} \left\{ 4 \frac{d^2\phi}{d\lambda^2} + \frac{2.\dot{J}(\lambda)}{J(\lambda)} \frac{d\phi}{d\lambda} \right\};$$

the whole expression of $-\nabla_e^2 \phi$ in terms of $\lambda\mu\nu$ requires the addition of corresponding terms in μ and ν, and $\dot{u}_\lambda$ is to be expressed in terms of $\lambda\mu\nu$. The connexion between $\lambda\mu\nu$ and

* "Æolotropic Potential," Phil. Mag. April 1905. References written as Æ. (86).

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xyz is made through

$$u_\theta \mathrm{J}(\theta) = \mathrm{J}(\theta) + \mathrm{J}_3(\lambda-\theta)(\mu-\theta)(\nu-\theta), \quad . \quad . \quad . \quad (1)$$

where

$$\mathrm{J}(\theta) = 1 + \theta\Sigma(pa + 2p'a') + \theta^2\Sigma(\mathrm{PA} + 2\mathrm{P'A'}) + \theta^3\Delta_p\Delta_a,$$

or

$$1 + \theta\mathrm{J}_1 + \theta^2\mathrm{J}_2 + \theta^3\mathrm{J}_3, \text{ say.}$$

As (1) is true for all values of θ we may differentiate it,

$$\dot{u}_\theta \mathrm{J}(\theta) + u_\theta \dot{\mathrm{J}}(\theta) = \dot{\mathrm{J}}(\theta) + \mathrm{J}_3 \frac{d}{d\theta} \overline{\lambda-\theta}\,\overline{\mu-\theta}\,\overline{\nu-\theta};$$

and then write λ for θ in the result and use $u_\lambda = 1$. Thus

$$\dot{u}_\lambda \mathrm{J}(\lambda) = -\mathrm{J}_3(\lambda-\mu)(\lambda-\nu). \quad . \quad . \quad . \quad . \quad (2)$$

If corresponding results for μ and ν are used, the æolotropic equation of Laplace is

$$\mathrm{J}_3\nabla_e^2\phi = \frac{1}{(\lambda-\mu)(\lambda-\nu)}\left\{4\mathrm{J}(\lambda)\frac{\partial^2\phi}{\partial\lambda^2} + 2\dot{\mathrm{J}}(\lambda)\frac{\partial\phi}{\partial\lambda}\right\} + \text{terms in } \mu \text{ and } \nu. \quad (3)$$

If the double brackets are proportional to ϕ in each case, or to $\lambda\phi$, $\mu\phi$, $\nu\phi$ in the respective cases, the sum vanishes. Thus we have solutions of $\nabla_e^2\phi = 0$ in the form of products of similar functions of λ, μ, ν respectively, the first of which satisfies the equation

$$4\mathrm{J}(\lambda)\frac{d^2\phi}{d\lambda^2} + 2\dot{\mathrm{J}}(\lambda)\frac{d\phi}{d\lambda} = m(m+1)\,\mathrm{J}_3(\lambda-\epsilon)\phi. \quad . \quad (4)$$

We get rid of J_3 by putting

$$\mathrm{J} = \mathrm{J}_3(\lambda-\theta_0)(\lambda-\theta_0')(\lambda-\theta_0''),$$

and the mode of dealing with Lamé's functions proceeds on normal lines; but the coefficients of the cubic now depend on the constants of æolotropy as well as on those of the ellipsoid.

§ 2. It is proposed to go a step further, and show that the parametral equation can for each type of solution be presented in a form dependent on a single constant of the ellipsoid. In Heine's work this valuable property *appears* to belong exclusively to the solutions K and N (in Heine's notation), and his work for L and M would support that view. But K may be an odd polynomial in Heine's λ, or may contain $\sqrt{\lambda+a^2}$ in the present notation for the isotropic case, and so correspond to a solution with a linear factor x. Also N may be an odd polynomial, in which case there are three linear factors (x, y, z for isotropy); or may be even,

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when only two appear, viz. y, z. What is true for x as factor in the one case, for yz as factors in the other case, must apply also to other factors; and a method of establishing the property will now be given in a general form.

Consider the transformation of variable from λ to ξ through $\lambda-\theta_0=p\xi$. Then, since $J(\lambda)$ is of the third degree and $J(\theta_0)=0$, we have

$$J(\lambda)=(\lambda-\theta_0)\,\dot{J}(\theta_0)+\tfrac{1}{2}(\lambda-\theta_0)^2\,\ddot{J}(\theta_0)+(\lambda-\theta_0)^3\,J_3$$
$$=p\xi\{\dot{J}(\theta_0)+\tfrac{1}{2}p\xi\,\ddot{J}(\theta_0)+p^2\xi^2\,J_3\}.$$

Now write

$$\ddot{J}(\theta_0)=-2pJ_3,\quad \dot{J}(\theta_0)=p^2\rho J_3,\ \text{and so}\ 4J_3\,\dot{J}(\theta_0)=\rho\{\ddot{J}(\theta_0)\}^2. \qquad (5)$$

Then

$$\left.\begin{array}{l} J(\lambda)=p^2J_3\,\xi(\xi^2-\xi+\rho),\qquad \dot{J}(\lambda)=p^2J_3(3\xi^2-2\xi+\rho),\\ \ddot{J}(\lambda)=2pJ_3(3\xi-1),\ \text{and}\ J(\lambda)/(\lambda-\theta_0)=p^2J_3(\xi^2-\xi+\rho);\\ \text{while}\qquad \dfrac{d\phi}{d\lambda}=\dfrac{1}{p}\dfrac{d\phi}{d\xi},\quad \dfrac{d^2\phi}{d\lambda^2}=\dfrac{1}{p^2}\dfrac{d^2\phi}{d\xi^2}.\end{array}\right\}\quad . \quad (6)$$

Taking the various solutions of (4) according to the classification above, in (i.) ϕ is itself a polynomial L of degree n in λ, and $m=2n$ is the degree of the harmonic in xyz. The above substitution gives

$$4\xi(\xi^2-\xi+\rho)\frac{d^2L}{d\xi^2}+2(3\xi^2-2\xi+\rho)\frac{dL}{d\xi}=2n(2n+1)\left(\xi+\frac{\theta_0-\epsilon}{p}\right)L$$
$$=2n(2n+1)(\xi-\epsilon')L. \quad . \quad (7\ \text{i.})$$

The equation contains a parameter ϵ' and one constant ρ depending on the constants of the ellipsoid and those of æolotropy. Each root of $J(\lambda)=0$ may be made the basis of transformation for this case, and also for

(iv.) where $\phi=L\sqrt{J}$ or $\propto L\sqrt{(\lambda-\theta_0)(\lambda-\theta_0')(\lambda-\theta_0'')}$.

If the order of L is n, that of the harmonic is $2n+3$, Lamé's equation becomes

$$4J\frac{d^2L}{d\lambda^2}+6\dot{J}\frac{dL}{d\lambda}+2\ddot{J}L=(2n+3)(2n+4)J_3(\lambda-\epsilon)L,$$

and the transformed equation is

$$4\xi(\xi^2-\xi+\rho)\frac{d^2L}{d\xi^2}+6(3\xi^2-2\xi+\rho)\frac{dL}{d\xi}+4(3\xi-1)L$$
$$=(2n+3)(2n+4)(\xi-\epsilon')L. \quad . \quad . \quad . \quad . \quad (7\ \text{iv.})$$

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The transformation with the root θ_0 is to be applied when

(ii.) $\phi = \mathrm{L} \times \sqrt{\lambda - \theta_0}$.

The order of the harmonic is $2n+1$, the equation in L is

$$4\mathrm{J}\frac{d^2\mathrm{L}}{d\lambda^2} + 2\left(\dot{\mathrm{J}} + \frac{2\mathrm{J}}{\lambda-\theta}\right)\frac{d\mathrm{L}}{d\lambda} + \left(\ddot{\mathrm{J}} - \frac{\mathrm{J}}{\lambda-\theta}\right)\frac{\mathrm{L}}{\lambda-\theta}$$
$$= (2n+1)(2n+3)\,\mathrm{J}_3(\lambda-\epsilon)\,\mathrm{L},$$

which transformed is

$$4\xi(\xi^2-\xi+\rho)\frac{d^2\mathrm{L}}{d\xi^2} + 2(5\xi^2-4\xi+3\rho)\frac{d\mathrm{L}}{d\xi} + (2\xi-1)\mathrm{L}$$
$$= (2n+1)(2n+2)(\xi-\epsilon')\mathrm{L}. \quad . \quad . \quad . \quad . \quad (7\text{ ii.})$$

The transformation with θ_0 is to be applied also when

(iii.) $\phi \propto \mathrm{L}\sqrt{(\lambda-\theta_0')(\lambda-\theta_0'')} = \mathrm{L}\sqrt{\mathrm{J}/(\lambda-\theta_0)}$.

The order of the harmonic is $2n+2$, the equation in L is

$$4\mathrm{J}\frac{d^2\mathrm{L}}{d\lambda^2} + 2\left(3\dot{\mathrm{J}} - \frac{\mathrm{J}}{\lambda-\theta}\right)\frac{d\mathrm{L}}{d\lambda} + \left(2\ddot{\mathrm{J}} - \frac{3\dot{\mathrm{J}}}{\lambda-\theta} + \frac{3\mathrm{J}}{\lambda-\theta^2}\right)$$
$$= (2n+2)(2n+3)\,\mathrm{J}_3(\lambda-\epsilon)\mathrm{L},$$

and the transformed equation is

$$4\xi(\xi^2-\xi+\rho)\frac{d^2\mathrm{L}}{d\xi^2} + 2(7\xi^2-4\xi+\rho)\frac{d\mathrm{L}}{d\xi} + (6\xi-1)\mathrm{L}$$
$$= (2n+2)(2n+3)(\xi-\epsilon')\mathrm{L}. \quad . \quad . \quad . \quad . \quad (7\text{ iii.})$$

The parametral equation in ϵ', or the equation which gives the condition that L may be a polynomial of degree n in λ, is of degree $n+1$ in ϵ' and in each case contains the single constant ρ in linear form. To connect the above with Heine's notation take the isotropic case

$$1 = p = q = r, \quad 0 = p' = q' = r', \quad 0 = a' = b' = c',$$

and suppose the root θ_0 to be $1 + \theta_0 a = 0$. Then

$$-2p\mathrm{J}_3 = \ddot{\mathrm{J}}(\theta_0) = 2(\mathrm{J}_2 + 3\theta_0\mathrm{J}_3) = 2(ab+bc+ca) - 6bc,$$
$$\text{or} \quad p = 2a^{-1} - b^{-1} - c^{-1};$$

and

$$p^2\rho\mathrm{J}_3 = \dot{\mathrm{J}}(\theta_0) = \mathrm{J}_1 + 2\theta_0\mathrm{J}_2 + 3\theta_0^2\mathrm{J}_3 = \Sigma a - 2a^{-1}\Sigma ab + 3a^{-2}\,.\,abc,$$
$$\text{or} \quad p^2\rho = (a^{-1}-b^{-1})(a^{-1}-c^{-1}).$$

When the principal semi-axes are used,

$$p = 2a^2 - b^2 - c^2 \quad \text{and} \quad p^2\rho = (a^2-b^2)(a^2-c^2);$$

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these are the quantities which Heine calls p and q, i.e. ρ is here the ratio $q:p^2$.

When $n=1$ or the polynomial is linear $=\lambda+k$, the results are—

$$\begin{array}{lll} \text{(i.)} & 6k=6\epsilon'-4, & (3\epsilon'-1)^2=1-3\rho\,; \\ \text{(ii.)} & 10k=12\epsilon'-9, & (12\epsilon'-5)^2=4(4-15\rho)\,; \\ \text{(iii.)} & 14k=20\epsilon'-9, & (20\epsilon'-5)^2=4(4-7\rho)\,; \\ \text{(iv.)} & 18k=30\epsilon'-16, & (15\epsilon'-5)^2=9(1-3\rho). \end{array}$$

The radical in the solutions (i.) and (iv.) is independent of the root used; it is $\sqrt{J_2^2-3J_1J_3}$ in the general case, and $\sqrt{\Sigma a^4-\Sigma b^2c^2}$ in the special notation of the isotropic case.

§ 3. We now give a brief sketch of some points in the Cartesian treatment, dwelling specially on the features of difference due to æolotropy*. With the notation in the preamble $f=u_\theta-1$, there is in the value of $\nabla_e^2(R_n \text{ or } f_1\ldots f_n)$ a term in which ∇_e^2 is applied solely to f_1 with the result

$$2\Sigma(p\alpha_1+2p'\alpha_1')=2\dot{J}(\theta_1)/J(\theta_1), \text{ cf. Æ. (86)},$$

and this multiplies the product $f_2\ldots f_n$. There is a term in which ∇_e^2 is applied jointly to f_1 and f_2, the term being

$$2f_3\ldots f_n\left[p\frac{\partial u_1}{\partial x}\frac{\partial u_2}{\partial x}+\ldots+2r'\left(\frac{\partial u_1}{\partial x}\frac{\partial u_2}{\partial y}+\frac{\partial u_1}{\partial y}\frac{\partial u_2}{\partial x}\right)+\ldots\right];$$

the bracket by (9) below is

$$\frac{4(u_2-u_1)}{\theta_1-\theta_2} \quad \text{or} \quad \frac{4(f_2-f_1)}{\theta_1-\theta_2},$$

and the product $f_2\ldots f_n$ has the multiplier $8/(\theta_1-\theta_2)$.

The whole result of $\nabla_e^2 R_n$ is a series of products of $n-1$ factors f and will vanish if the coefficient of each of such products vanishes, *i.e.* if

$$\frac{\dot{J}(\theta_1)}{4J(\theta_1)}+\frac{1}{\theta_1-\theta_2}+\ldots\frac{1}{\theta_1-\theta_n}=0, \quad . \quad . \quad . \quad (8\text{ i.})$$

and $n-1$ similar conditions are fulfilled. This is the result of writing in (4) the value $\lambda=\theta_1$, assuming that ϕ is there a polynomial L of degree n in λ, and so

$$\dot{L}(\theta_1)=\overline{\theta_1-\theta_2}\ldots\overline{\theta_1-\theta_n} \text{ and } \dot{L}(\theta_1)/\dot{L}(\theta_1)=2\left(\frac{1}{\theta_1-\theta_2}+\ldots\frac{1}{\theta_1-\theta_n}\right).$$

* The isotropic case is treated at length by W. D. Niven in a paper "On Ellipsoidal Harmonics," Phil. Trans. vol. 182 (1891) A.

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We now give the proof of the theorem

$$\frac{u_\lambda - u_\theta}{\lambda-\theta} + \frac{1}{4}\left[p\frac{\partial u_\lambda}{\partial x}\frac{\partial u_\theta}{\partial x} + \ldots + r'\left(\frac{\partial u_\lambda}{\partial x}\frac{\partial u_\theta}{\partial y} + \frac{\partial u_\lambda}{\partial y}\frac{\partial u_\theta}{\partial x}\right)\ldots\right] = 0, \quad . \quad (9)$$

the analogue of

$$\Sigma\frac{x^2}{a^2+\lambda} - \Sigma\frac{x^2}{a^2+\theta} + (\lambda-\theta)\,\Sigma\frac{x^2}{(a^2+\lambda)(a^2+\theta)} = 0,$$

required above.

To deal with the coefficient of x^2 we derive from Æ. (81)

$$\lambda\Delta_a\{\alpha_\theta(p\alpha + r'\gamma' + q'\beta')_\lambda + \gamma_\theta'(r'\alpha + q\gamma' + p'\beta')_\lambda + \beta_\theta'(q'\alpha + p'\gamma' + r\beta')_\lambda\}$$
$$= \Delta_a\alpha_\theta - \{\alpha_\theta(\mathrm{A}\alpha + \mathrm{C}'\gamma' + \mathrm{B}'\beta')_\lambda + \gamma_\theta'(\mathrm{C}'\alpha + \mathrm{B}\gamma' + \mathrm{A}'\beta')_\lambda$$
$$+ \beta_\theta'(\mathrm{B}'\alpha + \mathrm{A}'\gamma' + \mathrm{C}\beta')_\lambda\}.$$

In each double bracket the expression is symmetrical as regards λ and θ; if the formula is written with λ and θ interchanged and subtracted from the above, the result expresses the fact that the coefficient of x^2 in (9) vanishes. As (9) is true with any values of the parameters we may make $u_\lambda = 1$, and the first member is then $-f_\theta/(\lambda-\theta)$. If θ is made $=\mu$, the square bracket in (9) $=0$, this equation replacing the orthogonal condition in isotropic work.

The solution R_n applies to space within the ellipsoid; we shall prove that it is a normal solution by finding a function χ of λ such that $\chi\mathrm{R}_n$ is an external solution. Now, if ϕ is expressed in terms of xyz and λ, we have

$$-\dot{u}_\lambda\nabla_e^2\phi = -\dot{u}_\lambda\Delta e^2\phi\ (\text{explicit}) + 2\frac{\partial\phi}{\partial\lambda}\frac{\dot{\mathrm{J}}}{\mathrm{J}} + 4\frac{\partial^2\phi}{\partial\lambda^2}$$
$$+ 2\frac{\partial^2\phi}{\partial\lambda\partial x}\left(p\frac{\partial u_\lambda}{\partial x} + r'\frac{\partial u_\lambda}{\partial y} + q'\frac{\partial u_\lambda}{\partial z}\right) + \ldots = 0; \quad . \quad (10)$$

and therefore for $\phi = \chi\mathrm{R}_n$

$$\mathrm{R}_n\{4\ddot{\chi} + 2\dot{\chi}\dot{\mathrm{J}}/\mathrm{J}\} + 2\dot{\chi}\left\{\frac{d\mathrm{R}_n}{dx}\left(p\frac{\partial u_\lambda}{\partial x} + r'\frac{\partial u_\lambda}{\partial y} + q'\frac{\partial u_\lambda}{\partial z}\right) + \ldots\right\} = 0. \quad (10b)$$

Since $\dfrac{d\mathrm{R}_n}{dx} = \Sigma f_2\ldots f_n\dfrac{du_1}{dx}$, if we apply (9) the double bracket is $4\mathrm{R}_n\Sigma\dfrac{1}{\lambda-\theta_1}$; the factor R_n may be removed, and χ is given by

$$\left.\begin{aligned} &\frac{\ddot{\chi}}{\dot{\chi}} + \frac{\dot{\mathrm{J}}(\lambda)}{2\mathrm{J}(\lambda)} + 2\left(\frac{1}{\lambda-\theta_1} + \ldots\frac{1}{\lambda-\theta_n}\right) = 0, \\ \text{whence}\quad &\chi = \int_\lambda^\infty \frac{d\lambda}{(\lambda-\theta_1)^2\ldots(\lambda-\theta_n)^2\sqrt{\mathrm{J}(\lambda)}}. \end{aligned}\right\} \quad . \quad (11)$$

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In virtue of the condition (8) this integral may be referred to the form $\int(A\lambda^2+B\lambda+C)\frac{d\lambda}{J^{3/2}}$, or as $\int\dot{J}\,d\lambda/J^{3/2}$ is integrable to the forms $\int d\lambda(1 \text{ or } \lambda)/J^{3/2}$ [or, if we please, to $\int d\lambda/\sqrt{J}$ and $\int d\lambda/J^{3/2}$], *i. e.* such integrals as occur in the calculation of force. Thus from

$$\int_\lambda^\infty \frac{(\lambda+\gamma')d\lambda}{(\lambda-\theta)^2\sqrt{J}} = \frac{\lambda+\gamma'}{(\lambda-\theta)\sqrt{J}} + \int_\lambda^\infty \frac{d\lambda}{(\lambda-\theta)\sqrt{J}}\left\{1-\frac{(\lambda+\gamma')\dot{J}}{2J}\right\}$$

we see that the condition that $\lambda-\theta$ shall disappear from the denominator of the integral is $2J(\theta)=(\theta+\gamma')\dot{J}(\theta)$, or, if we write $\theta+\gamma'=\gamma$, the condition is $\gamma=2J(\theta)/\dot{J}(\theta)$ and the integral is

$$\int_\lambda^\infty \frac{d\lambda}{\sqrt{J}}\left(\frac{1}{\lambda-\theta}+\frac{\gamma}{\overline{\lambda-\theta_1}^2}\right) = \frac{\gamma}{(\lambda-\theta)\sqrt{J}} + \int_\lambda^\infty \frac{(2J-\gamma\dot{J})d\lambda}{2(\lambda-\theta)J^{3/2}}. \quad . \quad (12)$$

The condition as regards γ gives a special value to the ratio of numerators of $\overline{\lambda-\theta_1}^2$ and $\lambda-\theta_1$ when $1/\overline{\lambda-\theta_1}^2\ldots\overline{\lambda-\theta_n}^2$ is expressed by a series of partial fractions. Thus with $F(\lambda)=1/\overline{\lambda-\theta_2}^2\ldots\overline{\lambda-\theta_n}^2$, the condition is

$$\frac{\dot{J}(\theta_1)}{2J(\theta)_1} = \frac{1}{\gamma_1} = \frac{F'(\theta_1)}{F(\theta_1)} = -2\left\{\frac{1}{\theta_1-\theta_2}+\ldots+\frac{1}{\theta_1-\theta_n}\right\},$$

which is precisely (8). The value of χ or $\int_\lambda^\infty \frac{d\lambda}{L^2\sqrt{J}}$ is then

$$\Sigma\,\frac{1}{\{L(\theta_1)\}^2}\left[\frac{1}{(\lambda-\theta_1)\sqrt{J}(\lambda)}\right.$$
$$\left.+\frac{1}{2J(\theta_1)}\int_\lambda^\infty \frac{\{J(\lambda)\dot{J}(\theta_1)-\dot{J}(\lambda)J(\theta_1)\}d\lambda}{(\lambda-\theta_1)\{J(\lambda)\}^{3/2}}\right]. \quad . \quad (13)$$

§ 4. In the isotropic solution the linear factors which may occur are x, y, and z; for æolotropy they are to be found, *i. e.*, for example, $\phi=(lx+my+nz)R_n$ being assumed to be a solution, the ratios $l:m:n$ are required. Now

$$\nabla_e^2(lx+my+nz)R_n = (lx+my+nz)\nabla_e^2 R_n + 2\left[\frac{dR_n}{dx}(lp+mr'+nq')\right.$$
$$\left.+\frac{dR_n}{dy}(lr'+mq+np')+\frac{dR_n}{dz}(lq'+mp'+nr)\right],$$

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and $\frac{dR_n}{dx}$ is a sum of terms of the form $f_2 \ldots f_n \frac{du_1}{dx}$; thus a condition of solution is that the square bracket must contain the factor $(lx+my+nz)$, or

$$\left.\begin{array}{l} \alpha(lp+mr'+nq')+\gamma'(lr'+mq+np')+\beta'(lq'+mp'+nr)=\kappa l, \\ \gamma'(\quad\text{,,}\quad)+\beta'(\quad\text{,,}\quad)+\alpha'(\quad\text{,,}\quad)=\kappa m, \\ \beta'(\quad\text{,,}\quad)+\alpha'(\quad\text{,,}\quad)+\gamma(\quad\text{,,}\quad)=\kappa n. \end{array}\right\} \quad (14)$$

Moreover, the ratios $l:m:n$ must be independent of the particular θ underlying α, while κ may be expected to contain it. Supposing α to depend on the general parameter λ, treat (14) as a system of equations to determine the brackets: the solution gives

$$(lp+mr'+nq')\Delta_a=\kappa(lA_a+mC_a'+nB_a').$$

But $\qquad A_a/\Delta_a=(A+p\lambda\Delta_a)/\Delta_a$ [*cf.* Æ. (85)],

and therefore

$$(lp+mr'+nq')\Delta_a=\kappa(l\overline{A+p\lambda\Delta_a}+m\overline{C'+r'\lambda\Delta_a}+n\overline{B'+q'\lambda\Delta_a}),$$

or

$$l(A+p\Delta_a\overline{\lambda-\kappa^{-1}})+m(C'+r'\Delta_a\overline{\lambda-\kappa^{-1}})+n(B'+q'\Delta_a\overline{\lambda-\kappa^{-1}})=0\ ;$$

and there are two others. These equations are independent of λ, if

$$\lambda-\kappa^{-1}=\theta_0 \quad \text{or} \quad \kappa=1/(\lambda-\theta_0)\ ; \quad . \quad . \quad . \quad (15)$$

and the system is then

$$\left.\begin{array}{l} l(A+p\theta_0\Delta_a)+m(C'+r'\theta_0\Delta_a)+n(B'+q'\theta_0\Delta_a)=0, \\ l(C'+r'\theta_0\Delta_a)+m(B'+q\theta_0\Delta_a)+n(A'+p'\theta_0\Delta_a)=0, \\ l(B'+q'\theta_0\Delta_a)+m(A'+p'\theta_0\Delta_a)+n(C+r\theta_0\Delta_a)=0. \end{array}\right\} . \quad (16)$$

The determinant here vanishes, and θ_0 is therefore a root of $\Delta(\lambda)=0$ or $J(\lambda)=0$, and there are three types of linear factor corresponding to the several roots. Referring to (1), and the equation defining the α's, viz. Æ. (81), we see that the quadric u_{θ_0} is proportional to

$$(\lambda-\theta_0)(\mu-\theta_0)(\nu-\theta_0),$$

and as the quadric is then the square of a linear term, this latter is proportional to

$$\sqrt{(\lambda-\theta_0)(\mu-\theta_0)(\nu-\theta_0)}.$$

Thus by Cartesian work we infer the existence of Lamé's functions with the radicals $\sqrt{\lambda-\theta_0}, \ldots$

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Since

$$\frac{du_1}{dx}(lp+mr'+nq')+\ldots=2\kappa_1(lx+my+nz)=2(lx+my+nz)/(\theta_1-\theta_0),$$

$$\frac{dR_n}{dx}(lp+mr'+nq')+\ldots=2(lx+my+nz)\left(\frac{f_2\ldots f_n}{\theta_1-\theta_0}+\ldots\right),$$

and $$\nabla_e^2(lx+my+nz)\,R_n=0$$

if $$\frac{\dot{J}(\theta_1)}{4J(\theta_1)}+\frac{1}{2(\theta_1-\theta_0)}+\frac{1}{\theta_1-\theta_2}+\ldots\frac{1}{\theta_1-\theta_n}=0, \quad . \quad (8\text{ ii.})$$

and $n-1$ other similar conditions are satisfied. This being the modification of (8) required for one linear factor, a second linear factor will involve the addition of $1/2(\theta_1-\theta_0')$ to (8 ii.), and for three linear factors the condition is

$$\frac{3\dot{J}(\theta_1)}{4J(\theta_1)}+\frac{1}{\theta_1-\theta_2}+\ldots\frac{1}{\theta_1-\theta_n}=0. \quad . \quad . \quad . \quad (8\text{ iv.})$$

For the external harmonic of type (ii.) we seek a function χ of λ which makes

$$\nabla_e^2\chi(lx+my+nz)\,R_n=0.$$

The forms in (10 *b*) have the additional factor $lx+my+nz$, and there is a further term

$$2\dot{\chi}R_n\left[\frac{\partial u_\lambda}{\partial x}(lp+mr'+nq')+\ldots\right],$$

which

$$=4\dot{\chi}R_n(lx+my+nz)/(\lambda-\theta_0)$$

by using (14) and (15). Thus in lieu of (11) we have

$$\left.\begin{aligned}&\frac{\ddot{\chi}}{\dot{\chi}}+\frac{\dot{J}(\lambda)}{2J(\lambda)}+\frac{1}{\lambda-\theta_0}+2\left(\frac{1}{\lambda-\theta_1}+\ldots\frac{1}{\lambda-\theta_n}\right)=0,\\ \text{and}\quad &\chi=\int_\lambda^\infty\frac{d\lambda}{(\lambda-\theta_0)(\lambda-\theta_1)^2\ldots(\lambda-\theta_n)^4\sqrt{J(\lambda)}}.\end{aligned}\right\}\quad(11\text{ ii.})$$

The condition for the simplification of the integral is got by writing

$$F(\lambda)=1/(\lambda-\theta_0)(\lambda-\theta_2)^2\ldots(\lambda-\theta_n)^2$$

in the argument leading to (13), and is precisely the equation (8 ii.). The value of χ then shows the terms in (13) with the further divisor $\theta_1-\theta_0$, *i. e.*

$$\frac{1}{\theta_1-\theta_0}\,\frac{1}{\{L(\theta_1)\}^2}\quad\text{for}\quad\frac{1}{\{L(\theta_1)\}^2};$$

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and there is an extra term

$$\frac{1}{\{L(\theta_0)\}^2}\int_\lambda^\infty \frac{d\lambda}{(\lambda-\theta_0)\sqrt{J(\lambda)}}.$$

It is not necessary to pursue the modifications for the remaining cases. The essential point is that the only factors occurring in the denominator of the integral are those of $J(\lambda)$, so that the statement made above as to the general character of the integrals is applicable in all cases.

V. *A Simplification of the Mathematical Discussion of the Effect of a Long Straight Wire on Electric Waves.* By J. G. Leathem, *M.A.*, *D.Sc.**

1. THE use of Bessel's functions in the discussion of problems relating to the transmission of electric waves along wires is of course inevitable when precise mathematical formulation is required, but it has the disadvantage of presenting formidable difficulties to the student of rather limited mathematical attainments. Even one who has had considerable training in mathematical analysis may perhaps feel discouraged when, at the very outset of his study of a physical subject of great practical interest, he is faced with a function K_0 defined by the relation

$$K_0(x) = (\cdot 5772157 \ldots + \log 2 - \log x)J_0(x) \\ -2\{J_2(x) - \tfrac{1}{2}J_4(x) + \tfrac{1}{3}J_6(x) \ldots\ldots\},$$

and finds that upon his appreciation of the values of this function for different complex values of x must depend his right understanding of the subsequent investigation. To find even approximate formulæ for the values of this and of the J functions when x is very large or very small, recourse must be had to advanced treatises such as Heine's *Kugelfunctionen*.

Now while the definitions of the J functions as power series, and of the K functions as series of J functions, are from the mathematical point of view absolutely precise, yet the formulæ are of so difficult a character that they utterly fail to convey to the mind of the average reader a correspondingly precise idea, and so the precision theoretically attained is to a considerable extent ineffective. Even if this were not so, the precision of the mathematical formulation is ultimately spoiled by the substitution of values which are

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only approximately correct for the physical constants involved.

It seems desirable, therefore, to try to make the study of this part of the theory of Electric Waves easier and more attractive to the average student by simplifying the mathematical formulation; and while this must necessarily involve a sacrifice of that appearance of definiteness which attaches to the use of functions that have been at least partially tabulated, it will be shown that all those results of the mathematical theory which are of interest because of their admitting of direct experimental test can be obtained as well by quite elementary reasoning as by the elaborate and rather troublesome discussion which depends upon the use of standard functions.

The simplified treatment will be applied to the transmission of oscillatory currents by a wire, and to the scattering of waves by a perfectly conducting wire. The main idea of the method is that we take as specification of a solution of a differential equation, not an infinite series, but the differential equation itself, together with the limitations to which the solution is subject for indefinitely small or indefinitely great values of the argument. This specification leaves undone much that has to be done before the corresponding function can be evaluated for any assigned value of the argument; nevertheless it is a precise specification, and proves to be well adapted to the purposes to which we shall apply it.

2. *Solutions of the differential equation*

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+y=0. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We shall assume that there is a solution $g(x)$ which is finite for $x=0$ but possibly infinite for $x=\infty$; and that there is another solution $h(x)$ which is zero for $x=\infty$, but possibly infinite for $x=0$. The equation being linear, each of these solutions will contain an arbitrary constant factor.

Case of x *very small.*

In considering the case of x very small we take the equation in the form

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0,$$

and in attempting to get approximate solutions we note that the last term has the appearance of being unimportant in comparison with the middle term. We therefore, as an

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experiment, neglect the last term and consider, in the first instance, only the equation

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0.$$

That this step is justifiable cannot be guaranteed beforehand; it must be tested by the results obtained. Two independent particular solutions of the curtailed equations are $y=a$, and $y=b\log x$, where a and b are constants. The latter of these expressions for y is seen to be such as to make the neglected term very small in comparison with each of the terms retained, it therefore justifies the method by which it was obtained and is a genuine approximation to a solution; moreover it becomes very great as x decreases, so it must be an approximate form of that solution which we have called $h(x)$. The solution $y=a$, on the other hand, does not make the neglected term smaller than those retained; nevertheless it makes two of the terms zero and the third very small, so it is clearly an approximation to a solution. To improve the approximation we try $y=a(1+\lambda x+\mu x^2)$; substituting this in the complete equation, and neglecting powers of x higher than the first, we get

$$(4\mu+1)x+\lambda=0,$$

which is satisfied by $\lambda=0$, $\mu=-\frac{1}{4}$. So we get an approximate solution which, being finite for $x=0$, must be $g(x)$, namely,

$$g(x)=a(1-\tfrac{1}{4}x^2).$$

Differentiating we get further

$$g'(x)=-\tfrac{1}{2}ax,\quad h'(x)=b/x.$$

Case of x *very great.*

In seeking approximations to the solutions when x is very great, we note that the middle term of the equation has now the appearance of being relatively unimportant; so we neglect it provisionally, and consider only

$$\frac{d^2y}{dx^2}+y=0.$$

Two independent solutions of this are $y=a'e^{-ix}$ and $y=b'e^{ix}$, where a' and b' are constants and $i^2=-1$; these forms are found to make the neglected term of the differential equation small compared with each of those retained, and so the method by which they were arrived at is justified.

When x is complex one of these solutions is zero and the

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other infinite for $x=\infty$; the one which vanishes must be the approximation to $h(x)$, the other the approximation to $g(x)$. Thus if the real part of ix is negative, $g(x)$ is $a'e^{-ix}$ and $h(x)$ is $b'e^{ix}$. When x is real neither becomes infinite, and so there is nothing to indicate which solution or combination of the two solutions is $g(x)$; but when that case arises we shall find a method of picking out the particular approximation which we require.

An exponential infinity or zero is an infinity or zero of very high order, and might possibly be associated with a less important infinity or zero represented by a power of x; this power factor would escape notice in the first approximation, so in order to detect it if it exists in the present instance we assume

$$y=a''x^{\lambda}e^{-ix},$$

and try whether this will give a better approximation to solution of the differential equation (1). Substituting and neglecting $x^{\lambda-2}$ we get

$$(-2i\lambda-i)x^{\lambda-1}e^{-ix}=0,$$

which is satisfied by $\lambda=-\frac{1}{2}$. The other solution may be treated in the same way. Thus we have the better approximations

$$g(x)=a''e^{-ix}/\sqrt{x},\quad h(x)=b''e^{ix}/\sqrt{x},$$

and, by differentiation,

$$g'(x)=-ia''e^{-ix}/\sqrt{x},\quad h'(x)=ib''e^{ix}/\sqrt{x},$$

a'' and b'' being constants.

It is to be noticed that though a and a'' are arbitrary, the ratio a''/a is not arbitrary; it is a definite complex constant whose value we do not need to ascertain, and may be denoted by A. Similarly we denote b''/b by B. We assign to a and to b the value unity, and have finally the following table of approximate forms:—

	x small.	x large.
$g(x)$	$1-\frac{1}{4}x^2$	$Ae^{-ix}/\sqrt{x}$
$g'(x)$	$-\frac{1}{2}x$	$-iAe^{-ix}/\sqrt{x}$
$h(x)$	$\log x$	$Be^{ix}/\sqrt{x}$
$h'(x)$	$1/x$	$iBe^{ix}/\sqrt{x}$

. . . (2)

It is readily verified that $g(nx)$ and $h(nx)$ are solutions

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of the differential equation

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+n^2y=0.$$

Of course $g(x)$ and $h(x)$ can differ from Bessel's functions only by constant factors, but we can use them without any knowledge of the formulæ expressing properties of Bessel's functions; our great gain in simplicity depends upon our recognizing the fact that it is unnecessary to evaluate the constants A and B.

3. *Transmission of Waves by a Single Wire.*

Cylindrical coordinates r, θ, z are employed, the surface of the wire is taken to be $r=a$, the specific resistance and the magnetic permeability of the material of the wire are represented by σ and μ respectively, the dielectric is supposed to be non-magnetic and to have specific inductive capacity K, the velocity of radiation in free æther is denoted by c, that in the dielectric by v, the electromagnetic system of units is used, and the displacement current in the wire is neglected. The magnetic force is in circles round the axis of z and is denoted by η, the electric intensity is in planes through the axis of z, and has components R parallel and P perpendicular to that axis. Suffixes $(_1)$ and $(_0)$ refer to the metal and to the dielectric respectively.

From the equations of the electromagnetic field the following facts are deduced in the usual manner, it being assumed that all components of electric and magnetic intensity depend on the time t and on z only through the factor $e^{i(mz+pt)}$.

$$\left.\begin{aligned}&\eta_0=\frac{ip\mathrm{K}}{\mathrm{c}^2\kappa^2}\frac{\partial \mathrm{R}_0}{\partial r},\quad \mathrm{P}_0=-\frac{im}{\kappa^2}\frac{\partial \mathrm{R}_0}{\partial r},\\ &\frac{\partial^2\mathrm{R}_0}{\partial r^2}+\frac{1}{r}\frac{\partial \mathrm{R}_0}{\partial r}-\kappa^2\mathrm{R}_0=0,\end{aligned}\right\}\quad . \quad . \quad (3)$$

where $\kappa^2=m^2-\mathrm{K}p^2/\mathrm{c}^2=m^2-p^2/v^2$;

$$\left.\begin{aligned}&\eta_1=\frac{4\pi}{\sigma n^2}\frac{\partial \mathrm{R}_1}{\partial r},\quad \mathrm{P}_1=-\frac{im}{n^2}\frac{\partial \mathrm{R}_1}{\partial r},\\ &\frac{\partial^2\mathrm{R}_1}{\partial r^2}+\frac{1}{r}\frac{\partial \mathrm{R}_1}{\partial r}-n^2\mathrm{R}_1=0,\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad (4)$$

where $n^2=m^2+4\pi\mu ip/\sigma$,

$=4\pi\mu ip/\sigma$ very approximately for oscillations less rapid than those of light.

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Having regard to the fact that R_1 must not be infinite for $r=0$, and that R_0 must not be infinite for $r=\infty$, we see that R_1, R_0 must be of the forms

$$R_1 = Eg(inr), \quad R_0 = Fh(i\kappa r),$$

where E and F are constants. Then also

$$\eta_1 = E\frac{4\pi i}{\sigma n} g'(inr), \quad \eta_0 = -F\frac{pK}{c^2\kappa} h'(i\kappa r),$$

the exponential factor being, for brevity, omitted.

The boundary conditions at the surface $r=a$ are continuity of R and of η; these require

$$Eg(ina) = Fh(i\kappa a),$$

$$\frac{4\pi i}{\sigma n} Eg'(ina) = -\frac{p}{\kappa v^2} Fh'(i\kappa a),$$

and the elimination from these of the ratio E/F gives the relation

$$\frac{\sigma n}{4\pi i}\frac{g(ina)}{g'(ina)} = -\frac{\kappa v^2}{p}\frac{h(i\kappa a)}{h'(i\kappa a)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now the form of R_0 shows that $1/\kappa$ is comparable with the change in the value of r requisite to produce a sensible change in the value of R_0; this can never be of a smaller order of magnitude than the wave-length in the dielectric, and may be much greater, as in the case when the waves are nearly plane; thus κa is either smaller than or comparable with the ratio of a to the wave-length, and is therefore small when the wave-length is great compared with the radius of the wire. So κa is small, and we may use for the functions of κa the approximations obtained above. We therefore put

$$h(i\kappa a)/h'(i\kappa a) = i\kappa a \log(i\kappa a),$$

and get

$$(i\kappa a)^2 \log(i\kappa a)^2 = -\frac{a\sigma n p}{2\pi v^2}\frac{g(ina)}{g'(ina)} \quad . \quad . \quad . \quad (6)$$

4.

Slowly Varying Currents.

In this case na is small, and $g(ina)/g'(ina) = -2/ina$, so equation (6) becomes

$$(i\kappa a)^2 \log(i\kappa a)^2 = -i\sigma p/\pi v^2. \quad . \quad . \quad . \quad . \quad (7)$$

The right side of this equation is small, and an approximate solution may be obtained by the method used in Prof. J. J. Thomson's article on 'Electric Waves' in the *Encyclopædia Britannica*, or in a paper by Prof. A. Sommerfeld in the

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Annalen der Physik, 1899, vol. lxvii. p. 255. The result is, for equation (7),

$$m^2 = \frac{p^2}{v^2} + \kappa^2 = \frac{p^2}{v^2} + i\frac{p\sigma/\pi v^2 a^2}{\log(p\sigma/\pi v^2 a^2)}, \quad . \quad . \quad . \quad (8)$$

in which p^2/v^2 is relatively so small that it may be neglected.

Prof. Thomson's result is

$$m^2 = \frac{p^2}{v^2} + i\frac{p\sigma/\pi v^2 a^2}{\log(p\sigma/4\pi v^2\gamma^2)}, \quad . \quad . \quad . \quad . \quad (8')$$

where $\log\gamma = \cdot 5772157 \ldots$. Now as the complete quantity whose logarithm is taken is very small and therefore has a large logarithm, the difference between the denominators in formulæ (8) and (8′) is small compared with either, so the one formula is as good an approximation for m^2 as the other.

High frequency currents.

When na is great $g(ina)/g'(ina) = i$, and equation (6) becomes

$$(i\kappa a)^2 \log(i\kappa a)^2 = -ia\sigma np/2\pi v^2 ; \quad . \quad . \quad . \quad (9)$$

for frequencies sensibly less than that of Bose's most rapid vibrator the right side of this equation is small. The approximate solution is

$$m^2 = \frac{p^2}{v^2}\left[1 - i\left(\frac{\mu\sigma}{2\pi\rho a^2}\right)^{\frac{1}{2}} \Big/ \log\left\{\frac{\sqrt{2a}}{v^2}\left(\frac{\mu\sigma p^3}{2\pi}\right)^{\frac{1}{2}}\right\}\right]. \quad . \quad (10)$$

Prof. Thomson's result differs from this only by the presence of an additional factor $1/4\gamma^2$ in the argument of the logarithm; this implies the addition of a relatively very small term to the denominator, so that the two approximations are equally good.

The case of a wire with a return circuit in the shape of a coaxial cylinder may be treated by the same method. It is unnecessary to discuss it here, for the reasoning is the same as that given by Prof. Thomson in the *Encyclopædia Britannica* and in 'Recent Researches.' The only difference is that the g and h functions are to be used instead of Bessel's functions, and the approximate values are to be taken from Table (2). The final results are found to differ from Prof. Thomson's only by the absence of relatively very small terms.

The approximate distribution of currents in the wire, and the restriction of high frequency currents to a certain thin "skin," can be discussed as easily by the present mathematical formulation as by the use of Bessel's functions.

The present note makes no claim to be regarded as a complete account of the particular problems in the transmission

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of waves by wires with which it deals. On the contrary it omits the most interesting parts of the discussion, namely, the substitution of numerical values for the physical constants with a view to ascertaining what quantities in the formulæ are negligibly small, and the interpretation of results such as formulæ (8) and (10); these are to be found in the works to which reference has been made above. It is only to the preliminary mathematical analysis that our attention is now directed.

5. *The Scattering of Waves by a Perfectly Conducting Wire.*

Let the axis of the wire be the axis of z, and a the radius. Consider first the case in which the incident wave, propagated in a direction perpendicular to the wire, has its electric intensity perpendicular to the wire and therefore its magnetic force parallel to the wire. Taking the direction of propagation as axis of x, we have for the magnetic force in the incident wave an expression of the form $\gamma_0 = H_0 e^{i(pt-\kappa x)}$.

In cylindrical coordinates the space exponential is $e^{-i\kappa r\cos\theta}$, and may be expanded by Fourier's method in a series of cosines of multiples of θ; no sines occur because the function is an even function. Let the expansion be

$$e^{-i\kappa r\cos\theta} = \sum_s f_s(r)\cos s\theta. \quad . \quad . \quad . \quad . \quad (11)$$

Now as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \kappa^2\right) e^{-i\kappa x} = 0, \quad . \quad . \quad . \quad (12)$$

it follows that

$$0 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \kappa^2\right)\Sigma f_s(r)\cos s\theta,$$

$$= \Sigma\left\{\frac{d^2 f_s}{dr^2} + \frac{1}{r}\frac{df_s}{dr} + \left(\kappa^2 - \frac{s^2}{r^2}\right) f_s\right\}\cos s\theta;$$

whence for each value of s

$$\frac{d^2 f_s}{dr^2} + \frac{1}{r}\frac{df_s}{dr} + \left(\kappa^2 - \frac{s^2}{r^2}\right) f_s = 0. \quad . \quad . \quad . \quad (13)$$

Put $f_s(r) = g_s(\kappa r) = g_s(\rho)$ say, and we get

$$\frac{d^2 g_s}{d\rho^2} + \frac{1}{\rho}\frac{dg_s}{d\rho} + \left(1 - \frac{s^2}{\rho^2}\right) g_s = 0. \quad . \quad . \quad . \quad (14)$$

Clearly $g_s(\rho)$ is not infinite for $\rho = 0$; we may assume that there is another solution $h_s(\rho)$ of equation (14), which is not infinite for values of ρ other than zero, and is zero for $\rho = \infty$.

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It is readily seen that the scattered wave will have its magnetic force γ_1 parallel to z, and that γ_1 must satisfy the differential equation of wave propagation, which is of the same form as (12). Hence we assume

$$\gamma_1 = \Sigma A_s h_s(\kappa r) \cos s\theta \, e^{ipt}; \quad . \quad . \quad . \quad . \quad . \quad (15)$$

so that the total magnetic force in the dielectric is

$$\gamma = \gamma_0 + \gamma_1 = e^{ipt} \Sigma [\{ H_0 g_s(\kappa r) + A_s h_s(\kappa r) \} \cos s\theta]. \quad . \quad (16)$$

From the circuital relation between displacement current and magnetic force, it is readily seen that the component of electric intensity perpendicular to r is a constant multiplied by $\partial\gamma/\partial r$; the surface condition is that this shall vanish for $r=a$, since the wire is a perfect conductor.

Thus for each value of s

$$H_0 g_s'(\kappa a) + A_s h_s'(\kappa a) = 0,$$

and therefore the scattered wave is given by

$$\gamma_1 = -H_0 \Sigma g_s'(\kappa a) \frac{h_s(\kappa r)}{h_s'(\kappa a)} \cos s\theta. \quad . \quad . \quad . \quad (17)$$

It may be assumed that the radius of the wire is very small compared with the wave-length, so that κa is extremely small. Perhaps the simplest way of arriving at an approximation to $g_s(\kappa a)$ for κa very small is by the usual method of evaluating a coefficient in a Fourier series. Multiply both sides of equation (11) by $\cos s\theta$ and integrate from $\theta=0$ to $\theta=2\pi$; after dividing by π, the result is

$$g_s(\kappa r) = \frac{1}{\pi} \int_0^{2\pi} e^{-i\kappa r \cos\theta} \cos s\theta \, d\theta$$

$$= \frac{1}{\pi} \int_0^{2\pi} \cos s\theta \, \Sigma \frac{1}{n!} (-i\kappa r \cos\theta)^n d\theta. \quad . \quad . \quad . \quad (18)$$

Clearly the terms corresponding to values of n less than s give zero on integration, so when κr is small the most important term is

$$\frac{(-i\kappa r)^s}{\pi \, . \, s!} \int_0^{2\pi} \cos^s\theta \cos s\theta \, d\theta. \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Thus $g_s(\kappa a)$ is of the order of magnitude of $(\kappa a)^s$, and $g_s'(\kappa a)$ is of the order of magnitude of $(\kappa a)^{s-1}$ except in the case of $s=0$. So the successive terms in the series for γ_1

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decrease rapidly in importance, and we get a close approximation by taking only the first two, namely,

$$\gamma_1 = -H_0\left[g_0'(\kappa a)\frac{h_0(\kappa r)}{h_0'(\kappa a)} + g_1'(\kappa a)\frac{h_1(\kappa r)}{h_1'(\kappa a)}\cos\theta\right]. \quad (20)$$

Neglecting $(\kappa a)^2$, we get from (19)

$$g_1(\kappa a) = -\frac{i\kappa a}{\pi}\int_0^{2\pi}\cos^2\theta\, d\theta = -i\kappa a,$$

whence

$$g_1'(\kappa a) = -i. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Formula (18) does not hold for $s=0$: the corresponding formula is

$$g_0(\kappa r) = \frac{1}{2\pi}\int_0^{2\pi}\sum\frac{1}{n!}(-i\kappa r\cos\theta)^n d\theta,$$

whence approximately

$$g_0(\kappa a) = 1 - \tfrac{1}{4}(\kappa a)^2,$$

and

$$g_0'(\kappa a) = -\tfrac{1}{2}\kappa a. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Thus

$$\gamma_1 = H_0\left[\tfrac{1}{2}\kappa a\frac{h_0(\kappa r)}{h_0'(\kappa a)} + i\cos\theta\frac{h_1(\kappa r)}{h_1'(\kappa a)}\right]. \quad . \quad . \quad (23)$$

It remains to evaluate the h functions which, unlike the corresponding g functions, need not be precisely defined as to constant multipliers. Now $h_0(\rho)$ satisfies the equation

$$\frac{d^2h_0}{d\rho^2} + \frac{1}{\rho}\frac{dh_0}{d\rho} + h_0 = 0;$$

differentiation of this gives

$$\frac{d^2}{d\rho^2}\left(\frac{dh_0}{d\rho}\right) + \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{dh_0}{d\rho}\right) + \left(1 - \frac{1}{\rho^2}\right)\frac{dh_0}{d\rho} = 0,$$

which proves that $h_0'(\rho)$ satisfies the same differential equation as $h_1(\rho)$. Both are finite for $\rho = \infty$, and so we have a right to assume

$$h_1(\rho) = h_0'(\rho). \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The scattered wave is now given by

$$\gamma_1 = H_0\left[\tfrac{1}{2}\kappa a\frac{h_0(\kappa r)}{h_0'(\kappa a)} + i\cos\theta\frac{h_0'(\kappa r)}{h_0''(\kappa a)}\right]. \quad . \quad . \quad (25)$$

We may take h_0 to be the h function discussed in Article 2 above, and for values of r great compared with the wave-length we may use the approximations of formula (2), but with $-ix$ instead of ix, since x, $(=\kappa r)$, is real and we

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have to do with waves propagated outwards from the wire. Thus we get

$$h_0(\kappa r) = Be^{-i\kappa r}/\sqrt{\kappa r}, \quad h_0'(\kappa r) = -iBe^{-i\kappa r}/\sqrt{\kappa r},$$
$$h_0'(\kappa a) = 1/\kappa a, \quad h_0''(\kappa a) = -1/(\kappa a)^2.$$

Substituting these we obtain for the magnetic force in the scattered wave the approximate formula

$$\gamma_1 = BH_0\kappa^2 a^2(\tfrac{1}{2} - \cos\theta)e^{i(pt-\kappa r)}/\sqrt{\kappa r}, \quad . \quad . \quad (26)$$

valid for values of r which are great compared with the wave-length.

6. When the electric intensity in the incident wave is parallel to the axis of the wire, we denote it by

$$Z_0 = E_0 e^{i(pt-\kappa z)},$$

that is

$$Z_0 = E_0 e^{ipt}\Sigma g_s(\kappa r)\cos s\theta. \quad . \quad . \quad . \quad . \quad (27)$$

By reasoning similar to that of the previous Article, it is seen that the electric intensity Z_1 of the scattered wave will be of the form

$$Z_1 = e^{ipt}\Sigma B_s h_s(\kappa r)\cos s\theta. \quad . \quad . \quad . \quad . \quad (28)$$

The boundary condition at the surface of the wire is

$$Z_0 + Z_1 = 0,$$

whence

$$B_s = -E_0 g_s(\kappa a)h_s(\kappa a)$$

and

$$Z_1 = -E_0 e^{ipt}\Sigma g_s(\kappa a)\frac{h_s(\kappa r)}{h_s(\kappa a)}\cos s\theta. \quad . \quad . \quad . \quad (29)$$

When κa is very small the successive terms decrease rapidly in importance, and we may use the approximate formula

$$Z_1 = -E_0 e^{ipt}\left[g_0(\kappa a)\frac{h_0(\kappa r)}{h_0(\kappa a)} + g_1(\kappa a)\frac{h_1(\kappa r)}{h_1(\kappa a)}\cos\theta\right], \quad (30)$$

$$= -E_0 e^{ipt}\left[(1 - \tfrac{1}{4}\kappa^2 a^2)\frac{h_0(\kappa r)}{h_0(\kappa a)} - i\kappa a\frac{h_0'(\kappa r)}{h_0'(\kappa a)}\cos\theta\right]. \quad (31)$$

Introducing the approximations for the h_0 functions for κa small and for κr large, we get finally

$$Z_1 = BE_0\left\{\frac{\frac{1}{4}\kappa^2 a^2 - 1}{\log(\kappa a)} + \kappa^2 a^2\cos\theta\right\}\frac{e^{i(pt-\kappa r)}}{\sqrt{\kappa r}}, \quad . \quad . \quad (32)$$

for values of r great compared with the wave-length.

7. Results (26) and (32) should be compared with the

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corresponding formulæ in Prof. J. J. Thomson's 'Recent Researches,' Articles 367 and 364 respectively. It will be noticed that the great difference between his results and those obtained here is that the complex constant B is evaluated in 'Recent Researches,' but is left unevaluated in the present discussion. As a matter of fact, the evaluation of B is not required for the important inferences usually drawn from these results, such as the mode of dependence of the intensity of the scattered wave on the wave-length and on the diameter of the wire, the determination of the direction in which the scattered wave nearly vanishes, and the greater proportional scattering when the electric intensity in the incident wave is parallel to the wire than when it is perpendicular. For a full discussion of the interpretation of formulæ (26) and (32) the student naturally turns to 'Recent Researches.'

8. It need hardly be pointed out that the discussions of Articles 3 to 7 of the present paper are fundamentally the same as those given by Prof. Thomson. The functions used are, at most, different only by constant factors. The sole advantage, but that an important one, claimed for the present formulation is that it assumes no knowledge of Bessel's functions on the part of the reader and makes no appeal to formulæ, but obtains from first principles all the approximations required. The work is thus greatly shortened and simplified, and ought to be easily understood by many students to whom the standard formulation presents difficulty.

30th March, 1906.

VI. *On the Thermodynamical Theory of Radiation. By* J. H. JEANS, *M.A., F.R.S., Professor of Applied Mathematics in the University of Princeton* *.

1. THERE are two entirely distinct ways of attacking the problem of determining the intensity and partition of radiation from a body at a given temperature.

One method is based on what is commonly termed the "Thermodynamics of Radiation." Its fundamental conception is that of a hot body surrounded by æther, in which the partition of energy is such that the energy absorbed by the matter from the æther in any time is exactly equal to that yielded up by matter to æther. From the condition that equilibrium of this kind shall exist, it has been shown by Bartoli, Boltzmann, and Wien, that the law of partition of

* Communicated by the Author.

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radiant energy must be of the form

$$\lambda^{-4}T f(\lambda T) d\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and that the total energy of radiation per unit volume must be of the form

$$\sigma T^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This method, it is seen, arises essentially from the contemplation of a conservative dynamical system.

2. The second method consists in regarding the radiator and the æther surrounding it as a non-conservative system. This method is not concerned with the equilibrium between matter and æther, but with the rate of transfer of energy from the former to the latter. Assuming the source of radiation to be the motion of electric charges in the radiator, it can be shown by this second method also that the radiation must be given by formulæ of the type (1) and (2) *.

3. In the opinion of the present writer neither method has so far led to a valid determination of the form of the function f of formula (1). The two theories can, however, be differentiated by their treatment of the constant σ of formula (2).

4. In the second theory the quantity σ enters as a physical entity, dependent on the magnitude of the electric charges only (as soon as the temperature scale has been determined). From the observed value of σ it is found that the electric charges producing the radiation must, if the theory is valid, be of the order of magnitude of $1{\cdot}8 \times 10^{-10}$ C.G.S. electrostatic units †—a circumstance which points very clearly to electrons as the source of radiation.

5. In the first theory, on the other hand, the quantity σ enters mathematically as a constant of integration, and the theory itself does not supply the means of evaluating it. Since, however, σT^4 is to be the amount of energy per unit volume, the physical dimensions of σ are known.

The thermodynamical argument by which, in this first theory, the formula σT^4 is reached, is concerned only with phenonema taking place in the æther. Thus we should naturally expect that it would be possible to evaluate σ in terms of quantities which measure the properties of the æther. The properties of the æther, as contemplated by the argument which leads to the formula σT^4, are, however, all contained in the one constant which measures the velocity of wave-propagation, and obviously a quantity of the physical dimensions appropriate for σ cannot be constructed from this alone. Even if we attribute a further property, *e. g.* material density

* "On the Laws of Radiation," Roy. Soc. Proc. vol. 76 A. p. 545.

† *L. c. ante*, § 9.

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([mass] ÷ [length]^3) to the æther, it remains impossible to construct σ out of æther-constants.

The evaluation of σ can, however, proceed by taking advantage of the circumstance that there are physical constants associated with matter, which are common to all kinds of matter—in particular, the charge of the electron, e. If we make σ depend on the charge of the electron, the evaluation of σ may be borrowed from the second theory. We then find that σ is proportional to e^{-6}, and the numerical value of e deduced from the observed value of σ supplies strong confirmation that this is the right way of evaluating σ.

6. Thus if we agree to extend the first (thermodynamical) theory in the way here suggested, we may say that both theories lead to the formula σT^4, that both lead to the same numerical evaluation of σ, and that this evaluation is in agreement with experiment.

7. Next let us imagine an enclosure with perfectly reflecting walls, containing matter at temperature T. Let us refer to this piece of matter as R, and let us suppose that the matter is real matter in which each electron has the customary charge e. Let us suppose that we have also a mass of "ideal" matter I, in which each electron has a charge $\frac{1}{2}e$ only.

If we put R inside the enclosure we obtain, according to the thermodynamical argument, a state of equilibrium in which the æther has a volume density of energy σT^4.

Remembering that the constant σ has been found to be proportional to e^{-6}, it appears that if we take R out of the enclosure and insert I, we obtain equilibrium with a volume density of energy $64\sigma T^4$.

Now let us put R and I in the enclosure together. The thermodynamical argument might, I think, very fairly be used at this juncture to establish the equation $1=64$. Without pressing it thus far, however, we may say that it shows that the ideal matter I will give out more energy than it receives, while the opposite will be true of the real matter R. Thus the two bodies, originally at *equal* temperatures, will tend to assume *different* temperatures. This might or might not happen in nature if we had ideal matter to experiment with. What is of more importance is that the thermodynamical argument has led to a result which is opposed to the fundamental principles of thermodynamics. Or, to state the situation in another way, the thermodynamical argument first supposes the second law to be of such wide applicability that it may be applied to the æther, and then proves that it may not legitimately be applied even to different kinds of matter.

8. The difficulty may be illustrated by a second imaginary experiment, suggested to the writer in a conversation with

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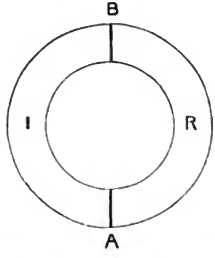
Prof. Lorentz. Let us join a piece of real and a piece of ideal matter, so as to form a closed circuit. Let one of the junctions, A, be a perfect conductor of heat devoid of radiating power, while each face of the other junction B is a perfect radiator, devoid of conducting power. Let all the rest of the surfaces of both R and I be supposed impervious to energy.



If junction A is thrown out of action the bodies R, I will as before assume different temperatures. If junction B is thrown out of action the temperatures will equalise by conduction of heat across A. If both junctions are in operation at once, we shall get a continuous flow of energy round the circuit in the direction I A R B, and a continuous rise of temperature as we pass through I from B to A, or through R from A to B.

These difficulties do not cast suspicion on the validity of the second form of the theory of radiation, which begins by presupposing a flow, rather than equilibrium, of energy, but to the present writer they seem fatal to the thermodynamical theory.

Princeton, N.J.,
April 24.

VII. *The H-Theorem and the Dynamical Theory of Gases. By* J. H. Jeans, *M.A., F.R.S., Professor of Applied Mathematics in Princeton University* *.

In the Phil. Mag. for April (pp. 455–465), Mr. Burbury again discusses my work on the Theory of Gases. While grateful for parts of his discussion which seem to be sympathetic and appreciative, I recognize that on the main question at issue Mr. Burbury is still inclined to disagree *in toto* with my views.

* Communicated by the Author.

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The aim of my work was *not* to restate the results of former writers in a new or simplified notation (without, as Mr. Burbury says, "in any way altering the logical position of the Kinetic Theory"). My aim was to give a new discussion of the whole question, based on no physical foundations except the laws of dynamics. This I believe to be possible. The molecules of a gas have probably never heard of Mr. Burbury's Assumption A, or been warned to remain "molekular-ungeordnet." But by passive obedience to the laws of dynamics, they arrange themselves in some definite way. The mathematician, from a knowledge solely of the laws of dynamics, ought to be able to follow the motion of the molecules and calculate their final arrangement.

This is the problem which I have tried to solve. Mr. Burbury states clearly that he thinks I have failed; while I, to be quite frank, still think I have succeeded. I cannot, therefore, let Mr. Burbury's article pass without reply.

I do not find that Mr. Burbury brings any new charges against my work, beyond those which he has brought in previous articles *; and to these I have already replied †. I will confess, however, that I have found a generality and vagueness about these charges, which makes them difficult to refute satisfactorily. If, as Mr. Burbury seems to think, my argument is wholly unsound, he can surely pick out one definite point and say: "Here, at this particular point, you introduce an assumption from outside your premisses"; or, "Here, in this particular statement, your logic is at fault." Or if, as Mr. Burbury perhaps thinks, the number of weak points is legion, I ask him to pick out the weakest one, and concentrate his attack on that—only *one* link need be broken to destroy a chain. I cannot, within reasonable limits of space, defend a whole theory against vague expressions of opinion, or against a general condemnation, unsupported by specific arguments.

Meanwhile may I reply to those two of Mr. Burbury's objections, which I understand most clearly?

Objection I. "The real defect . . . in every proof yet given of Maxwell's Law, consists in my opinion in this, that it ignores the continuity of the motion altogether."

Answer. Continuity of motion enters most proofs, including my own attempted proof, through the dynamical equations of motion. In fact, these equations simply express that the continuity assumed by the theory is of the kind observed in nature.

* Phil. Mag. [6] vi. p. 529; vii. p. 467.
† Phil. Mag. [6] vi. p. 720; vii. p. 468.

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OBJECTION II. "If the new method leads, or appears to lead, to any physical results not attainable by the ordinary method, that can only be because along with the new method, we have introduced some new hypothesis unawares."

ANSWER. In point of fact the physical result *is* attainable by the ordinary method. For "ρ" let Mr. Burbury read "total number of systems for which the variables $x_a, y_a, z_a, x_b, \ldots w_n$ lie between the limits x_a and $x_a + dx_a$, y_a and $y_a + dy_a$, z_a and $z_a + dz_a$, x_b and $x_b + dx_b, \ldots w_n$ and $w_n + dw_n$, divided by $dx_a\ dy_a\ dz_a\ dx_b \ldots dw_n$," &c., and the result will have been attained, although perhaps without much elegance or conciseness.

But if the ultimate test of any "physical result" is to be whether or not it is attainable by the "ordinary" method, then let physicists be warned against new methods, and let us admit that Nature herself must be "introducing a great many hypotheses unawares."

Princeton, N.J.

VIII. *Letter from* Mr. BARBER STARKEY *describing remarkable results of a Discharge of Lightning upon an Oak-Tree; with Note by* Lord KELVIN.

DEAR LORD KELVIN, June 4th, 1906.

ABOUT 5 o'clock on the afternoon of August 29th last year during a severe thunderstorm, three Oak-trees were struck in the Park; and in one case the effect on the tree was so remarkable that I think I ought to describe it to you. A large number of shreds of wood, probably not fewer than two or three dozen, and varying from 1 to 3 feet in length, and of an irregular breadth of $\frac{1}{16}$ to $\frac{1}{4}$ of an inch, were torn out of the bole of the tree, and remained with one end fixed in the ground to a depth of 1 inch or more, standing in a more or less nearly vertical position inclined towards the tree, at a distance of about 15 feet from the bole of the tree on the west side. These irregular shreds were so light and flexible that the act of falling could not have placed them in the position described. Apparently they had been sucked into the ground. Could this have been caused by any electrical action?

When I visited the tree a few days ago, I still found one of these shreds embedded in the ground.

I am, Yours very truly,
W. I. S. BARBER STARKEY.

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In answer to Mr. Barber Starkey's question: I think it probable that each shred, during its passage through the air from the tree to the earth, was kept stiff and straight by mutual electric repulsion between its own parts, and was shot into the earth as an arrow might be in virtue of its velocity before touching the earth. I scarcely think that after it touched the earth there could have been enough of electric attractive force to "suck it into the ground" much farther than its own velocity and inertia took it. The velocity which it acquired after leaving the tree was certainly given to it initially by electric repulsion from the tree, due to large difference of potential between the tree and the earth. During its flight it must have been strongly electrified; and must have experienced therefore an oblique force, becoming nearly perpendicular to the ground before it struck. It is quite certain that each shred must have been strongly electrified (whether vitreously or resinously we cannot say) at the instant of its leaving the tree. K.

IX. *On the Solution of Problems in Diffraction by the Aid of Contour Integration.* By Henry Davies, *B.Sc., Technical Institute, Portsmouth* *.

THE general problem of diffraction consists of finding solutions of the equation

$$\frac{\delta^2 V}{\delta t^2} = a^2 \nabla^2 V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The solutions must remain finite throughout the space considered and must satisfy certain specified boundary conditions.

This equation is modified when assumptions are made concerning the light vector.

Consider the case of a wedge of angle α, and assume that the electric force—taken as the light vector—is parallel to the edge of the wedge. Assume also that $V \propto \epsilon^{ikt}$. Then the general equation reduces to

$$\frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + m^2 V = 0, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

provided the origin of co-ordinates is taken in the edge of the wedge, the latter being assumed to occupy the space $\alpha < \theta < 2\pi$.

The boundary conditions are that V shall vanish at $\theta = 0$ and at $\theta = \alpha$, and shall become infinite at a point $(r' \,.\, \theta')$.

* Communicated by the Physical Society: read June 8, 1906.

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2. The proper solution for unbounded space is

$$V = K_0(mR), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$R = \sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')}$$

and $K_n(x)$ is Bessel's function of the second kind and of order n.

At this point it is necessary to introduce the relations which hold between the various functions which will be used.

The Bessel's functions of the second kind are related to those of the first kind by the equation

$$K_n(x) = \frac{\pi}{2 \sin n\pi} \cdot \left[J_{-n}(x) - \epsilon^{-in\pi} J_n(x) \right]. \quad . \quad (4)$$

When n is large the value of $J_n(x)$ is given very approximately by

$$J_n(x) = \left(\frac{x}{2}\right)^n \frac{1}{\Pi(n)}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\Pi(n)$ represents an infinite product.

Taking the asymptotic value of $\Pi(n)$ we have

$$J_n(x) = \left(\frac{x}{2}\right)^n \frac{1}{\sqrt{2\pi n}\, \epsilon^{n \log n - n}}. \quad . \quad . \quad . \quad (6)$$

From (6) it is evident that $J_n(x)$ vanishes at infinity when the real part of n is positive.

To obtain a value for $J_{-n}(x)$ proceed as follows:—

$$\Pi(n)\,\Pi(-n) = \frac{n\pi}{\sin n\pi}$$

$$\therefore \quad \frac{1}{\Pi(-n)} = \frac{\sin n\pi}{n\pi}\,\Pi(n).$$

Therefore

$$J_{-n}(x) = \left(\frac{x}{2}\right)^{-n} \frac{1}{\Pi(-n)}$$

$$= \left(\frac{x}{2}\right)^{-n} \frac{\sin n\pi}{n\pi} . \Pi(n)$$

$$= \left(\frac{x}{2}\right)^{-n} \frac{\sin n\pi}{n\pi} . \sqrt{2\pi n}\, \epsilon^{n \log n - n}. \quad . \quad . \quad (7)$$

This does not vanish at infinity when the real part of n is positive. This difficulty can be overcome as follows.

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From (4) by multiplication by $J_n(x')$

$$K_n(x)\,J_n(x') = J_n(x')\,.\,\frac{\pi}{2\sin n\pi}\left[J_{-n}(x) - \epsilon^{-in\pi}\,J_n(x)\right]$$

$$= \left(\frac{x'}{2}\right)^n \frac{1}{\Pi(n)}\cdot\frac{\pi}{2\sin n\pi}\left\{\left(\frac{x}{2}\right)^{-n}\frac{\sin n\pi}{n\pi}\cdot\sqrt{2\pi n}\,.\,\epsilon^{n\log n - n}\right.$$

$$\left. - \epsilon^{-in\pi}\left(\frac{x}{2}\right)^n\cdot\frac{1}{\sqrt{2\pi n}\,\epsilon^{n\log n - n}}\right\} \quad . \quad . \quad . \quad (8)$$

With substitution of the value of $\frac{1}{\Pi(n)}$ this reduces to

$$K_n(x)\,J_n(x') = \frac{1}{2n}\cdot\left(\frac{x'}{2}\right)^n\left(\frac{x}{2}\right)^{-n}$$

$$-\epsilon^{-in\pi}\left(\frac{x}{2}\right)^n\left(\frac{x'}{2}\right)^n\frac{\pi}{2\sin n\pi}\left\{\frac{1}{\sqrt{2\pi n}\,\epsilon^{n\log n - n}}\right\}$$

Hence when the real part of n is positive this becomes

$$K_n(x)\,J_n(x') = \frac{1}{2n}\left(\frac{x'}{x}\right)^n, \quad . \quad . \quad . \quad (9)$$

since the second part vanishes at infinity.

The expression

$$\frac{\cos n\{\pi - (\theta - \theta')\}}{\sin n\pi}$$

vanishes at infinity if $\theta - \theta'$ lies between 0 and 2π.

3. Consider the integral

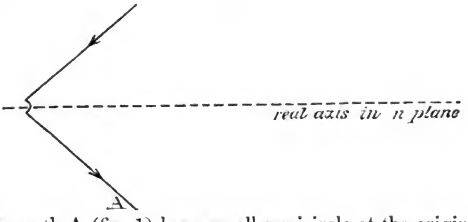
$$\int_A \frac{\cos n(\pi - \bar{\theta} - \theta')}{\sin n\pi}\,J_n(mr')\,K_n(mr)\,dn, \quad . \quad (10)$$

where $r > r'$ and $\theta > \theta'$;

the integral being taken over the path A in the n-plane.

The integral is equal to $2\pi i\,\Sigma B$, where ΣB is the sum of the residues of this function.

Fig. 1.



The path A (fig. 1) has a small semicircle at the origin.

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If, however, we remove the small semicircle then we easily find that

$$\frac{1}{i\pi}\int_{A'}\frac{\cos n(\pi-\overline{\theta-\theta'})}{\sin n\pi}\,J_n(mr')\,K_n(mr)du$$

$$=\frac{1}{\pi}\left\{J_0(mr')\,K_0(mr)+2\sum_{n=1}J_n(mr')\,K_n(mr)\times\cos n(\theta-\theta')\right\}. \quad . \quad (11)$$

The path A′ is that with the circle removed. There is a well-known addition theorem for $K_0(mr)$ which gives

$$K_0(mR)=J_0(mr')\,K_0(mr)+2\sum_{n=1}J_n(mr')\,K_n(mr)\times\cos n(\theta-\theta'). \quad (12)$$

From (11) and (12) there results

$$K_0(mR)=\frac{1}{i}\int_{A'}\frac{\cos n(\pi-\theta-\theta')}{\sin n\pi}\,J_n(mr')\,K_n(mr)\,du. \quad (13)$$

This equation gives a solution for the unbounded space. It is necessary now to add terms which shall satisfy the boundary equations while introducing no new singularities. After many trials I have found that the following equation satisfies the conditions completely:—

$$V=\frac{1}{i}\int_{A'}\left[\cos n(\pi-\overline{\theta-\theta'})-\cos n(\pi-\overline{\alpha-\theta'})\frac{\sin n\theta}{\sin n\alpha}\right.$$

$$\left.-\cos n(\pi-\theta')\frac{\sin n(\alpha-\theta)}{\sin n\alpha}\right]$$

$$\times\frac{J_n(mr')\,K_n(mr)}{\sin n\pi}\,du. \quad . \quad . \quad (14)$$

If this is tested term by term it will be found to satisfy the differential equation and the boundary conditions. With some laborious work the trigonometry can be simplified, and the final result appears as

$$V=2i\int_{A'}\frac{\sin n(\alpha-\theta)\sin n\theta'}{\sin n\alpha}\,J_n(mr')\,K_n(mr)du \quad . \quad (15)$$

$$r>r' \quad \text{and} \quad \alpha>\theta>\theta'.$$

Since in (15) there is no pole at the origin, then A and A′ are identical.

By Cauchy's residue theorem the whole solution is now

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obtainable as an infinite series for all values of θ in the space $0>\theta>\alpha$.

The series is

$$V=\frac{4\pi}{\alpha}.\Sigma \sin\frac{s\pi}{\alpha}\theta \sin\frac{s\pi}{\alpha}\theta' J_{\frac{s\pi}{\alpha}}(mr') K_{\frac{s\pi}{\alpha}}(mr), \quad . \quad (16)$$

when $r>r'$, and for the case when $r<r'$ it is only necessary to interchange these quantities.

A solution of the same problem is given by Macdonald in his book on Electric Waves, which depends on a theorem in an earlier portion of his book. That solution is in agreement with the above.

The method can be applied to three-dimensional problems, and some interesting results are being obtained which I hope soon to send in.

X. *Interference Colours of Chlorate of Potash Crystals and a New Method of Isolating Heat Waves. By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University**.

[Plate I.]

THE remarkable colours exhibited by certain twinned crystals of chlorate of potash have been studied by Stokes and Lord Rayleigh.

The crystals are easily prepared by making a hot saturated solution of the potash, and allowing it to cool slowly in a large flat-bottomed tray. On breaking up the crystalline mass, and shaking the tray gently in front of a window, numerous highly coloured flakes will appear, which can be lifted out by means of a small bent spatula, made of thin sheet copper or brass. They should be dried on filter-paper, and mounted in balsam, preferably between two glass prisms of about ten degrees angle. The thin flakes have the remarkable property of reflecting practically all of the incident light of a certain colour or colours, while freely transmitting the remainder of the spectrum. The reflected colours are of great spectral purity, the width of the band being sometimes not greater than the distance between the yellow mercury lines. Seen by transmitted light the flakes often appear delicately tinted, and the spectrum of the transmitted light will be found to be crossed by one or more narrow and intensely black bands.

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Lord Rayleigh has shown that the peculiarity of the crystals consists in the occurrence of a large number of twin planes, parallel to the surfaces of the flake, the narrowness of the reflected band resulting from multiple reflexions, as in the Fabry and Perot interferometer.

In the present paper the behaviour of the crystals in the infra-red and ultra-violet regions will be considered.

Stokes observed that many crystals reflected two or even three narrow bands in the visible spectrum, and inferred that, in these cases, there must be several sets of multiple twin planes. It seems curious that he was led to adopt this hypothesis, as the more obvious explanation would be to refer the colours to different orders.

A thin film, or a number of parallel thin films of equal thickness D, reflect at normal incidence light of wave-lengths given by the equation $2D=0$, $2D=\lambda$, $2D=3\lambda$, $2D=4\lambda$, &c., if we consider no phase change to occur. If $2D=\cdot0006$ mm. the film will reflect red light, and ultra-violet light of wave-lengths ·00030 and ·00020. If, however, 2D is equal to some wave-length in the infra-red region, say at ·0012 mm. ($1\cdot2\,\mu$), it is clear that we shall have more than one reflected colour in the visible region, the maxima occurring at wave-lengths ·0006, ·0004, ·0003, ·00024, &c. Such a film should appear purple, since it reflects red and violet light. If the first-order colour is at $2\,\mu$, the higher orders will fall at $1\,\mu$, $\cdot66\,\mu$, $\cdot50\,\mu$, $\cdot40\,\mu$, &c., and the films will reflect narrow regions in the red, green, and violet.

We can moreover, by measuring the wave-lengths of the bands in the visible and ultra-violet spectrum, calculate the position of the first-order band in the infra-red. An examination of the crystals by transmitted light, derived from a cadmium spark, with a small quartz spectrograph, showed that they were perfectly transparent down to the very end of the spectrum*.

A number of the films were then mounted in glycerine between two quartz prisms, by which device all light except that reflected from the equidistant laminæ is eliminated. The colours appear much more saturated when the films are thus mounted, for the light reflected from their upper and lower surfaces is not destroyed by interference, and dilutes the selectively reflected colours. This circumstance may be due to the fact that the distances of the surfaces from the neighbouring twin planes is not equal to the distances between

* This property is worth remembering, as a thin lamina, or plate, transparent to ultra-violet light is sometimes required in optical work; they are very fragile however, and require careful handling.

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the parallel strata, or possibly to the large amplitudes of the disturbances reflected from the outer surfaces.

The films, after having been prepared in the manner described above, were mounted in front of the slit of a small quartz spectrograph, and oriented so as to throw the light from a cadmium spark into the instrument.

A series of photographs obtained in this way is reproduced in fig. 1 (Plate I.). In some cases a number of bands appear in the ultra-violet, and in other cases only one or two. It is obvious that the thicker the reflecting strata, the larger will be the number of regions selectively reflected in a given spectral range. The approximate values of the lengths of the reflected waves are given in the following table :—

	1st Order Colour.	Higher Order Colours.					
A ...	1·6 μ	·80	·54	·40	·30	·26	·23
B ...	·617	·31					
C ...	·53	·28					
D ...	1 μ	·53	·28				
E ...	·64	·325	·22				
F ...	1·26 μ	·633	·435	·32	·23		
G ...	1·13	·565	·39	·235			
H..	1·23	·614	·42	·326	·266	·226	
I ...	·44	·23					
J ...	·60	·30					

In all except A and J the cadmium-spark spectrum is recorded for comparison.

It will be noticed that certain films (spectra F and G for example) reflect double bands, which are often as close together as the yellow mercury lines. One film which I examined showed a double line in the red the components of which were just barely resolvable. Its appearance reminded one of the sodium lines when on the point of fusing together owing to the density of the vapour. There seems to be no way of explaining cases of this sort except by assuming a double set of twin planes. In the case above referred to there may have been two thin crystalline plates stuck together. The specimen unfortuately was destroyed so that further examination was impossible.

A search for the predicted bands in the infra-red region was undertaken at my request by Mr. A. H. Pfund, Fellow in Physics. The light from a Nernst lamp was reflected from one of the purple films, into the slit of a reflecting spectrometer furnished with a rock-salt prism, and radiometer. This particular film showed a sharp band in the red at wave-length

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60, another in the violet at wave-length 40, and others in the ultra-violet. The first-order colour should be found at about $1{\cdot}2\,\mu$. The radiometer gave a deflexion of several centimetres, when illuminated with the red band, immediately dropping back to zero, as the prism was so turned as to bring the infra-red region upon the slit. At about $1\,\mu$ a large deflexion was obtained, which at $1{\cdot}2\,\mu$ was "off the scale." The energy curve was then plotted from a large number of readings, the slit having been narrowed in the meantime. This curve is reproduced in fig. 2. The width and asymmetry of the infra-red, or first-order band, are due to the fact that the energy curve in this region of the spectrum is very steep and asymmetrical.

A crystal reflecting red, green, and violet bands at wave-lengths 66, 50, and 40, should show two maxima in the infra-red, one at $2\,\mu$, the other at $1\,\mu$.

It seems probable that large sized flakes of chlorate of potash may at times prove useful in research work when it is desired to isolate a single spectral line, or cut out a narrow region of a continuous spectrum. The advantage of the flake over the spectroscope would lie in the large amount of light available, which is often of great advantage, for example in the study of the fluorescence of substances when stimulated with monochromatic light obtained from metallic arcs. The power of the crystals to reflect a comparatively narrow band in the infra-red might sometimes be made use of as well. In making crystallizations on a small scale, I have seldom obtained satisfactory flakes much over $0{\cdot}7$ cm.^{2} Larger ones could perhaps be made by working on a large scale.

XI. *The Effect of Radium in Facilitating the Visible Electric Discharge in Vacuo.* By A. A. Campbell Swinton *.

As has been shown by Edison, Fleming, and others, the passage of the electric discharge in vacuo is much facilitated by heating the cathode.

More recently, as has been shown by Owen (Phil. Mag. viii. 1904) and by Wehnelt (Phil. Mag. x. 1905), the passage of the discharge is still further facilitated by coating the heated cathode with oxides of the alkaline metals; the effect in this case being so great that large and highly luminous discharges amounting to several amperes can,

* Communicated by the Physical Society: read June 22, 1906.

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under favourable conditions, be passed through the vacuum tube, using pressures of only 30 to 300 volts.

It is generally held that the efficacy of the hot oxides in this direction is due to their giving off negatively charged ions or corpuscles.

It therefore occurred to the writer to ascertain whether similar effects could not be obtained by painting the cathode with radium, and as radium gives off corpuscles when cold, it was anticipated that it might not be necessary to heat the cathode.

In the first experiment, however, this was found not to be the case, as with a cold cathode, and using continuous current up to 400 volts pressure, the radium did not seem to have any appreciable effect in producing a visible discharge. When, however, the radium-coated cathode was heated to redness, the radium was found to have a very marked action in facilitating the production of a luminous discharge.

In the experiments a strip of platinum foil was used for the cathode, which, before mounting in the tube, was dipped into a solution of radium bromide and dried. The amount of radium on the foil must have been extremely small, but with a suitable vacuum, and with the cathode heated to a bright red colour, the discharge passed, and the gas in the tube lighted up brightly with electrical pressures as low as about 80 volts.

In order to determine whether it was necessary that the radium should be on the cathode itself or whether the mere presence of radium in the tube would be sufficient to produce the effect, a tube was constructed, as shown in the illustration (p. 72), with two similar platinum-foil electrodes, either of which could be used as a cathode, and both being arranged so that they could be heated to redness by passing a current through them, the anode being mounted midway between the two cathodes. One of the cathodes was dipped before mounting in a solution of radium bromide, while the other was not so treated.

Under these circumstances, it was found that with pressures up to 400 volts a visible discharge passed only when the platinum strip that had been treated with radium was used as cathode and was heated to redness, and that when both platinum strips were heated to redness there was no visible discharge when the untreated strip was used as a cathode. Furthermore, it was found that the tube would only allow visible discharges to pass in the direction that made the treated platinum strip cathode, the tube acting as a unidirectional valve in the same way as do tubes with cathodes coated with oxides.

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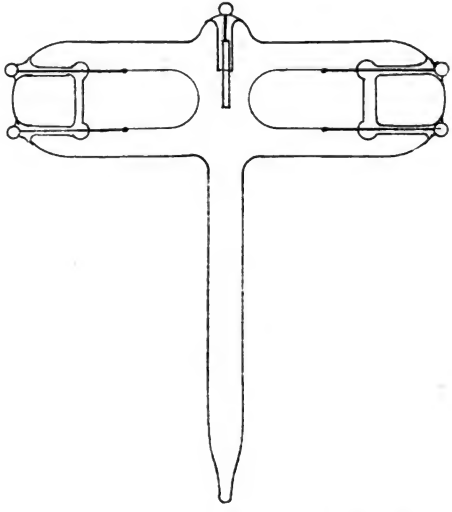
It was not found possible to obtain as large currents through these radium tubes as through oxide tubes under similar conditions as regards voltage, degree of exhaustion,



and temperature of cathode; but, having regard to the very small quantity of radium employed in comparison with the quantities of oxide, this is not surprising.

Experiments were next made without any heating of the cathode, but using alternating currents of higher voltages than were available in continuous current.

Employing the tube with one central electrode and two symmetrically placed loop electrodes, one of the latter being treated with radium and the other being plain, the following results were obtained, the voltages being measured by means of a Duddell thermo-galvanometer.

It was found that, using the untreated electrode, it required from 800 to 900 volts to get a visible discharge to pass; whereas, using the radium treated electrode, a visible discharge could be got to pass with from 700 to 800 volts. The exact voltages required in each case to cause a visible discharge were somewhat variable; but it was always found that a visible discharge could be got to pass, using the radium treated

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electrode, with about 100 volts less than when the untreated electrode was employed. It was further very noticeable that, using the treated electrode and gradually reducing the voltage, a much fainter luminosity of discharge could be obtained without actual extinction of the discharge than was the case when the untreated electrode was employed, in which case the transition from a visible discharge to no luminosity was much more abrupt. This fact goes to show that the differences in the minimum voltage required to produce a visible discharge in the two cases was actually due to the presence of radium, and not to any slight difference in the distances between the electrodes or other want of symmetry in the tube.

A d'Arsonval mirror galvanometer was next inserted in the circuit. With a plain alternating current this galvanometer would, of course, give no deflexion. It was, however, found that whether the radium treated or the untreated electrode was used, there was always a slight deflexion of the galvanometer, due no doubt to some unidirectional valve action on the part of the vacuum-tube. It was found, however, that using the treated electrode, the galvanometer deflexion was between two to three times as great as when the untreated electrode was used, this being evidence that the presence of radium increased the valve action to a considerable extent.

The thermo-galvanometer was next substituted for the d'Arsonval mirror galvanometer, and it was then found that, using the radium treated electrode, the amount of current that passed through the tube was from one and a half times to twice as large as when the untreated electrode was employed, this showing that the presence of radium on the cathode materially increases the amount of current that passes through the tube with any given voltage.

The writer is indebted to Mr. J. C. M. Stanton and Mr. R. C. Pierce for their assistance in making the above investigations.

XII. *On the Periods of Transformation of Radium A, B, and C.* By Howard L. Bronson, *Ph.D.**

THE present investigation was originally undertaken in order to determine whether or not radium A was the parent of radium B. Rutherford (Phil. Trans. 1904, p. 198) had previously pointed out that the experimental β-ray curve seemed to agree better with the theoretical curve calculated on the assumption that the products A and B were not

* Communicated by Professor E. Rutherford, F.R.S.

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successive. He also stated that this problem was of too great theoretical importance to be settled without further data.

During the course of this investigation, occasion was taken to redetermine with care the periods* of the three products A, B, and C, and also to investigate the effect of high temperatures on these periods. As a result of these experiments it has already been shown (Amer. Journ. of Science, July 1905, and Phil. Mag. Jan. 1906) that radium C does not have a longer period than radium B, as had been previously supposed. It was also shown that 21 and 28 minutes, the generally accepted values for the periods of the two products B and C respectively, were both too large, and that 26 and 19 minutes must be very close to the true periods. These results have recently been confirmed by von Lerch (*Sitz. Akad. d. Wiss. Wien.* Feb. 1906), who separated the products B and C by electrolysis and obtained 26·7 and 19·5 minutes as their respective periods.

After making such corrections in the calculations as the above changes made necessary, there still existed larger differences between the theoretical and experimental curves than could be accounted for by errors of measurement. These differences were especially marked in the early portion of the β-ray decay curve.

While the investigation was still in progress, Schmidt (*Physikalischen Zeitschrift*, Jan. 1906) showed that radium B was not rayless as had been supposed, but that it emitted β rays of much less penetrating power than those from radium C. It was at once seen that this would probably account for the observed discrepancies, since the β ray activity will evidently decay more rapidly if part of it comes from radium B, than if it is entirely due to radium C. Thus the β-ray decay curve will not be a function merely of the periods of the three active products, but will also depend upon the ratio of the activities due to the two products B and C. In this paper the word ratio means the ratio when the products are in equilibrium.

Relative Amounts of Ionization produced by α and β Rays.

Since the β rays from the products B and C have different penetrating powers, the ratio of the ionization produced by these two kinds of rays will not be a constant, but will depend upon the amount of material through which they have to pass, and also upon the shape and size of the testing-vessel. The

* In this paper the word "period" always means the time required for the activity of any product to decay to half value.

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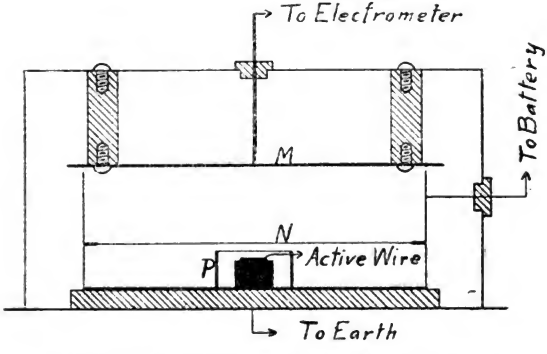
necessity of determining this ratio for every β-ray decay curve, and the difficulties involved in obtaining it with sufficient accuracy, seemed to make the β-ray decay curve untrustworthy for the purposes of analysis. It seemed worth while, however, to determine approximately the maximum value for this ratio, and at the same time to get an approximate value for the ratio of the ionization produced by the β rays to that produced by the α rays under certain definite conditions.

The conditions of the problem were such that the method of measurement and the testing-vessel employed had to be different from those which were found to give the best results in the measurement of decay curves. The arrangement of the apparatus is seen in fig. 1. The insulated plate M was

Fig. 1.



connected to an electrometer. The plate N was of aluminium-foil about 0·00029 cm. thick. The active wire was placed beneath N and arranged so that it could be easily covered with an aluminium cap P, about 0·005 cm. in thickness. This was just sufficient to cut off all the α rays. The ionization-chamber between M and N was so small that the α particles, after passing through the foil N, would practically penetrate to the side-walls in every direction. The difference between the ionization produced by the α rays and that produced by the β rays was so large that the constant-deflexion method was not adapted for the measurements. The ionization currents were therefore measured in the ordinary way by means of an electrometer with suitable capacities in parallel. Measurements were taken alternately with and without the

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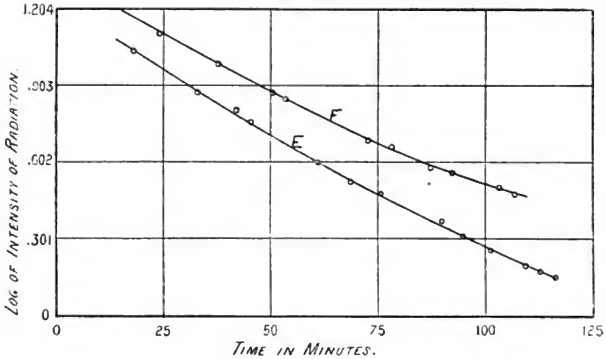
cap P over the active wire. Fig. 2 shows the two curves obtained in this way from the same active wire. E represents

Fig. 2.



the α-ray activity divided by 100, and F represents the β-ray activity. From these curves it is easily seen that the ratio of the β-ray to the α-ray activity continues to decrease with the time.

Now let x be the ratio of the β-ray ionization of radium B to the α-ray ionization of radium C, and let y be the ratio of the β-ray ionization of radium C to the α-ray ionization of radium C. If then we assume that the periods of radium B and C are 26 and 19 minutes respectively, we obtain from fig. 2 a series of equations of the following type:

$$ax + by = rb,$$

where a and b are the calculated percentages of radium B and C remaining at any time, and r is the ratio of the total β-ray ionization to the α-ray ionization, which is calculated directly from fig. 2. A number of such equations can be obtained from fig. 2 by determining r at different times. Each equation represents a straight line, and all should be satisfied by the same values of x and y. The graphical method is the simplest way of obtaining a solution. Such a solution is given by the intersections of the lines in fig. 3. The

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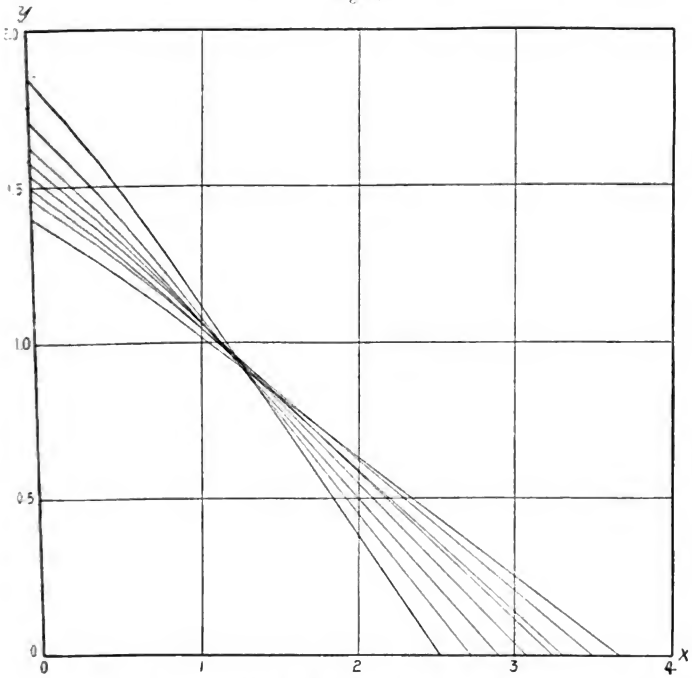
intersections are not at exactly the same point, but approximately $x=1{\cdot}2$, $y=0{\cdot}96$, and $\frac{x}{y}=1{\cdot}25$. Other values obtained under similar conditions were:—

$$x = 1{\cdot}10,\ 1{\cdot}50,\ 1{\cdot}00;$$
$$y = 0{\cdot}90,\ 1{\cdot}00,\ 0{\cdot}65;\ \text{and}$$
$$\frac{x}{y} = 1{\cdot}22,\ 1{\cdot}50,\ 1{\cdot}55.$$

Fig. 3.



There is considerable variation in the values obtained in the different experiments, which is due to the fact that a very small variation in either of the curves of fig. 2 causes a large

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variation in x and y, and the nature of the experiment made it very difficult to obtain great precision in these curves. In order to get a check on these results, the ratio of the β-ray to the α-ray activity was measured after the greater part of radium B had been removed by volatilization. In this case the α- and β-ray curves were approximately parallel, which means that x is nearly zero and that y is nearly equal to r. The values obtained for r in the two experiments tried were 0·80 and 1·00, a very satisfactory agreement with the other results. As has been mentioned above, the results are not accurate; and furthermore there would be no particular advantage in having them so, for they would be quite different under different experimental conditions. However, under ordinary experimental conditions, when there is no covering over the active wire, it would seem safe to say that the β rays from radium C produce nearly 1 per cent. as much ionization as the α rays, and that the β rays from radium B produce somewhat more than this, when the two products are in equilibrium.

Measurement and Calculation of Decay Curves.

In obtaining the experimental decay curves of the active deposit, the most satisfactory results were obtained by using a cylindrical testing-vessel with a central electrode, and by measuring the ionization current with an electrometer, using the constant-deflexion method. A saturation-potential of 200 volts was in general used on the testing-vessel. This was found to produce practical saturation in all cases. In fact, almost no difference in the curves could be detected when 600 volts were used instead of 200. In the case of the β-ray curves, the active wire was carefully covered with lead or aluminium-foil and used as the central electrode. In the case of the α-ray curves the best results were obtained by putting the emanation itself into the testing-vessel and then blowing it out suddenly, after a long or short exposure, depending on the experiment. The short exposure was never over 30 seconds. In this paper the time is always reckoned from the removal of the emanation.

The calculations were all made on the assumption that the three products radium A, B, and C are successive. The theory of three successive changes for both long and short exposure has been given by Rutherford (Phil. Trans. 1904, p. 181, and Radioactivity, 2nd ed. p. 331). The periods of the three products were taken as 3, 26, and 19 minutes respectively. These values for the periods had been previously obtained by direct experiment. The period of radium

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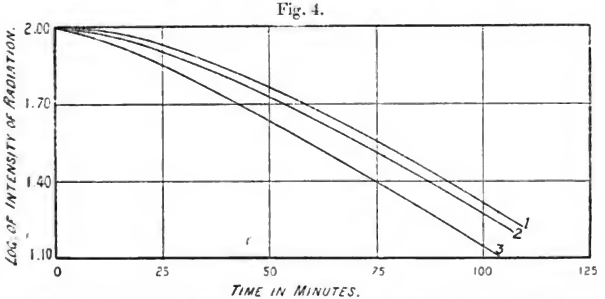
A had been obtained from the early portion of the short-exposure α-ray curve. The period of radium B had been determined from the final portion of the long-exposure curve. The period of radium C had been determined by measuring the period of the product which remained after radium B had been removed by volatilization. In the calculation of the α-ray curves the assumption was also made that the β rays of radium B gave about 2 per cent. of the total ionization produced by the two products B and C, when they were in equilibrium. This is a slightly larger value than that found in the previous experiments; but the change in the experimental conditions would seem to warrant the assumption. In any case the total effect of this factor is very small.

β-Ray Decay Curves.

As has been stated already, the β-ray curve is not suitable for analysis, because the ratio $\frac{x}{y}$ is not a constant, and cannot easily be determined with accuracy. Nevertheless, it seemed of interest to see in a couple of different cases how much effect the ionization due to radium B had upon the β-ray decay curves. Fig. 4 shows three curves. Curve 1 was

Fig. 4.



calculated on the assumption that the activity was entirely due to radium C. Curve 2 was the experimental decay curve obtained when the active wire was surrounded with about 0·047 cm. of lead. In this case the β rays from radium B were largely absorbed. This is clearly seen from Table I., where the calculated values for curve 1 were obtained by assuming that $\frac{x}{y} = 0{\cdot}2$. Curve 3 was the experimental curve obtained when the active wire was covered with about 0·005 cm.

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of aluminium. The calculated values for this curve given in Table I. were obtained by assuming that $\frac{x}{y}=1{\cdot}2$. This is about the same value for this ratio as was obtained experimentally in the previous experiments, where a similar thickness of foil was used. The variation among the three curves is very marked, and in itself would furnish additional evidence, if such were needed, that radium B emits β rays, which are less penetrating than those from radium C.

TABLE I.

Time in minutes.	Curve 1.	Curve 2.		Curve 3.	
	Calc.	Obs.	Calc.	Obs.	Calc.
0.....	100·0	100·0	100·0	100·0	100·0
5.....	99·7	99·0	98·9	97·0	97·1
10.....	98·1	95·7	96·0	91·1	91·1
15.....	95·1	91·2	91·9	84·7	84·3
20.....	91·0	86·0	86·8	78·1	78·1
25.....	86·2	80·3	81·5	70·8	70·8
30.....	80·9	74·9	75·9	64·2	64·5
40.....	69·8	63·9	64·7	53·0	53·0
50.....	58·8	53·5	54·0	42·8	43·0
60.....	48·8	44·0	44·5	34·7	34·7
70.....	40·0	35·8	36·2	27·9	27·8
80.....	32·4	29·3	29·3	22·1	22·1
100.....	20·9	18·8	18·8	13·9	13·8

α-Ray Decay Curves.

The observed and calculated α-ray decay curves of the active deposit from radium are given in Table II. The results are given in this way because it is more accurate than a curve, and because the experimental and theoretical results can thus be easily compared.

The observed values represent the mean of a very large number of curves taken at different times during the past year and a half.

The same cylindrical testing-vessel was used in obtaining both the long- and short-exposure curves. We should therefore expect to find the ratio of the α-ray ionization due to the two products A and C the same for both curves. In the calculations for the long-exposure curve this ratio was taken as 1·24, and for the short-exposure curve as 1·26, a very satisfactory agreement. This ratio is only constant under exactly similar experimental conditions, and with a sufficiently large testing-vessel would become less than unity, since an α particle from radium C has a much longer ionizing path than one from

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TABLE II.

Time in minutes.	Long Exposure.		Short Exposure.	
	Obs.	Calc.	Obs.	Calc.
0	.....	100·0	.....	100·0
2	80·3	79·6	64·0	63·2
4	66·9	66·8	39·9	40·2
6	.....	.....	25·9	25·9
7	55·7	55·6	.....	.....
9	.....	.....	13·8	14·0
10	49·6	49·6	.....	.....
12	.....	.....	8·44	8·35
15	44·1	44·5	5·73	5·72
18	.....	.....	4·60	4·56
20	41·2	41·4	.....	.....
21	.....	.....	4·12	4·11
25	38·6	38·8	3·92	3·91
30	36·2	36·4	3·92	3·93
40	31·2	31·3	3·91	3·91
50	26·4	26·3	3·65	3·65
60	21·7	21·9	3·27	3·26
70	17·7	17·9	2·81	2·84
80	14·4	14·5	2·36	2·40
100	9·32	9·30	1·60	1·64
120	5·91	5·85	1·03	1·07
140	3·64	3·63	·66	·68
160	2·23	2·22	·41	·43
180	1·35	1·34	·25	·26
200	·812	·812	·15	·16
220	·481	·486	...	...
240	·286	·290	...	...

radium A. In the above experiments the radius of the testing vessel was about 2·5 cms. and its length about 10 cms. Therefore a large part of the α particles from both radium A and C had an ionizing path of less than 4 cms. Now Bragg and Kleeman (Phil. Mag. Sept. 1905) have clearly shown that an α particle from radium A produces a considerably larger number of ions per cm. during the first 4 cms. of its path than is produced by an α particle from radium C during the same distance. This fact accounts satisfactorily for the mean value, 1·25, obtained for the ratio in the present experiments.

The remarkable agreement between the experimental and theoretical curves over so long a period of time would seem to be conclusive proof that the products A, B, and C are successive, and that the periods, which were obtained by direct experiments and used in these calculations, are very nearly correct. It would not seem probable that any one of the above periods was incorrect by more than two or three tenths of a minute, inasmuch as a change of three tenths of a minute in the period of radium B would make a 6 per cent. change in the calculated value of the ionization current at the time 240 minutes.

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Conclusions.

The above analysis of the decay curves of the active deposit from radium clearly establishes the following points:—

(1) That, within the limits of experimental error, the experimental decay curves agree with the theoretical decay curves, calculated on the assumptions that the three products A, B, and C are successive, and that their periods are respectively 3, 26, and 19 minutes.

(2) That radium B (as Schmidt observed) emits β rays of less penetrating power than those from radium C; and that on this account the β-ray decay curves are unsatisfactory for the purposes of analysis.

(3) That these β rays from radium B completely explain the divergence which Rutherford found between the experimental and theoretical β-ray curves.

In conclusion, I take pleasure in expressing my indebtedness to Professor Rutherford for his continued guidance throughout this work.

Macdonald Physics Building,
McGill University, Montreal.
May 1, 1906.

XIII. *On some Properties of the α Rays of Radiothorium.* (II.) *By* O. Hahn, *Ph.D.*

THE first communication on this subject (Phil. Mag. June 1906) dealt with the properties of the α particles emitted by the active deposit of thorium. It was shown that the active deposit consists besides thorium A of two α-ray products, thorium B and thorium C, which have quite distinct ionization ranges in air and other substances.

The present paper deals with the ionization ranges of the other α-ray products of thorium, namely radiothorium, thorium X, and the emanation. The method used, with the exception of the experiments with the thorium emanation, was very similar to that employed by Bragg and Kleeman† for the different radium products. The apparatus, however, had to be slightly altered for reasons described below.

In the case of thorium some special difficulties are at once encountered, as there are two essential conditions to be fulfilled in order to obtain well-defined ionization curves :—

(1) The material to be investigated has to be present in a very thin layer, so as to allow all the α particles to escape

* Communicated by Professor E. Rutherford, F.R.S.

† Bragg & Kleeman, Phil. Mag. Dec. 1904; and Sept. 1905.

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into the gas without appreciable absorption in the active material itself.

(2) The thin layer of the material has to be strongly radioactive, so as to obtain well-marked effects in the electrometer using a narrow ionization-chamber.

In the case of pure radium salts, on account of their extremely high activity, the condition (2) does not present any difficulty. In the present case, the strength of the radiothorium preparation available for this investigation was of the order of magnitude of about $\frac{1}{1000}$ of that of pure radium salts, with the exception of fig. 1, where a stronger preparation was used. One, therefore, could not expect to obtain as accurate results as in the case of radium.

Another difficulty arises from the presence of the emanation, which so readily escapes from solid radiothorium preparations. The emanation diffuses into the air some distance from the radioactive source, and, unless this emanation is removed, the ionization curves are not at all clearly defined, since the α particles from the emanation, which enter the ionization-chamber, come from varying distances above the radioactive source.

In order to avoid the disturbance caused by the emanation, the following arrangement was found to be fairly satisfactory. Instead of the little platform, on which the active material was placed, as described in the first communication, a funnel was used. A little dish, generally of porcelain, sometimes of platinum, contained the thin layer of the active material, and this dish was placed inside the funnel. Some strips of metal prevented the dish from fitting too closely to the walls of the funnel. The lower, narrow end of the funnel was connected with rubber-tubing, which passed through the bottom of the metal box to a water-pump. By means of the pump, a constant air-current was pulled through the funnel, and this air-current removed most of the emanation given off by the active source as fast as it was formed, and prevented the diffusion upwards of the emanation.

This arrangement proved satisfactory for large distances between the active source and the ionization-chamber. For short distances, disturbances due to the emanation could never be completely got rid of. The disturbance due to the emanation could be entirely avoided by hermetically covering the active dish with a thin uniform sheet of mica. This method was sometimes successfully employed. The sheet of mica reduces the range of ionization of the α particles in the air, and the stopping power of the α particles for the mica has to be determined in terms of air by a separate experiment. For

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this purpose, a wire coated with the active deposit of thorium was used. The ionization-distance curve was determined with and without the mica screen, in the manner explained in the previous paper. From these curves the decrease of range of the α rays in passing through the mica screen could be expressed in terms of the corresponding distance of air.

The ionization-chamber was the same as used in the case of the active deposit, the two plates being 0·5 centim. apart from each other. In order to obtain a fairly narrow cone of rays a great number of brass tubes, each 1·1 centim. high and 0·4 centim. in diameter, were placed side by side, connected together, and then placed over the active material. The cones of rays were narrow enough to allow the whole cross-section of the cone of rays for the greatest distance employed to enter the ionization-chamber.

For the reasons described above, it is not easy to determine with accuracy the range in air of the α particles emitted from the different thorium products. Since, however, the experiments in every case were repeated several times with freshly-prepared active material and under different conditions, the final results obtained are probably close to the truth.

Ionization Curve of a Radiothorium Preparation in Radioactive Equilibrium.

The ionization-distance curve was first obtained for a strong radiothorium preparation in radioactive equilibrium. A few drops of a radiothorium solution, activity over 100,000, were placed on a little flat porcelain dish and evaporated to dryness. The layer obtained was fairly thin.

If the α particles, expelled from each of the five α-ray products present in the radiothorium, had different ranges in air, theoretically one should expect to observe five distinct breaks in the ionization curve. The result obtained is shown in fig. 1 *a*.

Only two well-marked changes in the curve are to be observed. The upper part, beginning at about 8·6 centims. indicates the presence of thorium C, the range of which had been shown previously to be 8·6 centims. A second set of α particles appears to enter the ionization-chamber at a distance slightly less than 6 centims., but the point of change of the curve is not very clearly defined. The distance PQ represents the ionization due to thorium C alone (range 8·6 centims.). The distance PR represents the ionization due to the five α-ray products of thorium together, and is seen to be more than ten times as great as PQ. Theoretically PR should be equal to 5PQ, since five successive products

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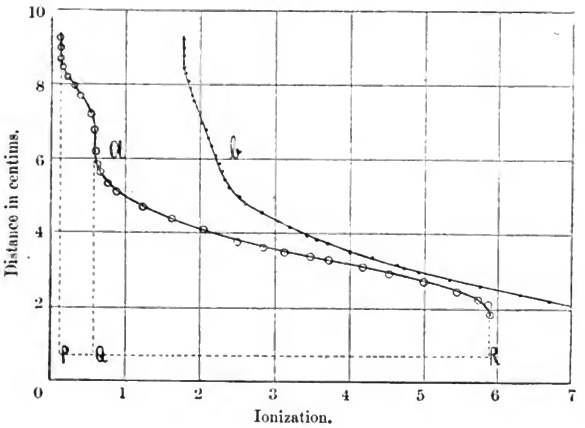
are present. The divergence between theory and experiment indicates that at least half of the emanation produced in the preparation escapes into the air.

The absence of any well-marked breaks in the curve renders it impossible to determine with certainty by this method the ranges of the various sets of α particles emitted by the products.

Fig. 1.



For comparison fig. 1 *b* shows a curve obtained from a radiothorium preparation in a thick layer. From the results obtained by Bragg and Kleeman with a thick layer of radium, one should expect to observe fairly abrupt changes in the slope of the ionization curve at points where a new set of α particles enters the ionization-chamber. The part of the curve due to thorium C is well-defined as in the case of a thin layer. A change of slope begins below 6 centims., but the rest of the curve does not give any definite information in regard to the ranges of the other products.

An examination of the ionization curves of radiothorium in equilibrium, whether in the form of a thin or of a thick layer, thus did not give any information of value in regard to the ranges of the α-ray products.

It was, consequently, necessary to obtain as far as possible each product free from mixture of the others, and to separately

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determine the range of the α particles emitted from it. This method of procedure, although somewhat laborious, proved successful.

Ionization Curve of Radiothorium itself, temporarily freed from all its products.

The radiothorium preparation used for this and the following experiments was not very active at first, for it contained a large amount of iron and other impurities. The activity was certainly not more than a few thousand times that of uranium. In order to obtain a product free from thorium X and containing as little inactive substance as possible, the following method was used. The acid solution was nearly neutralized by means of ammonia. A small precipitate fell down, leaving the greatest part of the thorium X and a large amount of iron &c. in solution. After filtering, the precipitate was dissolved in nitric acid, and the solution again treated in the same way, namely, ammonia was added to such an extent that only a small fraction of the former precipitate appeared, the greater part of the former remaining again in solution. This process of solution and precipitation was again repeated. The writer has stated * that in this way a concentration of the radiothorium may possibly be carried out, since the radiothorium seems to be carried down to a larger extent in the first part of an ammonia precipitate than in the latter part. On the other hand, no trace of thorium X was likely to be still contained in the very first precipitate, since thorium X remains in the solution. The nearly invisible precipitate on the filter was again dissolved in nitric acid and the solution evaporated in a flat porcelain dish. The layer obtained was very thin and showed comparatively strong α-ray activity. The ionization-distance curve was taken almost immediately afterwards. The result is shown in fig. 2, curve *a*. The curve has the characteristic shape to be expected for a single α-ray product. The ionization of the α rays begins at about 3·9 centims., increases rapidly, reaches a maximum and then decreases with the distance from the source. Another curve was taken three hours afterwards, and was practically the same.

We therefore have to conclude, that the α particles of radiothorium itself cease to ionize the air at a distance of about 3·9 centims.

That the curve is really due to radiothorium is seen by its later behaviour. Curve 2 *b* was obtained four days later

* *Jahrbuch für Rad. u. Elektr.* ii. 3, p 233.

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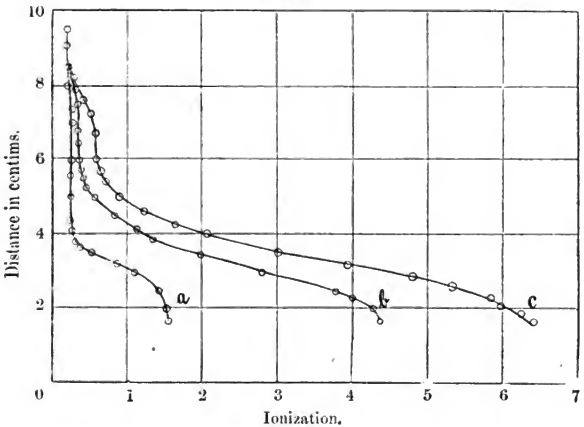
with the same preparation under exactly the same conditions. The shape of the curve is very different. The first effect of α particles begins at a much greater distance, namely, about 8·6 centims. From previous work, it is known that this new effect is due to the rays of thorium C. Below 6 centims. there is a change in the ionization curve, showing that the particles of another product add their effect. The α rays of this product have less range than those of thorium C, but more than those of thorium B and radiothorium. As there

Fig. 2.



is only the emanation and thorium X still unaccounted for, we conclude that this range belongs to the α particles of one of these products. Since a greater part of the emanation escaped from the preparation and was removed by the air-current, it appears probable that the range of somewhat below 6 centims. belongs to the α particles of thorium X. This conclusion is confirmed by later results.

In the four days' interval, more than half of the maximum amount of the thorium X is formed by the radiothorium. The thorium X produces the emanation, which in turn gives rise to thorium B and C. The ionization due to the latter should, consequently, be observed, and this we have seen is the case.

There must, naturally, be the same amount of thorium B

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as of thorium C present, but as the range of the α particles of the former is only 5 centims., its effect simply adds to that of the thorium X, without showing any distinct break in the curve.

Fig. 2, curve *c*, shows the curve obtained from the same product 21 days after preparation. The whole ionization effect is now more than four times as strong as at the beginning, the part due to thorium C and thorium X being far more marked than in curve *b*.

It will be seen that curve *c* in fig. 2 closely resembles curve *a* in fig. 1. This is to be expected, since both represent the ionization of radiothorium in equilibrium.

It is difficult to obtain accurately the slope of the curve close to the radiating source on account of the disturbances due to the released emanation. For example, the final bend of the curve *c*, fig. 2, could not be obtained with accuracy. It was for this reason that the curves in some cases are not continued closer to the radiating source. The disturbances due to the emanation are naturally most marked in strongly emanating substances, and increase rapidly as the ionization-chamber is brought close to the active matter. Since, however, it is the object of this paper to obtain the *maximum* range of the α particles from each particular product, it is of no great importance to determine the slope of the curve close to the active material.

Ionization Curve of Thorium X.

The thorium X used for the investigation of the ionization curve was obtained in the usual way by precipitating a thorium, or, in this case, a radiothorium solution with ammonia, the filtrate containing a large amount of the thorium X. In the case of radiothorium one does not at once succeed in obtaining a filtrate completely free from the radiothorium itself. The thorium X generally contains a small quantity of radiothorium, and its activity consequently does not decay to zero. If there is added to the filtrate a little iron solution, and the precipitation repeated, the separation of the radiothorium from the thorium X is more complete; but some of the thorium X is precipitated at the same time. Care has to be taken to use pure chemicals and good distilled water, since all solid and non-volatile impurities remain with the thorium X. The presence of such impurities renders it difficult to obtain the very thin layer which is required for the experiment.

After driving off the ammonia salts, the thorium X was

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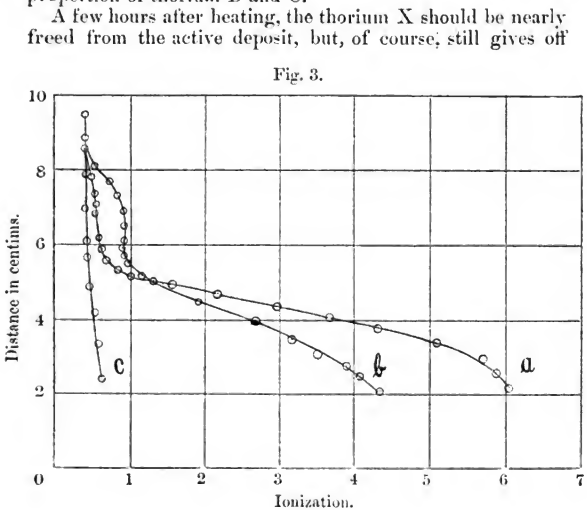
dissolved and the solution poured into a flat platinum cover, where it was carefully evaporated. Then the platinum cover was strongly heated for some time by means of a blowpipe. Thorium A was practically all driven off, and also a large proportion of thorium B and C.

A few hours after heating, the thorium X should be nearly freed from the active deposit, but, of course, still gives off

Fig. 3.



the emanation in a large amount. Curve A, in fig. 3, shows the ionization of thorium X under the above described conditions. It is seen that at a distance of about 5·7 centims. the ionization increases rapidly, showing that the α particles from a strong radioactive product enter the ionization-chamber at that distance. Above 5·7 centims., and continuing to 8·6 centims., there is a weak ionization due to a small quantity of thorium C, which still remained with the thorium X. The α particles of range 5·7 centims. undoubtedly belong to the thorium X, but it is difficult to fix with certainty the actual range closer than one or two millims., on account of the disturbances, even with a strong air-current, due to the large amount of emanation which is continuously released.

In order to see how far the emanation disturbed the accuracy of the determinations, the measurements were

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repeated, when the active matter was hermetically covered by a thin sheet of mica of absorbing power equal to 2·0 centims. of air. Allowing for the absorption of the mica, the maximum range was found to be practically identical with that observed with the unscreened source.

We may, therefore, conclude that the α rays of thorium X cease to ionize the air at a distance of about 5·7 centims. Curves *b* and *c*, in fig. 3, show the ionization curves 5 and 28 days later respectively. It is clearly seen that in this case, where the active material was strongly heated, not such a large fraction of the emanation escapes as in the preparation used to determine the curves of fig. 2. The part of the ionization curve due to thorium C is very clearly marked, showing that there is a greater proportion of the active deposit present in the heated thorium X than in the unheated radiothorium (fig. 2 *c*). As the thorium X decreases in activity, the ionization due to its successive products decreases in like ratio. The very small residual activity, represented in the curve *c*, fig. 3, shows that the thorium X was nearly free from radiothorium or radium.

Ionization Range of the α Particles of the Thorium Emanation.

On account of the strong emanating power of preparations of radiothorium or thorium X, the electrical method cannot be used with advantage to determine accurately the range of the α particles from the emanation. Even when the emanating preparation is completely covered by means of a mica screen, no distinct break in the curve due to the emanation could be observed, indicating that the range of the α particles of the emanation was not very different from those of the other α-ray products with a range below 6 centims.

The range of the α particles due to the emanation could, however, be measured by observing under special conditions the distance of air through which the α particles produced scintillations on a sensitive zinc-sulphide screen.

The general arrangement was the same as that used for the determination by the scintillation method of the maximum range of the α rays of the active deposit of thorium, as described in the first paper on this subject. The active wire was replaced by a shallow metal dish about 2·5 centims. square and a few millims. high. A thin mica screen, which is impervious to the emanation, was waxed down on the upper surface of the dish. Two narrow tubes were attached to the metal. A slow current of air, which was passed over a

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strong radiothorium preparation, carried the emanation slowly through the flat vessel and then escaped through a long rubber tube so as to completely remove the emanation from the neighbourhood of the screen.

As the air carried the emanation into the vessel the scintillations of the α particles, which had passed through the mica screen, could be easily observed on the little zinc-sulphide screen placed close above the vessel. By altering the distance between the vessel and the screen the distance at which the scintillations disappeared could be determined. It was, however, not very easy to fix this point, as the scintillations became very weak near the end of the ionization range of the α particles. The scintillations, finally observed, were due to the α particles coming from the top of the vessel, while the scintillations of the α rays from the emanation in the lower parts of the vessel had already disappeared.

This *gradual* falling off in the number and intensity of the scintillations makes an accurate determination of the range very difficult. In addition, there is another point of uncertainty. After a large amount of emanation has been blown through the vessel, the active deposit begins to appear to a small, but always increasing extent, and, as the rays of the active deposit of thorium contain the most penetrating α rays yet known, one soon notices a few scintillations at distances much greater than that which corresponds to the maximum range of the α particles of the emanation itself.

Consequently, the numbers obtained by the scintillation method for this maximum range are uncertain within small limits, but, as the results of several series of measurements were in good agreement, the value of the range observed is probably not much in error. For the arrangement used, the maximum distance, at which scintillations due to the α particles of the thorium emanation were still visible, was found to be 4·0 centims. But since the α rays had already passed through a thin mica screen, which corresponded in stopping power to 1·2 centim. of air, the maximum range in air of the α particles from the emanation by the scintillation method is $4{\cdot}0 + 1{\cdot}2 = 5{\cdot}2$ centims.

I pointed out in my previous paper that the electrical method is somewhat more sensitive than the scintillation method for determination of the range of the α particles. For example, the ranges in air for the α particles from radium C and thorium C are 7·06 and 8·6 centims., respectively, using the electrical method, while for the scintillation method the values found are 6·8 and 8·3 centims. The real range in

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air of the α particles from the emanation, if determined by the electrical method, would, consequently, be about 5·2 + 0·3 or 5·5 centims.

In the previous paper I determined the ranges of the α rays from thorium B and thorium C to be 4·7 and 8·6 centims. respectively. The range 4·7 centims. was deduced by direct inspection of the ionization-distance curve of the active deposit of thorium. Since, however, the curve due to the rays from thorium C slopes towards the origin at the point distant 4·7 centims., the range 4·7 centims. for thorium B is too small and is more probably 5·0 centims.

The ranges of the α particles from the five α ray products of thorium are collected in the following table. For comparison, the ranges of the α particles from the radium products determined by Bragg and Kleeman are also added:—

Product.	Range of the α particles in centims. in air at atmospheric pressure and temperature.	Product.	Range of the α particles.
Thorium	rayless.	Radium.....	3·50 centims.
Radiothorium ...	3·9 centims.	Emanation ..	4·23 ,,
Thorium X	5·7 ,,	Radium A...	4·83 ,,
Emanation	5·5 ,,	,, B...	only β rays.
Thorium A	rayless.	,, C...	7·06
Thorium B	5·0 centims.	,, D...	rayless.
Thorium C	8·6 ,,	,, E...	only β rays.
		,, F...	3·86 *

* The range of the α particles from radium F has recently been determined by Dr. Levin in this laboratory.

The results obtained in this investigation of the α rays from thorium are in complete harmony with the theory of absorption of the α rays advanced by Bragg and Kleeman. The α rays emitted by any single product all travel at the same speed and cease ionizing the gas fairly suddenly after passing through a definite distance of air. The α particles from the different products differ only in the speed of projection, which determines the range of the ionization of the α particles in air.

If we assume that the α particles emitted by the thorium and radium are identical in all respects, the results show that the α particles from the thorium products are, on the average, projected with greater velocity than the α particles from the radium products. In each case, the velocity of the α particles from the corresponding products radium C and thorium C

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is much greater than that of the α particles from the other products.

Note.—In the previous paper I mentioned that some experiments were in progress to settle definitely whether the activity of radiothorium decays appreciably with the time or not. It was stated that two standard watch-glasses showed a decrease of activity and that another sealed watch-glass was under investigation. The activity of all these specimens decays slowly, though apparently not very regularly, and sufficient time has not yet elapsed to give the period of decay with any certainty.

In addition, I wish to state that in the first paper on radiothorium in the *Jahrbuch der Radioaktivität und Elektronik*, there occurred a misprint, giving the time for radiothorium to be half transformed as 109 years. The number was meant to be 10^9 years, and gives the period of thorium itself, not of radiothorium.

In conclusion it is with much pleasure that I express my gratitude to Professor Rutherford for suggesting this work, for his kind assistance, and for the interest which he took in the experiments.

McDonald Physics Building,
McGill University, Montreal,
May 15, 1906.

XIV. *Notices respecting New Books.*

Die Fortschritte der Kinetischen Gastheorie. Von Dr. G. JÄGER, Professor der Physik an der technischen Hochschule in Wien. Mit 8 eingedruckten Abbildungen. Braunschweig: F. Vieweg und Sohn. 1906. Pp. xii + 122.

THIS monograph, which forms No. 12 of the series now being published under the general title of *Die Wissenschaft*, is one of the clearest expositions which we have come across of the kinetic theory of gases and its recent advances. It is just the sort of account to satisfy the demands of a physicist as distinguished from a pure mathematician. The kinetic theory has recently assumed fresh importance in view of the developments of the electronic theory. The first half of the book under review contains a delightfully lucid and remarkably simple account of the older portions of the kinetic theory, while the remainder is concerned with its more recent developments, and includes such subjects as the temperature-coefficient of viscosity, discontinuity of temperature in heat conduction, the intrinsic pressure of fluids, the viscosity of ideal fluids and the size of molecules. We can heartily

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recommend this useful and timely little work to all physicists, and hope that an English translation of it will before very long be forthcoming.

Baumert's Lehrbuch der Gerichtlichen Chemie. Zweiter Band: *Der Nachweis von Schriftfälschungen, Blut, Sperma, u.s.w.* *Von* Prof. Dr. M. DENNSTEDT *und* Dr. F. VOIGTLÄNDER. Mit 98 Abbildungen. Braunschweig: F. Vieweg und Sohn. 1906. Pp. vii+248.

THIS is the second volume of the second edition of Dr. Baumert's treatise on forensic chemistry, and deals with those aspects of the subject which are not of a purely chemical nature. The work is addressed not only to professional chemists and medical men, but also to lawyers and judges, and the educated public in general. For this reason, it is written in a style which is clear and simple, and as free from technicalities as the nature of the subject will allow. The greater portion of the work deals with the methods for detecting forgeries, and in this connection very special attention is devoted to photographic methods. The remaining two sections of the book deal with the detection of blood and blood-stains, and of sperm. The book is illustrated with a large number of instructive photographic reproductions of actual cases within the experience of the authors, and will be found extremely useful to all interested in the investigation of criminal cases.

Philosophisches Lesebuch. Zum Gebrauch an Höheren Schulen und zum Selbststudium. *Von* Dr. BASTIAN SCHMID. Leipzig: B. G. Teubner. 1906. Pp. viii+166.

To those who take a keen interest in philosophical questions, but who cannot spare the time to wade through the voluminous writings of the leading philosophers, or to sound the innermost depths of their metaphysical subtleties, we can warmly recommend the present volume, which will enable any intelligent reader to acquire at least a general notion of the trend of philosophic thought in all ages, from the earliest period of Greek philosophy to its most modern developments. The author has made a careful selection from the writings of the leaders of philosophic thought, and has, by means of judicious explanatory notes, welded the whole together to form a more or less connected account of the various schools of philosophy. It is certainly a bold and effective plan to allow each philosopher to plead his own cause, as it were, the author merely acting the part of introducer, and doing his best to make the meaning of each as clear as possible. The book contains some passages which form difficult reading, but the selection as a whole has been carried out admirably, and a perusal of the book should certainly have the effect of stimulating those with a metaphysical turn of mind to further study and research.

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Die Theorie, Berechnung und Konstruktion der Dampfturbinen. Von GABRIEL ZAHIKJANZ. Mit 23 Textfiguren. Berlin: M. Krayn. 1906. Pp. iv + 179.

ENGINEERS interested in the theory and design of steam turbines will find in the small book under review an admirable account of these subjects, the treatment being thoroughly scientific, and at the same time clear and practical, the author being careful not to introduce extraneous theoretical matter having no immediate connexion with the subject in hand. No very advanced knowledge of mathematics is assumed, and any reader having a knowledge of trigonometry and the elements of the calculus should find no difficulty in reading the book; in fact, one of its chief merits is that it appeals to a very wide circle of readers, and that the author takes so very little for granted. All the more important principles and rules of design established by the author are liberally illustrated by fully worked numerical examples.

The title of the book is, however, to some extent misleading: for in a book which professes to deal with the construction of turbines as well as their theory and design, one might not unreasonably expect to find some account of details of construction and examples of modern types of steam turbine. Instead of this, however, the part devoted to "construction" consists of a single concluding sentence, containing, no doubt, a sound moral precept, but hardly to be taken seriously as an account of the "construction" of steam turbines.

XV. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from Vol. xi. p. 616.]

February 16th, 1906.—J. E. Marr, Sc.D., F.R.S., President, in the Chair.

THE President delivered his Anniversary Address, in which he described the Influence of the Geological Structure of English Lakeland upon its present Features.—A Study in Physiography.

After a brief account of the light thrown upon the structure of Lakeland by the writings of Otley, Hopkins, Clifton Ward, Goodchild, and Strahan, he considered his subject under the following heads:—

I. Events prior to the Uplift which produced the Dome.
II. Production of the Dome.
III. Initiation of the Drainage-Lines.
IV. Effects of the Three Types of Rocks upon the Scenery.
V. Modification of old Drainage-Lines.
VI. Depression of the Outskirts.
VII. Effects of Meteorological Conditions. (1) General.
VIII. Do. Do. Do. (2) The Glacial Period.
IX. Conclusion.

Of the events prior to the dome-shaped uplift he laid greatest

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stress upon the movements of Devonian times, which had caused the Lower Palæozoic rocks to be affected by a series of fractures forming a roughly-rhomboidal network, the fissures being marked by belts of broken rock along their courses. These belts were spoken of as 'shatter-belts,' and their nature and distribution described.

He accepted Hopkins's view of the formation of a dome comparable in shape to a 'caddy-spoon' with the short handle to the east, and argued that a subsidiary uplift occurred over the site of the Howgill Fells and adjoining country, and other subsidiary uplifts possibly over the Skiddaw and Helvellyn tracts. Between these uplifts and the Pennine Chain were the depressions of Edenside on the north-east, and of Morecambe Bay and the neighbouring lowlands on the south-east.

He gave further reasons in support of the view, previously put forward independently by Goodchild and himself, that the uplift of the dome and the final movements of the Pennine Chain were of Tertiary date.

After commenting on the now generally-received view that rocks of New-Red-Sandstone age, and perhaps of later date, extended over the district, he discussed the nature of the radial drainage impressed upon these newer rocks during the uplift of the dome, and the removal of these rocks in the district itself by denudation, producing a superimposed drainage on the Lower Palæozoic rocks.

He then discussed the changes which took place in the valleys as the result of the imposition of the rivers upon the ancient rocks, and maintained that diversion of the river-courses had largely taken place owing to the easier erosion along the shatter-belts, describing in detail the cases of the Langdale, Duddon, and Borrowdale drainage-areas in support of his views.

In this part of the address the position of the principal hanging-valleys was indicated, and it was pointed out that there were two sets, namely those which 'mouthed' into valleys that had been deepened in softer rocks, and those which 'mouthed' into portions of main valleys that had been deepened along shatter-belts.

When discussing the effects of meteorological conditions he commented on hill-outlines, which he had elsewhere mentioned, where the upper parts of hill-slopes presented a convex outline towards west and south, and a concave curve towards east and north. This he attempted to explain as due to the more profuse growth of vegetation on the slopes facing west and south.

In discussing glacial changes, he admitted that in former papers he had over-estimated the importance of glacial dams in holding up the waters of lakes, and now recognized that Watendlath Tarn, Elterwater, and Thirlmere occurred in true rock-basins, and that other lakes of the district were therefore, in all probability, partly rock-basins.

He believed that some of his views would be proved capable of more than local application, and that they proved the importance of the study, on the part of the geographer, of geological details in addition to general geological structure.

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THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

AUGUST 1906.

XVI. *On the Experimental Determination of the Ratio of the Electrical Units.* *By* Lord RAYLEIGH, *O.M., Pres. R.S.**

AN able discussion of the principal determinations of the above quantity, usually denoted by v, has been given in the Reports of the Paris Physical Congress (1900) by H. Abraham—himself a contributor to the series. This ground it is unnecessary to retraverse, but I desire to place on record one or two suggestions which have occurred to me but which I may probably have no opportunity of myself putting into practice.

The most approved methods involve the construction either of a condenser or of an electrometer, of which in the first case the capacity, and in the second the potential, can be calculated in electrostatic measure. The first method, on the whole, offers the greatest advantages, and I preferred it when (about 1882, and with the advice of Prof. Stuart) the Cambridge condenser was designed †. In this method two currents are compared by a galvanometer. The first is that due to a given electromotive force in a resistance whose value is known in electromagnetic measure. The second is the intermittent current due to the same electromotive force charging n times per second a condenser whose capacity is known from the data of construction in electrostatic measure. The comparison may be conducted by the aid of Wheatstone's bridge.

* Communicated by the Author.

† For description see J. J. Thomson, Phil. Trans. 1883, p. 711; Thomson and Searle, Phil. Trans. 1890, p. 586.

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There are, however, one or two matters as to which doubts may arise. Thus it is essential that the commutator by whose action the condenser is periodically charged and discharged, should introduce no electromotive force on its own account. A more serious doubt hangs over the behaviour of the galvanometer. It is assumed that this instrument indicates exactly the *mean* current, whether the current be steady or intermittent. The principal error to be feared, arising from a somewhat oblique position of the needle and its temporary magnetization under the condensed charging currents, would be eliminated by reversing the battery. But is it certain that the axial magnetization remains constant, even when this axis is strictly perpendicular to the magnetic forces due to the currents *?

Another question relates to the leads connecting the condenser with the remainder of the apparatus. These must themselves have capacity, and the effect is easily allowed for † if the capacity is definite. It is here that a doubt arises. Consider for example the coaxial cylinders of the Cambridge condenser. When the condenser is to be in action, a leading wire is brought into contact with the inner surface of the inner cylinder. A rupture of this contact throws the condenser out of action; but whatever be done with the end of the lead, its electrical situation is not the same as before. It is only in very special cases, if at all, that capacities can be *added* by simply making contacts.

Passing on to the condenser itself, we may notice that in almost all cases it has been necessary to provide a guard-ring, on the principle first introduced by Lord Kelvin. This leads to complications, though perhaps not very serious ones. Thomson and Searle have shown how to allow for the guard-ring in the calculation of electrostatic capacity, and how to connect it with the bridge in the electromagnetic measurement. It is a further slight complication that the potential is not quite the same for the guard-ring and for the main part of the condenser.

It has occurred to me that a condenser, not very different from the Cambridge one, may be so arranged as not only to dispense with the guard-ring, but also to eliminate all questions connected with the capacity of the leads. The principle is that of the variable condenser described in Maxwell's

* It is possible that the difficulty arising from the uncertain behaviour of steel magnets might be obviated by the use of a galvanometer of the so-called d'Arsonval type. The string galvanometer of Einthoven (Drude, *Ann.* xii. p. 1059, 1903) would appear to be specially suitable.

† Thomson and Searle, *loc. cit.*

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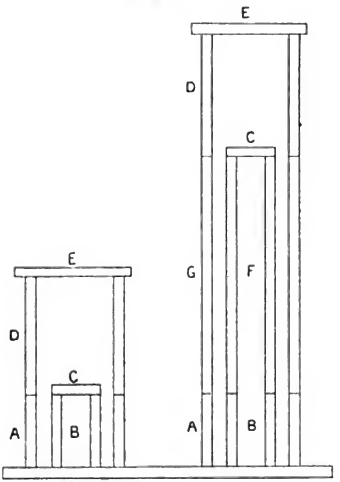
'Electricity' § 127, and further considered below. There are three outer A, G, D and two inner cylinders B, F, the components of two pairs being of the same length; and the outer surfaces of the inner cylinders and the inner surfaces of the outer cylinders being accurately worked to the same diameters. One pair A and B are mounted coaxially upon an insulating base and remain undisturbed. The other parts are movable and allow of the formation of two condensers. In the first of these the third outer cylinder D is mounted upon A so that the inner surfaces correspond. Upon the accurately worked top of B is placed a disk C of the same diameter, and D is also closed above by a plate E. The leads make contact with the cylinders A, B at their bases. Of this condenser and its leads the capacity is unknown.

Fig. 1.



In the second arrangement the long pair of cylinders F, G are interpolated, G resting directly upon A and F upon B, while C is removed so as to close F in place of B. The third outer cylinder D with its cover E now rests upon G instead of A. In this way we obtain a second condenser. Although its capacity is unknown, the *increase* of capacity is accurately that of the intermediate cylinders F, G considered as forming

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parts of infinitely prolonged wholes. That is, if l be the length, b the larger and a the smaller radius, the increase of capacity is $\frac{1}{2}l \div \log(b/a)$.

The circumstance that in this method the smaller capacity is much greater than that of the leads alone is scarcely an objection. In the approximate formula the electromagnetic capacity is proportional to the resistance of the opposite member of the Wheatstone quadrilateral, so that it is merely with the *difference* of resistances needed in this branch that we are mainly concerned. The resistance that must be *added* as we pass from one condenser to the other can be determined with full accuracy.

The length l and the smaller diameter $2a$ are readily measured. The inner diameter $2b$ of the outer cylinder is less easily dealt with; and even if the error were no greater than for $2a$, it would be seriously multiplied in $\log(b/a)$, which is approximately proportional to $(b-a)$. In the Cambridge apparatus the interval between the cylinders was intended to be found by gauging the space with water, and the process is described by Thomson and Searle (p. 600). If this plan be adopted, there is no need to measure b otherwise. If v be the included volume,

$$v=\pi l(b^2-a^2),$$

and

$$C=\frac{\frac{1}{2}l}{\log(b/a)}=\frac{l}{\log\left(1+\frac{v}{\pi l a^2}\right)},$$

or approximately

$$C=\frac{\pi l^2 a^2}{v}.*$$

It is to be remarked that by this method we determine what we really require, *i. e.* the *mean* value of $b-a$.

In carrying out the necessary measurements there should be no difficulty over l or a. The evaluation of v is more troublesome, and the principal uncertainty would seem to arise out of the possible presence of air-bubbles. Thomson and Searle used a vacuum towards the later stages of the filling. Perhaps it would be an improvement to have a vacuum (? from carbonic acid) from the first, and to introduce the previously boiled water from *below*. It would be

* In the Cambridge condenser $l=61$ cm., $2a=23\frac{1}{3}$ cm., and $2b-2a=1{\cdot}1$ cm. I do not know that these dimensions are susceptible of much improvement.

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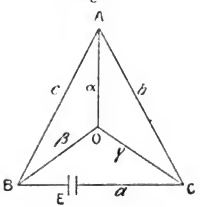
possible, though probably more elaborate, to determine v without water by the behaviour of included air.

The investigation of the formula for the electromagnetic measure of the capacity as derived from observations with Wheatstone's bridge is given in Maxwell's 'Electricity,' §§ 775, 776, but so succinctly that the full bearing of it may easily be misunderstood. Thus Thomson * speaks of it as "only an approximation," and substitutes a fuller treatment. After pointing out that in simple circuit the combination of commutator (period T) and condenser (capacity C) is equivalent to a resistance R, where $R=T/C$ †, Maxwell proceeds to consider the bridge arrangement. "Let us suppose that....a zero deflexion of the galvanometer has been obtained, first with the condenser and commutator, and then with a coil of resistance R_1 in its place, then the quantity $T \div [2]C$ will be measured by the resistance of the circuit of which the coil R_1 forms part, and which is completed by the remainder of the conducting system including the battery. Hence the resistance R, which we have to calculate, is equal to R_1, that of the resistance-coil, together with R_2, the resistance of the remainder of the system (including the battery), the extremities of the resistance-coil being taken as the electrodes of the system."

"Using the notation of Art. 347 [see figure], and supposing the condenser and commutator substituted for the conductor AC in Wheatstone's Bridge, and the galvanometer inserted in

Fig. 2.



OA, and that the deflexion of the galvanometer is zero, then we know that the resistance of a coil, which placed in AC would give a zero deflexion, is

$$b=\frac{c\gamma}{\beta}=R_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* Phil. Trans. 1883, p. 708.

† Maxwell has 2 C in place of C, inasmuch as he supposes the charge of the condenser to be *reversed* instead of merely annulled.

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The other part of the resistance, R_2, is that of the system of conductors AO, OC, AB, BC, and OB, the points A and C being considered as the electrodes. Hence

$$R_2 = \frac{\beta(c+a)(\gamma+\alpha)+ca(\gamma+\alpha)+\gamma\alpha(c+a)}{(c+\alpha)(\gamma+a)+\beta(c+a+\gamma+\alpha)} \quad . \quad (4)^*$$

"In this expression a denotes the internal resistance of the battery and its connexions, the value of which cannot be determined with certainty; but by making it small compared with the other resistances, this uncertainty will only slightly affect the value of R_2.

"The value of the capacity of the condenser in electromagnetic measure is

$$C = \frac{T}{[2](R_1+R_2)} \quad . \quad . \quad . \quad . \quad . \quad (5)"$$

Apart from the difference of notation, (5) is the same as the formula arrived at by Prof. Thomson. Maxwell's idea would appear to have been that it makes no difference to the galvanometer in OA whether in AC we have the resistance R_1, which gives the ordinary balance, or the commutator and condenser, provided that the condition be satisfied that the same integral current passes from A to C in both cases †. In considering the fulfilment of this condition we must remember that the difference of potential $(A-C)$ at A and C under the steady current is not the same as that $(A'-C')$ to which the condenser is charged. The latter corresponds to the rupture of AC, so that no current there passes. The condition may be expressed:

$$\frac{\text{Capacity}}{T}(A'-C') = \frac{A-C}{R_1};$$

and what we have further to consider is the relation between $A'-C'$ and $A-C$.

Let E' be the electromotive force which must act in R_1 in order to stop the current in it. Then $E'=A'-C'$. From another point of view the zero current in AC may be regarded as the resultant effect of two independent electromotive forces E, E' acting in the system composed of R_1 and the other

* In Maxwell's statement a and α are interchanged in the first term of the denominator.

† *E. g.* in the case of steady currents the introduction of an electromotive force into AC has no effect, provided the resistance of that branch be so altered as to satisfy the above condition.

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resistances. Thus

$$\frac{E'}{R_1+R_2}=\frac{A-C}{R_1},$$

so that

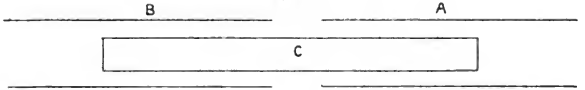
$$\frac{A'-C'}{R_1+R_2}=\frac{A-C}{R_1}.$$

And

$$\frac{\text{Capacity}}{T}=\frac{1}{R_1+R_2}, \text{ simply.}$$

But although the condenser method may be the best, it is not so perfect but that a desire remains to see results so obtained confirmed otherwise. The construction of an absolute electrometer is beset with difficulties, some of which have been remarked upon by M. Abraham. In point of theory the best arrangement is that described by Maxwell ('Electricity,' § 127, 1873) in which (fig. 5) an inner cylinder

Fig. 5.



C moves coaxially in the interior of fixed coaxial cylinders A, B. It will suffice to suppose that C and A are at potential zero while B is at electrostatic potential B.

"The capacities of the parts of the cylinders near the [gap] and near the ends of the inner cylinder will not be affected by the [motion] provided a considerable length of the inner cylinder enters each of the hollow cylinders. Near the ends of the hollow cylinders, and near the ends of the inner cylinder *, there will be distributions of electricity which we are not yet able to calculate, but the distribution near the [gap] will not be altered by the motion of the inner cylinder provided neither of its ends comes near the [gap], and the distributions at the ends of the inner cylinder will move with it, so that the only effect of the motion will be to increase or diminish the length of those parts of the inner cylinder where the distribution is similar to that on an infinite cylinder."

* A solution of this problem for the case of *two dimensions* has been given by Prof. J. J. Thomson (Recent Researches, § 237, 1893).

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Thus if a be the radius of the inner cylinder and b of the outer, the force with which the former is drawn into B is

$$F = \frac{\frac{1}{4}B^2}{\log(b/a)}.$$

It appears that F depends only on the *ratio* of the diameters of the cylinders B, C. Suppose for example that $2a=2$, $2b=4$ (perhaps in inches), then $\log(b/a)=\log 2=\cdot 69$. If the potential B correspond to 2000 volts,

$$B = \frac{2\times 10^{11}}{v} = \frac{2\times 10^{11}}{3\times 10^{10}} = \frac{20}{3},$$

and $F=16$ dynes, or mgs. weight. This is rather small; but since $F \propto B^2$, we get 64 mgs. for 4000 volts, and 144 mgs. for 6000 volts.

As regards the effect of errors in the fundamental measurements b, a, we have if $y=\log(b/a)$,

$$\frac{dy}{y} = \frac{db}{by} + \frac{da}{ay};$$

or with the above proportions

$$\frac{dy}{y} = \frac{d(2b)}{\cdot 7(2b)} - \frac{d(2a)}{\cdot 7(2a)}.$$

The outer cylinder is the more difficult to measure, but a given absolute error in it is less important. If we suppose $2b=4$ inches, and $d(2b)=\frac{1}{1000}$ inch,

$$\frac{d(2b)}{\cdot 7(2b)} = \frac{1}{3000} \text{ about,}$$

and the proportional error in y is halved when we pass to that of B.

It should be borne in mind that what we have to do with here is *not* the *mean* diameters of the cylinders, if such diameters vary.

This form of absolute electrometer was employed in the researches of Hurmuzescu *, who mounted the cylinders on a torsion balance, so that the motion was horizontal and not strictly axial. Some advantages are attained in this arrangement, especially perhaps that of being able to reverse the force and so to double the rather inadequate value of the subject of measurement; but upon the whole it appears to

* *Ann. d. Chim.* x. p. 433, 1897. This author erroneously attributes Maxwell's reasoning above, by which the unknown parts of the electrical distribution are eliminated, to Bichat & Blondlot (1866).

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me preferable to suspend the moving cylinder C vertically in an ordinary balance, C and the upper fixed cylinder A being in connexion with earth by wires which may be very fine*. The force is then evaluated in gravitation measure; and it may of course in effect be doubled by duplicating the cylinders on the other side of the balance.

When we come to actual design the question at once obtrudes itself as to how long the cylinders really need to be. In theory it is easy to treat them as infinite, but in practice some concession must be made. In particular the weight of C must not be increased unnecessarily.

The penetration of potential arising at the gap between B and A into the annulus between C and A is easily investigated. For practical purposes it suffices to treat the problem as in two dimensions. If r be the radius and z (the axial co-ordinate) be measured from the end of A, the potential V in the annulus may be taken to be

$$V = \Sigma H e^{-m\pi z/(b-a)} \sin \frac{m\pi(r-a)}{b-a},$$

where m is an integer and H an arbitrary constant variable with m. At the surfaces of the cylinders where $r=a$ or $r=b$, V vanishes. The term whose influence extends furthest is that for which $m=1$. Limiting ourselves to this, we take, since $e^{\pi}=23{\cdot}2$,

$$V = H_1 (23{\cdot}2)^{-z/(b-a)} \sin \frac{\pi(r-a)}{b-a},$$

showing that when $z=b-a$ the value of V is already reduced to one twenty-third part of that at $z=0$. When $z=4\ (b-a)$, that is at a distance from the end equal to four times the thickness of the annulus, this term is attenuated 290,000 times, and it is safe to conclude that the whole disturbance of potential may be neglected. A similar argument applies to the annulus between B and C; so that a total length of 8 or 9 times the thickness of the annulus—8 or 9 inches in the example spoken of above—should amply suffice.

There is less objection to increasing unnecessarily the length of the fixed cylinders, but even here there must be some limit. It is easy to see that the prolongation of the upper cylinder A at zero potential above C, also at zero potential, is of little importance. But the case of the charged cylinder B requires further consideration.

* Further screens in connexion with earth may be introduced to protect the moving parts of the balance more effectually from the influence of the electricity upon B.

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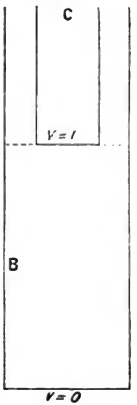
For the sake of definiteness it is convenient to suppose B provided with a metallic bottom as in the figure annexed, and the question is as to the effect of the finite distance l of this bottom. If we regard the bottom as a moveable piston at the same potential (taken as zero) as the cylinder B which it closes, we may regard it as mechanically attached to the suspended cylinder C, in spite of electrical attachment to B. In this case, *i. e.* when C and the bottom of B move together as a rigid body, Maxwell's argument applies exactly as if the cylinder B were infinite. The correction of which we are in search is therefore equal to the electrical force of attraction which tends to draw the piston towards C.

Fig. 3.



The potential V in the interior of B is expressible by means of Bessel's functions in terms of the values of V over the plane which includes the bottom of C. Thus, if r be the radius vector and z the vertical axial coordinate measured downwards from the above plane, we may write

$$V = \Sigma A J_0(kr) \sinh k(l-z),$$

where k has in succession a series of values such that

$$J_0(kb) = 0,$$

b being the radius of B. Each term in the above satisfies Laplace's general equation and reduces to zero on the walls and on the bottom of the cylinder. If σ denote the density of electricity on the bottom $(z=l)$, we have

$$4\pi\sigma = dV/dz(=l),$$

and for the force of attraction upon the bottom as a whole

$$\text{Force} = \frac{1}{8\pi}\iint\left(\frac{dV}{dz}\right)_l^2 dS,$$

dS representing an element of area. Now

$$\frac{dV}{dz(=l)} = -\Sigma k A J_0(kr);$$

and thus, since the products of the various terms must vanish

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when integrated over the circle $r=b$,

$$\text{Force}=\frac{1}{4}\int_0^b r\,dr\Sigma k^2A^2[J_0(kr)]^2=\tfrac{1}{8}\Sigma k^2b^2A^2J_0'^2(kb),$$

by a known theorem. The values of kb are 2·404, 5·520, 8·654, &c.

When $z=0$, $\quad V=\Sigma AJ_0(kr)\sinh kl$;

and the values of A can be found if we know that of V over the whole circle $r=b$. As usual we have

$$\int_0^b VJ_0(kr)r\,dr=A\sinh kl\int_0^b J_0^2(kr)r\,dr$$
$$=\tfrac{1}{2}b^2A\sinh kl\,J_0'^2(kb);$$

so that

$$\text{Force}=\Sigma\frac{\frac{1}{2}k^2\left[\int_0^b VJ_0(kr)r\,dr\right]^2}{b^2\,.\,\sinh^2 kl\,.\,J_0'^2(kb)}.$$

This would determine the force if we knew the value of V over the whole circle $r=b$. We know that $V=1$ from $r=0$ to $r=a$, and that from $r=a$ to $r=b$ it falls from 1 to 0, but we do not know the precise law of this fall. A pretty good estimate of the integral would be made by taking $V=1$ up to a radius $\frac{1}{2}(a+b)$ and afterwards $V=0$. In this case we get*

$$\int_0^b VJ_0(kr)r\,dr=-\frac{\frac{1}{2}(a+b)}{k}J_0'[\tfrac{1}{2}k(a+b)];$$

and

$$\text{Force}=\Sigma\frac{(a+b)^2J_0'^2[\frac{1}{2}k(a+b)]}{8b^2\,.\,\sinh^2 kl\,.\,J_0'^2(kb)}.$$

If the object be merely to find an upper limit to the force of attraction, we may suppose the value $V=1$ to extend up to $r=b$, and this in practical cases will not really alter the result very much. Writing $a=b$ in the above formula, we get the simple expression

$$\text{Force}=\tfrac{1}{2}\Sigma[\sinh kl]^{-2}.$$

This is the force with which the piston is attracted towards C when the potential-difference is unity, and it expresses the excess of the force by which C is drawn in over that which

* See for example 'Theory of Sound,' § 204, equation (8).

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would act if $l=\infty$. In the case above considered of $b=2a$, the latter is equal to about $\frac{1}{3}$.

As has been stated, the principal value of kl is $2{\cdot}404\ l/b$. If we suppose that the distance between C and the bottom of B is equal to the diameter of the latter, $l=2b$, and sinh ($4{\cdot}808$) may be identified with $\frac{1}{2}e^{4{\cdot}808}$, while the other terms corresponding to higher values of k may be neglected. Hence

$$\text{Correction to force} = 2e^{-9{\cdot}6} = (6500)^{-1}.$$

This compares with $\frac{1}{3}$: so that if the correction be neglected altogether the error would be less than 1 in 2200. This error, although finally halved, is too great. It would be necessary either to increase the value of l a little, or to calculate the correction and allow for it.

Perhaps the weakest point in the use of an absolute electrometer on these lines is the rather high potential required to attain the necessary sensitiveness. There should be no difficulty over $\frac{1}{20}$ mg., especially if the necessary changes could be made without taking the moving parts of the balance off their knife edges. But even then 2000 volts would scarcely suffice, and it is likely that 3000 or 4000 would prove necessary*.

The objection to a high potential is not so much the difficulty of obtaining it with steadiness as the risk of a brush-like discharge through the air, the occurrence of which would probably be fatal to the success of the measurements. In order to diminish the risk, the edges of the cylinders A, B, C should be rounded off, as can be done without theoretical objection.

I scarcely know whether the necessity of measuring a high potential in electromagnetic measure (say in volts) is to be regarded as a disadvantage in this method of determining v. It would seem that such measurements are needed in any case and that they constitute a separate problem.

Upon the whole, while still disposed to give the preference to the condenser, I am of opinion that the electrometer method is worthy of further trial.

Terling Place, Witham, June 1906.

* Hurmuzescu employed about 2000 volts.

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XVII. *The Effect of Electrical Oscillations on Iron in a Magnetic Field.* By W. H. Eccles, *D.Sc., A.R.C.S.**

THE following investigation of the action of high-frequency oscillations upon iron held magnetized in a magnetic field was carried through in July and August of last year, and was undertaken to supplement in a quantitative manner the rather vague information available. Since Marconi † announced in 1902 that feeble electrical oscillations were capable of altering the flux of induction in a piece of magnetized soft iron, a great deal of work has been done in the endeavour to determine what it is that really happens in this process. A glance through the chronicles of these labours reveals a surprising lack of precise data, and shows, in fact, that most of our knowledge of this subject is merely qualitative. This is due, no doubt, to the complicated possibilities accompanying any experimental method that can be devised. One, for instance, among these difficulties, is the shielding of the greater mass of an iron wire, or other sample, by the eddy currents that are generated in the metal by the oscillations. This skin-effect was, in a way, overcome by Maurain by using oscillations sufficiently vigorous to penetrate to the very core of the iron or steel wire experimented upon. By this means he has shown that if the I H curve of a sample be drawn in the ordinary manner while these very vigorous oscillations are in operation, the curve obtained is not looped but is the single path obtained by the coalescence, as it were, of the branches due to increasing and decreasing fields. The single curve thus obtained he called, after Duhem, the "normal curve of magnetization." Duhem had previously shown from thermodynamical reasoning that the effect of a long train of slowly damped oscillations superposed on iron at any stage of the cyclic process would be to bring the representative point on the I H diagram towards a unique locus—a locus, he concluded, exactly the same as that obtained by mechanically agitating the iron throughout the cycle. This theory of Duhem's takes no account, of course, of the shielding of the inner parts of the iron from the action of the oscillations; and thus cannot be demonstrated experimentally except by using, as Maurain ‡ has done, very powerful oscillations.

In the experiments to be described, an endeavour has been made to turn the difficulty arising from the skin-effect. Oscillations so feeble have been used that they affected only

* Communicated by the Physical Society: read June 22, 1906.

† Roy. Soc. Proc. lxx. p. 341, July 1902.

‡ Maurain gets slightly different curves by mechanical agitation and by oscillations.

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the outermost layers of the iron wires employed. The cores of the iron wires have therefore not been used.

Other and great difficulties arise in the matter of producing oscillations of determinate and invariable character. Maurain, in the greater part of his work on this subject, appears to have used the oscillations that passed through a helix in series with a Leyden jar kept sparking strongly and continuously by means of an induction-coil. Russell *, in some recent experiments, applied to his iron the oscillations passing through a coil connected directly in series with a small induction-coil. This last method appears to the writer to subject the iron to very violent treatment of a nature not easily described accurately; for how far the mere surgings of secondary current overwhelm in importance the genuine oscillations, it will be difficult to say. Piola †, again, worked with highly damped oscillations, because, he found, such oscillations produce the highest effects. He has, indeed, following Rutherford, used this fact in determining the damping factor of a circuit. But in the present investigation these sources of indeterminateness have, as far as might be, been avoided by using a single train of waves instead of a continued torrent of such trains, and by using oscillations as little damped as possible.

It would not be to the point to extend this brief survey of previous work till it included that of Walter, of Ascoli, of Arnò, and of Foley; for all these have paid especial attention to oscillations whose magnetic field was in some degree transverse to the main magnetic field. The present paper deals solely with oscillations whose magnetic field is along the direction of the principal field.

In these experiments fairly soft Swedish charcoal iron, aged, and not freshly annealed, was taken a large number of times round any chosen magnetic cycle (as in ordinary magnetic testing) till what may be called a cyclic state was attained. The field was then given any desired value, and the iron submitted to a single train of oscillations. This was managed by generating the oscillations on a helix surrounding the iron wire. The consequent alteration in pole strength was observed by the deflexion of a magnetometer mirror. These processes were all repeated a number of times for each selected point of the cyclic curve. The figures given in the tables below are thus each the mean of a number of observations.

In detail, the apparatus adopted consisted of two straight

* Proc. Roy. Soc. of Edin. Nov. 20, 1905.

† *Elettricista*, iv. p. 145, May 15, 1905.

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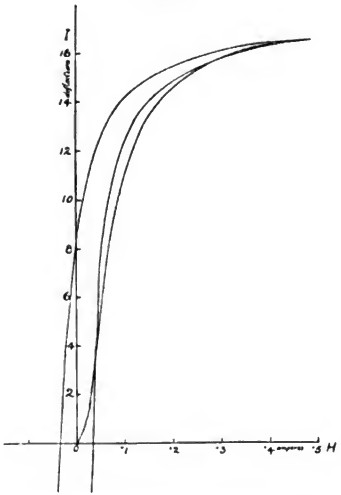
solenoids each of 3270 turns of No. 20 copper wire wound in 6 layers on a length 59 cm. of split brass tube. They were placed horizontally on one and the same magnetic east-west line, but on either side of a mirror magnetometer. The magnetometer-needle was in the common axis of the two solenoids. The solenoids were connected in series, and so adjusted in position that when a large current was passed through them the magnetometer-needle was undisturbed. A liquid resistance, a Weston milliammeter, suitable switches and commutators, and a battery of six secondary cells, completed the solenoid circuit. The coil destined to be the seat of the oscillations consisted of 1252 turns of No. 26 copper wire wound in a single layer on 70 cm. of a glass tube 0·5 cm. in external diameter, and had a resistance of about 3 ohms. The whole coil was wrapped in paraffined paper and pushed into the brass tube of the east solenoid.

Fig. 1.



The iron wire examined was unannealled Swedish charcoal iron, diameter 0·749 mm., and was used always in lengths of 56 cm. Its characteristic curve is given in fig. 1. The magnetometer was a silk-suspended mirror carrying four very

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small magnets; readings were taken on a scale distant 88 cm.

The object in employing twin solenoids is clear. By placing equal amounts of iron wire in each solenoid and adjusting their positions carefully, the magnetometer deflexion could be kept very slight whatever magnetic variations the iron was taken through, the iron in the one solenoid compensating that in the other.

It was then permissible to exalt greatly the sensibility of the magnetometer. This was done by reducing the controlling field at the needle by means of an auxiliary permanent magnet. As a matter of fact, slight inequalities existing in the construction of the solenoids made perfect balance unattainable; there was always a remanent deflexion, which was different in sign and magnitude for different values of the current through the solenoids, and which in the main proved rather useful by furnishing a check on the proper carrying out of the commutating operations involved in a magnetic cycle.

In the experiments, three iron wires insulated from one another and tied in a bundle were used in each solenoid, the group in the east coil lying, of course, in the glass tube on which the oscillation-coil was wound. The inner ends of the wires were 9·3 cm. distant from the magnetometer needle. The field at the needle was reduced to 0·034 C.G.S. units. With these arrangements, such a sensibility was attained that on certain days of August last the strokes of distant lightning, not visible in London *, were easily perceptible in the laboratory by their effects on the iron in the oscillation-coil.

The oscillations used were produced by leading the free east end of the oscillation-coil, that is the end distant from the magnetometer, to one side of a micrometer spark-gap. The other end of the oscillation-coil was left insulated. The poles of the spark-gap were connected to the terminals of a diminutive influence machine, that pole not connected to the oscillation-coil being, besides, earthed. Thus, when the handle of the influence machine was turned through a certain angle (depending on the spark-gap) at a speed easily learned, a spark occurred which set up oscillations in the coil. The spark-length finally settled upon was about half a millimetre. It was found possible in this way to get over and over again practically the same magnetometer deflexion for every spark, provided the effect of previous oscillations was wiped out by taking the iron through a cycle. As these small sparks were usually inaudible, their occurrence was recognized by the sudden deflexion of the magnetometer-needle.

* The newspapers gave accounts of thunder-storms in Hertfordshire.

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The calculated period of the stationary waves on the oscillation-coil is $5{\cdot}7 \times 10^{-7}$ second.

A typical set of operations was as follows. The iron was demagnetized by reversals. An amplitude of cycle being then decided upon, the iron was taken a number of times through that cycle. This last operation was stopped at the point settled upon for the observations, and the magnetometer reading taken. A spark was passed, and the magnetometer again read after 30 seconds. A complete cycle was now performed ending at the same point as before, and another spark passed, and so on. Usually, about twenty points on a cycle were examined by 4 or 6 observations at each. In this paper, however, the effects of the spark are recorded as if the observations had been taken only on the ascending half of the hysteresis curve; the figures given being in fact the means of the measured effects at points symmetrical with regard to the origin on the ascending and descending branches.

TABLE I.

Current in milliamperes.	−150	−100	−75	−50	−40	−30	−20	0	20	30	40	50	75	100	150	200
50 cycle	...	...	...	...	0·06	0·10	0·13	0·28	0·38	0·45	0·60	0·55				
100 cycle ...	...	...	...	0·27	...	...	...	0·68	1·08	...	...	1·26	1·30	1·18		
150 cycle ...	...	0·20	...	0·44	...	...	...	0·92	...	1·32	...	1·51	1·62	1·57	1·19	
200 cycle ...	0·20	0·36	...	0·68	...	...	...	1·22	...	1·57	...	1·71	1·75	1·55	0·93	0·83

In this table the figures given can be reduced to absolute measure as follows:—The numbers at the top of each column and down the left column, representing the current through the solenoids in milliamperes, yield when multiplied by 0·0696 the magnetic field in C.G.S. units applied to the iron. The deflexions in the body of the table give the change in pole-strength, due to the spark, by multiplying by 0·017; or yield the volume-average change of intensity of magnetization by the factor 1·27; or give change of total magnetic moment of the affected specimen by the reducing factor 0·940.

Fig. 2 is plotted from the above table. The curves show clearly how for increasing cyclic amplitudes, within the range here attempted, the effect of the same spark is increased. The curves show very distinct maxima. It is evident, moreover, that the magnitude of the effect at any point is closely

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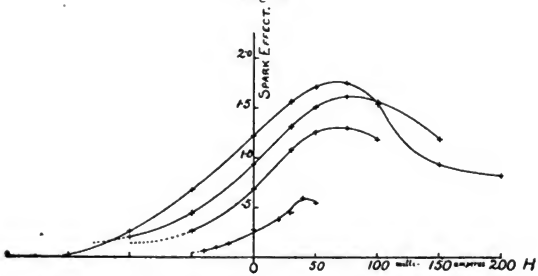
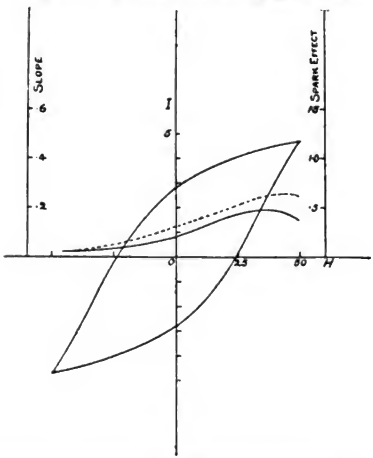
connected with the slope of the hysteresis curve. On the whole the curves tend to corroborate the fact—first noted by

Fig. 2.



E. Wilson in his 1902 patent specification—that the sensibility of the iron to oscillations is greatest when in the magnetic condition represented by the point of inflexion on the hysteresis curve. To examine this matter more closely

Fig. 3.—The dotted curve shows spark-effect.



these curves representing the effect of the spark are repeated in figs. 3, 4, 5, 6, each alongside the gradient curve, that is,

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Fig. 4.—The dotted curve shows spark-effect.

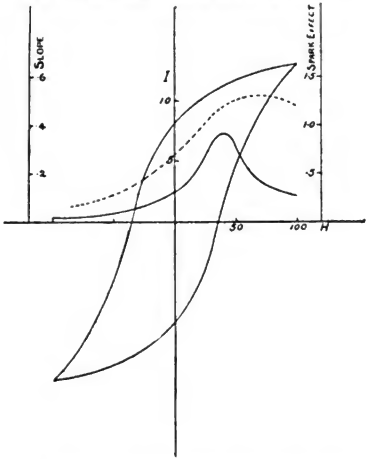
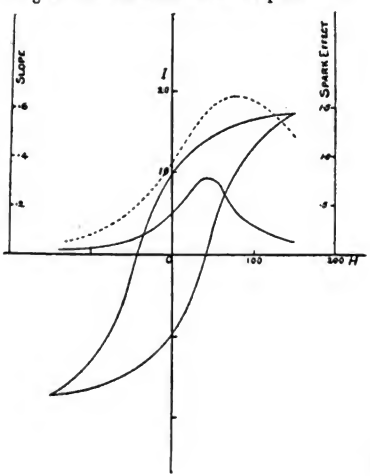


Fig. 5.—The dotted curve shows spark-effect.



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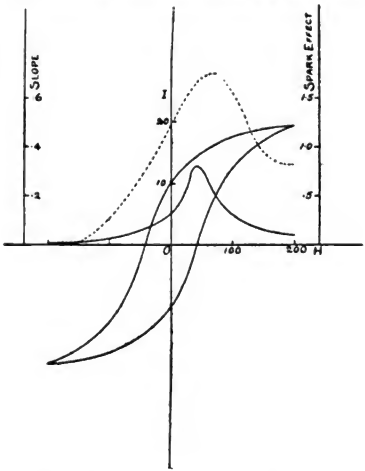
first derived curve, of the corresponding cyclic curve. To assist in picturing the magnetic state of the iron there is plotted on each diagram the proper hysteresis loop. These loops were obtained by independent experiments with the earth's field controlling the magnetometer-needle. (Reducing factor for I is 67·5).

Fig. 6.—The dotted curve shows spark-effect.



In these figures the abscissæ for all the curves are milliamperes (multiply by 0·0696 to get C.G.S. field). The figures have been so constructed that, except in the cyclic curves, equal ordinates on the different figures represent absolutely equal effects. The reducing factors for the spark-effect curves are as given in connexion with Table I. In all the slope curves the ordinates give at once very approximately the change of intensity of magnetization per ·001 C.G.S. increment of field. But the ordinates of the cyclic curve are reduced 21 times in fig. 3, 42 times in fig. 4, 63 times in fig. 5, 84 times in fig. 6; and are thus not directly comparable with one another or with the companion curves.

Returning to the spark-effect and the gradient curves, however, it is seen that the spark-effect is by no means a

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simple function of the gradient of the cyclic curve. The curves show, moreover, that the maximum effect of the spark is greater in the cycles of larger amplitude; and not only because the hysteresis curves are steeper in large cycles than in small, but also because in small cycles the effect of the oscillations is, for some unknown reason, much smaller than the diminished gradient would lead one to expect.

It must be mentioned here that during the above measurements the effect of changing the sign of the initial charge given to the oscillation-coil was repeatedly tried. On no occasion was there any perceptible difference in the magnetometer deflexion. The damping of the oscillations must therefore have been very trifling.

A matter quite different, but one of considerable interest, is examined in the following short table of the effect on the magnetometer deflexion of varying the length of the spark-gap from which the oscillation-coil was excited. The observations here recorded were all made at that point of the 0·200 ampere cycle where the magnetic field is zero; observations made at other points of this or other cycles gave results the same in character.

TABLE II.

Length of Spark-gap in centimetres.	Magnetometer Deflexions.
0·024	0·8
039	0·8
053	1·1
070	2·6
087	4·4

This table is plotted in fig. 7. On the same diagram appears the curve of Paschen's* observations of potential difference and spark-length.

The most important question to which the results set forth in this paper can be applied, is that bearing on the real nature of the phenomena observed. A number of observers have summed up their views by the phrase "hysteresis is annulled by oscillations." The present experiments at first sight bear out that conclusion anew. The effect of the oscillations at every point of the cyclic curve is such as to carry the representative point towards the curve that would be yielded by a substance

* Table in J. J. Thomson's 'Recent Researches,' p. 78.

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devoid of hysteresis. This might be interpreted in the same manner as is, usually, the effect of mechanical vibration on iron undergoing varying magnetic forces—the intermolecular bonds that hold the magnetic molecules in their instantaneous configuration are loosened for a moment so that the resultant field can have full play in rearranging the molecular groupings. Looking further into the matter, however, on this view that hysteresis is temporarily annulled, it is evident that on the ascending loop of the cyclic curve the external applied field is permitted to do work when the magnetic shaking occurs. On the other hand, similar alterations of intensity of magnetization occur when the applied field is zero; and, in addition, on the descending loop of the cyclic curve there are alterations in intensity opposite in sense to the applied field. It seems clear, then, that there must exist in the magnetized material intrinsic forces that tend to drag the substance into that presumably more stable magnetic state defined by the "normal curve of magnetization" before mentioned. Thus this view that hysteresis is annulled, taken with the experimental facts of this paper, implies that the store of intrinsic energy of magnetization may possibly be allowed to run down through and during the action of the oscillations. In other words, the action of the oscillations may be analogous to the pulling of a trigger. These considerations threaten to lead us into the often discussed question of the intrinsic energy of a magnetized material having hysteresis. In the present case there is, besides, the difficulty arising from the fact that different layers of the wire used are doubtless differently affected. Notwithstanding, it is perhaps worth while attempting to answer the question: Is the alteration of stored magnetic energy greater than the initial energy of the oscillations producing the alteration?

The latest word on the subject of the intrinsic energy of magnetized iron is contained in a paper by Larmor*. There the expression $2\pi\int I^2 d\tau$, where I is the intensity of magnetization and $d\tau$ an element of volume, is deduced for the part of the energy we are here concerned with. Taking the observations recorded in this paper for that point of the ·200 ampere cycle where the field is zero, the wastage of the intrinsic energy instigated by a spark is, if it be legitimate to apply here this formula, 3600 ergs. Now the initial energy of the oscillations, which exists as the electrostatic energy of the charge on the oscillation-coil just before the spark-gap breaks down, is about 5700 ergs. These figures are too close to enable a decisive conclusion to be reached; that is, to say

* Roy. Soc. Proc. lxxi. p. 229, Feb. 1903.

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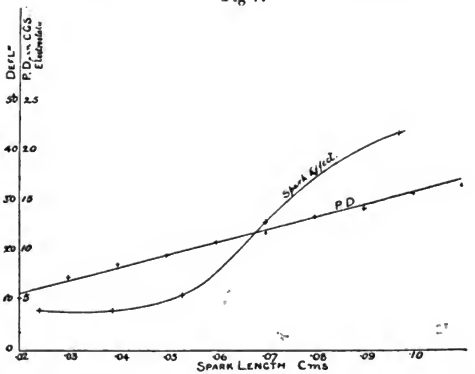
finally whether or not any considerable amount of magnetic energy is available and drawn upon. Allowing mentally for the oscillation energy lost in Joule effects and in radiation, it seems probable that magnetic work is done by the oscillations—whose efforts, therefore, would appear to be attended with hysteresis loss, just as are those of slower variations of magnetic field.

Table II. and fig. 7 give, in some degree, support to this

Fig 7.



conclusion. The bent (experimental) curve may be taken as representing the magnetic work done by sparks of length shown by the abscissæ, other circumstances being the same. The almost straight line represents the potential to which the oscillation-coil was charged at the corresponding spark-length. The relation between the two curves shows that for some portion, at any rate, of the diagram the magnetic work done varies as the square of the charging potential, that is, the electrostatic energy of the initial charge. The bending away of the first curve towards the right is explicable by the consideration, that as the applied oscillations get more violent the outermost layers of the iron wire attain, so to speak, a saturated condition; that is, become incapable of being more greatly impressed—with the result that the magnetic effect of the oscillations begins to fall off in amount. It was for this reason that in the experiments of Table I. the spark-gap was kept at the value 0·53 millimetre, well within the part of the curve free from this saturation effect.

I have to thank my colleague Mr. J. Lister for deducing the gradient curves and drawing all the figures in this paper.

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XVIII. *Colour Phenomena in Photometry.* *By* J. S. Dow, *A.C.G.I., B.Sc.**

THE discussion of Dr. Fleming's paper on Photometry, read before the Institution of Electrical Engineers in 1903, revealed great differences of opinion on the importance of colour phenomena in photometry, and it still does not seem to be generally known to what extent they are noticeable under ordinary working conditions. This may be due to the fact that most of the work done on this subject was carried out with special apparatus, such as the spectrophotometer, and not with the ordinary implements of photometry.

It therefore occurred to the author that some simple experiments on these points, carried out by him at the Central Technical College on an ordinary photometrical bench, might be of interest.

The sources of light were two similar glow-lamps, which could be screened with glass of different colours, and which were compared by means of one or other of several different photometers in the usual way. Four photometers were made use of during the experiments—the Lummer-Brodhun, the Grease-Spot, the Joly, and the Flicker.

The uncertainties which may be introduced by colour phenomena appear to be due to four separate effects:—

(1) The difficulty experienced in forming a judgment in the case of differently coloured lights, and the possibility that the judgments of different people may not be the same.

(2) The fact that the apparent relative brightness of two surfaces, illuminated by light of different colour, depends on the part of the retina on which the image of them is received.

(3) The Purkinje phenomena.

(4) The possibility, when mirrors are made use of, that the coefficient of reflexion may not be the same for different coloured lights.

(1) No doubt people differ in their capacity in this respect, but according to the author's experience it is chiefly a matter of practice. After a considerable amount of practice, he has found that it is possible to secure fairly consistent results even when comparing such colours as ruby-red and signal-green, while anyone unused to such work would be quite unable to do so. Extraordinarily consistent results were sometimes obtained, but it was found that if the observer stopped work for an hour or so, his readings afterwards

* Communicated by the Physical Society: reed May 25, 1906.

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would settle down to another very consistent value, but differing by perhaps 5 or even 10 per cent. or so from those obtained before. This seems to suggest that extreme consistency in reading is partly a matter of visual memory. We recall the impression previously received by the eye and involuntarily set the photometer, the next time, so as to produce the same appearance of the field of view.

The difference in sensibility of different eyes to a particular colour certainly introduces another disturbing factor in observations made by different people. The author has not met with any serious differences in judging the colour contrasts which ordinarily occur, but it might be supposed that in such an extreme case as that quoted above considerable divergences in judgment would exist, and have, indeed, been recorded by Sir Wm. Abney, Professor Rood, and others.

It must be noted, however, that the effect mentioned in (2) must be eliminated, in order to make any satisfactory comparisons. All the figures given in this paper were obtained by the author himself, though corroborative results from other eyes were obtained in many cases.

(2) This effect has been very completely dealt with by Sir William Abney in his investigation on Colour Vision.

It has long been known that the central portion of the retina—"the yellow spot"—is much more sensitive to the red end, and less sensitive to the blue end of the spectrum, than the surrounding portion of the retina.

It has been suggested that the "yellow spot," being yellow in colour, will obstruct the blue rays, but will allow the yellow rays to pass through practically unimpeded, to the light-perceiving organs. This explanation, however, does not explain why the differences observed are distinctly more noticeable at low illuminations.

There appears to be another physiological effect, which will be referred to later.

Suppose, now, that we compare a red and a green light with a Joly photometer. An image of the illuminated blocks is formed on the retina, and we adjust the position of the photometer until the red and green appear equally bright.

But if we now observe the photometer obliquely, or if we observe it with the eye at a different distance away, the image falls on a different part of the retina where the sensibility to red and green may be different. Consequently the red and green may no longer appear equally bright.

And if a photometer with blocks of a different size were used we might again come to a different conclusion; for,

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even if the eye were kept at the same distance, the size of the image would be different, and a new portion of the retina would be covered by it.

The position of the photometer, for which we obtain balance, therefore depends upon :—

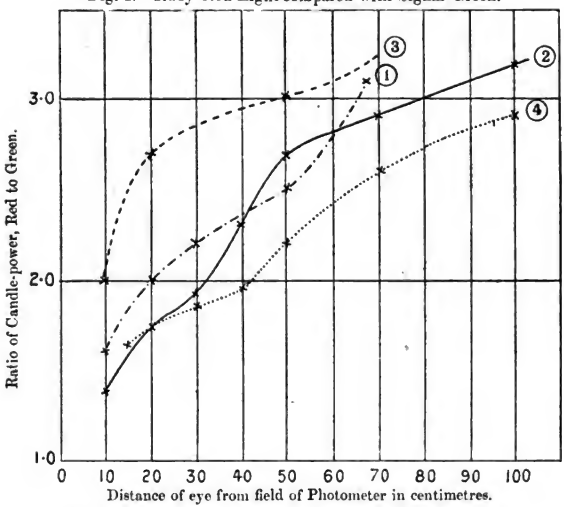
(*a*) The obliquity at which rays from the illuminated surface strike the eye.

(*b*) The distance away of the eye from the surface.

(*c*) The size of the surfaces.

The first point is not very important, for in focussing our eyes on the surfaces we involuntarily look straight at them. But (*b*) and (*c*) may easily affect the readings considerably. This is brought out in the curves in fig. 1.

Fig. 1.—Ruby-Red Light compared with Signal-Green.



(1) Lummer-Brodhun Photometer (telescope removed).
(2) Joly Photometer.
(3) Joly Photometer (linear dimensions of blocks reduced by half).
(4) Grease-Spot Photometer.

These curves were obtained as follows :—Two glow-lamps, screened with red and green glass respectively, were run at a constant P.D., and compared by means of each of the three photometers referred to above.

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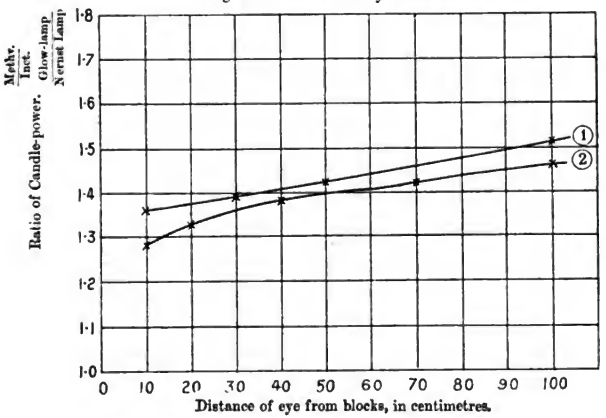
In each case a series of readings was obtained with the eye at different distances away from the illuminated surfaces. It should be mentioned that in the case of the Lummer-Brodhun photometer, the telescope was removed while the readings were taken. As will appear later, however, considerable variation is possible even with the telescope in position, as in use.

Curves (1), (2), and (3) speak for themselves. They bring out two points. Firstly, that the ratio of the candle-powers of the red and green lights, as thus observed, is quite different for each photometer.

Secondly, that this ratio depends on the distance of the eye from the photometer, the red becoming more and more accentuated as the eye recedes. Also the red is most accentuated and the curve is steepest for the Lummer-Brodhun photometer, in which the field is smallest.

Curve (4) was obtained by placing in front of the blocks of the Joly photometer a paper screen which reduced its linear dimensions by one half. The reduction in size of the blocks corresponds with a still further accentuation of the red.

Fig. 2.—Curves for Joly Photometer.



(1) Glow-lamp compared with Nernst Lamp.
(2) Methven Gas Standard compared with incandescent mantle.

This effect has been found to be quite distinctly observable in several commonly occurring comparisons. Fig. 2 exhibits

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the connexion between the apparent relative candle-power and the distance away of the eye with a Joly photometer, when a Nernst lamp was compared with a glow-lamp (running at 3·7 watts per c.p.), and when an incandescent mantle was compared with a Methven burner.

In the Joly photometer the distance away of the eye is left entirely to the inclination of the observer, and it will be seen from the above that differences of 5 per cent. or more might easily be introduced between the readings of different observers in this way.

In the Lummer-Brodhun photometer the distance of the eye is limited, to some extent, by the use of the telescope. But the position of the telescope can be varied between wide limits without putting the field out of focus, and this latitude allows of a considerable difference in the readings.

In the table below the extreme differences are given for these two limiting positions of the telescope :—

Nature of Lights compared.	Ratio of C.P.		Percentage Difference.
	Telescope in.	Telescope out.	
Ruby-Red to Signal-Green.	1·65	2·20	25·0
Glow-lamp (3·7 watts per c.p.) to Nernst lamp.	1·38	1·43	3·5
Methven Gas Standerd to Incandes. mantle.	0·129	0·135	4·4
Harcourt 10 c.p. Pentane Standard to Fleming Standard Glow-lamp.	0·652	0·657	0·77

Here again a distinct difference in reading is produced in several practical cases.

It is also remarkable that a small but distinct effect was produced in the last case, even though the flame of the Harcourt lamp is only very slightly redder to the eye than the light from the Fleming glow-lamp.

It is difficult, of course, to speak with certainty of such a small change as this—a change which would be produced by moving a photometer, set midway between two lights 2 metres apart, a distance of less than 2 millimetres. But the writer has usually found that the mean of a set of readings, taken with the telescope out, worked out to a value slightly

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different to the mean of those taken with the telescope in, and the difference was always in favour of the redder of the two lights. It need hardly be said that the difference observed might be important in such work as these two standards are used for.

There is one other point that requires mention. It is, of course, often necessary to reverse a photometer in order to correct for any differences between the two sides of the screen &c. However, for lights of similar colour, very little difference is produced by doing so, as a rule.

But when the lights differ in colour, the Lummer-Brodhun behaves differently from the Joly and Grease-Spot photometer. In the case of the latter, the image the retina receives is unaltered by reversing. But, with the Lummer-Brodhun, the image is reversed. If, before reversing, we see a green disk with a red centre, after reversing we see a red disk with a green centre.

We should, therefore, expect a much greater difference on reversing the photometer in the case of the Lummer-Brodhun.

The following figures, obtained for similarly coloured lights, and for red and green lights, exhibit this:—

Photometers used.	Lights of similar colour.			Red to Green light.		
	Ratio of C.P.		Per cent. differ.	Ratio of C.P.		Per cent. differ.
	1st Pos.	2nd Pos.		1st Pos.	2nd Pos.	
Joly	0·93	0·91	2	2·24	2·12	5·5
Grease-Spot	0·94	0·91	3	1·81	1·70	6·0
Lummer-Brodhun	0·95	0·91	4	2·10	1·75	18·0

(3) The Purkinje phenomenon has often been referred to as the chief source of trouble in heterochromatic photometry, but it appears to be only troublesome at very low illuminations.

An experiment was shown illustrating the Purkinje effect. A series of coloured screens, diminishing in size from about four feet square, as shown in the diagram (fig. 3), were illuminated by a 32 C.P. glow-lamp at a distance of about ten feet away. All these screens were made from the same identical red and blue paper, but, even at normal illuminations, it could be seen that as the surfaces became smaller the red appeared brighter and brighter in comparison with the blue. The red,

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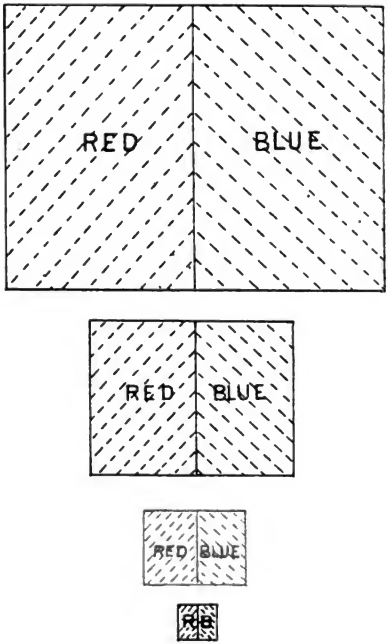
however, was distinctly the brighter, even in the case of the very large screen.

Fig. 3.



The illumination was now weakened by introducing resistance in series with the glow-lamp, and the blue began to appear brighter. A point was soon reached, when, for the very large screen, the blue was unquestionably brighter than the red. At this stage the difference in appearance of the different sized screens was much more marked than at the higher illumination, and, in the case of the smaller screens, the red was still much brighter than the blue.

As the light was still further weakened the colours began to disappear until, eventually, the red appeared as black, while the blue shone out with a phosphorescent white

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appearance. After this point the blue also fades away until nothing can be seen.

There is, however, a distinct difference between the behaviour of the very big screen and the very small ones. In the case of the latter the Purkinje effect is much weaker. Both colours seem to fade away together, and, by the time the Purkinje effect is really noticeable on the big screen, the very small screen can scarcely be seen at all.

In fact, to see the Purkinje effect really well, it is necessary to stand quite close up even to the big field.

A very interesting physiological explanation of these effects has been given by M. Sartori in a recent paper *.

Dotted about over the retina are two varieties of light-perceiving organs, known from their appearance as the "rods" and the "cones" respectively. The rods, it is thought, are sensitive to light but cannot perceive colour. Light of any colour appears to them white, but they are most sensitive to blue light. They are, moreover, sensitive to very weak light, but as the illumination is increased they become, as it were, saturated, and do not respond any further. The cones, on the other hand, perceive colour, but are most sensitive to yellow-green light, and while they do not respond at the low illuminations at which the rods can act, they continue to respond further to increased stimulus, once they have started, long after the rods have ceased to do so.

At normal illuminations, therefore, it is the cones which chiefly act, and we see colour. At very low illuminations the action of the rods is predominant, and we cannot see colour, while light of a bluish colour shines out with a whitish appearance. As the illumination is increased the cones suddenly begin to act and the colours appear. Then takes place what has been called "The Battle of the Rods and Cones." It is while this battle is in progress that the Purkinje effect is noticeable.

But the Purkinje effect is complicated by the fact that the rods and cones are unequally distributed over the retina. At the yellow spot the cones are predominant. Consequently, the Purkinje effect is much weaker when the field of view subtends a small angle at the eye.

This uneven distribution of the rods and cones will also explain the fact, referred to above, that at low illuminations the size of the field of view can produce much greater differences in the results.

In order to gain an idea at what illumination the effect

* *Electrotechnik und Maschinenbau*, March 18, 1906.

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becomes noticeable, in practice the following experiment was carried out :—

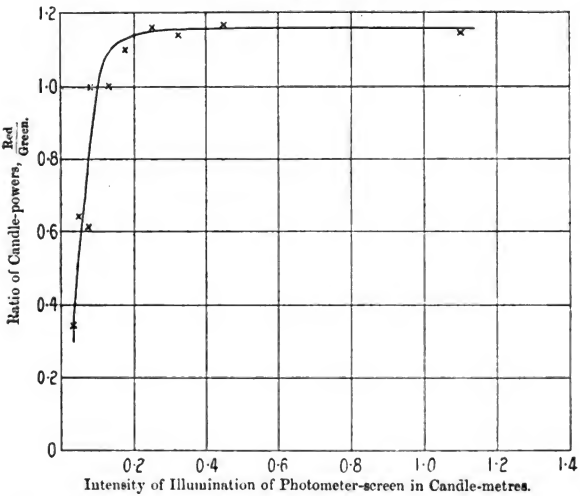
Two 100-volt 8 C.P. glow-lamps were run in series with a constant P.D. of 190 volts across them.

One was screened with red glass, and the other with green glass. The distance between the two lamps was varied from 20 to 250 inches, and the mean of a set of readings, giving their relative candle-power, taken in each case. In order to avoid the effect mentioned in (2) above, the Lummer-Brodhun photometer was used, and the telescope was kept in exactly the same position throughout the experiment.

Fig. 4 shows the result of plotting the ratio of the candle-power of the two lamps against the illumination of the photometer-disk, in candle-metres.

Fig. 4.

Fig. 4.—Curve exhibiting Purkinje Phenomena for Ruby Red and Signal-Green Lights.



From what has been said above, it is clear that we cannot, strictly, define the candle-power of a red light in terms of white-light standard, because this ratio depends upon the distance of the eye from illuminated surfaces.

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Nevertheless, in order to give an idea of the order of illumination used, the red lamp was compared against a 2 C.P. Methven gas standard, using the Lummer-Brodhun photometer, and its candle-power worked out to something of the order of 0·25 candle-power.

The relative illuminations used in the curve are expressed in terms of this figure.

It will be seen it is only when the illumination has been reduced to about 0·2 candle-metres, that the ratio of the red to the green light begins to decrease.

The same experiment was tried on several occasions with lights of different candle-power, and it was always found that fairly consistent results were obtained until the illumination fell to something of the order specified above, when the accentuation of the green light began to appear.

At such illuminations it is very difficult to obtain readings at all, and the mean of a great many should be taken. The eye seems to be in a state of uncertainty, and one receives the impression that the lights are continually varying. In fact the eye seems to be in a state analogous with that of unsaturated iron on the steep part of the magnetization curve.

It may safely be assumed, therefore, that in all ordinary cases, where the illumination would be at least ten or twelve times as great as that employed above, where the field of view would subtend a comparatively small angle at the eye, and where we should never meet with such an extreme colour contrast as in this case, the Purkinje phenomena will not materially influence the results.

(4) In order to obtain the curve of vertical distribution of light from arc lamps, &c., the beam of light is often reflected in the desired direction by means of a 45 degrees mirror*. When this is done, the question arises, whether the coefficient of reflexion of the mirror will be the same for lights of different colour.

The following experiment was carried out with a silvered glass mirror of this type. Two 200-volt 32 C.P. lamps were run in parallel off a constant P.D. of 200 volts. The two lamps were compared against each other direct. They were then compared when the light from one of them was reflected along the bench from the mirror. In this way, the coefficient of reflexion of the mirror is easily obtained.

This was repeated when the lamps were screened with red and green glass, successively.

* A mirror, that is, which rotates about an axis making an angle of 45° with its plane.

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The mean of one set of determinations gave :—

Coefficient of reflexion, 78·8 per cent. for unscreened lamps.
79·0 „ „ red light
77·8 „ „ green light.

Repetitions of the experiment gave slightly differing results, but it was found that the values for the three different colours did not differ among themselves by more than 2·5 per cent. Moreover, no connexion could be traced between these differences and the colours, for the difference was sometimes in favour of the red light, and sometimes in favour of the green.

We may safely conclude therefore, that in all ordinary cases, where the difference in colour is less pronounced than that adopted here, the effect is inappreciable.

In any case, the adoption of adjustable photometers of the Simmance-Abady type will probably render mirrors unnecessary in obtaining curves of distribution of light, in the future.

The only really important effect, under ordinary working conditions, therefore, is that described under the second heading. It has been shown that this may, apparently, give rise to a perceptible discrepancy even when lights so similar in colour as the Harcourt 10-candle Petane standard and the Fleming glow-lamp standard are compared.

Flicker Photometers.

The interesting question now arises whether photometers of the Flicker type are also influenced by these colour-phenomena.

If, as is claimed, the disappearance indicates that the two surfaces appear equally illuminated to the eye, one would suppose that all the conditions which determine this impression must also determine the point of disappearance of the flicker. On the other hand Mr. T. C. Porter, and others, have come to the conclusion that the flicker, at ordinary illuminations, is independent of colour.

Through the kindness of Messrs. Everett & Edgcombe the author has been able to make some experiments on the points referred to, with a photometer of this kind.

It is clear, in the first place, that when comparing lights of different colour, the readings of an ordinary photometer and a Flicker photometer may not agree even if the latter be independent of all colour effects. For instance, when two lamps giving light of exactly the same colour were compared,

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the results with the Joly photometer and the Flicker photometer were exactly the same. The two lamps were now screened with red and green glass respectively, and the ratio of the red to the green, obtained by the Flicker photometer, was about 1·34; but with the Joly photometer it was found possible to get readings from 0·7 to 1·4 for distances of the eye up to half a metre. Agreement between the photometers occurred when the eye was about 40 centimetres away from the blocks of the Joly.

One difficulty met with in this investigation was as follows. As the effects observed were comparatively small, it was advisable to use lights widely differing in colour. On the other hand the position of minimum flicker, though sufficiently sharply defined in most practical cases, is far from being so when such colours as red and green are observed. In such cases, the method followed was to note the position of the photometer in which a flicker was just visible on either side of balance, and take the mean. There is, however, another method of judging the position of balance for these two colours.

When the photometer is too near the red light, the field of view in the photometer appears reddish in tint. Similarly, a greenish tinge shows that the green illumination is the stronger. When the illumination of the two surfaces, illuminated by light of these two complementary colours, is the same, an intermediate greyish tinge is produced. To the writer's eye the transition from red to grey to green was sharper than the disappearance of the flicker, and a series of tests showed that the result was the same in each case. The method, however, is only applicable to complementary colours.

Some experiments were first made to discover whether a difference of reading could be produced by altering the telescope, as in the case of the Lummer-Brodhun photometer, but no distinct difference was observable. Next the telescope was removed, and readings were taken with the eye about 20 centimetres from the aperture. A brass tube was then inserted which allowed the aperture to be inspected from a distance of 60 centimetres from the eye. It was then observed that, as the eye was withdrawn, the field became distinctly redder, and readings taken by the "disappearance of flicker" method also showed a change of relative candle-power in favour of the red. The following table exhibits some of the results obtained :—

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Nature of Lights compared.	Eye 20 cms. from aperture.	Eye 60 cms. from aperture.	Percentage change.
Ruby-Red to Signal-Green.	$\frac{\text{Red}}{\text{Green}}=1{\cdot}52$	$\frac{\text{Red}}{\text{Green}}=1{\cdot}70$	12
Ruby-Red to Light from unscreened glow-lamp(white).	$\frac{\text{Red}}{\text{White}}=0{\cdot}70$	$\frac{\text{Red}}{\text{White}}=0{\cdot}75$	6
Methven Gas Standard to Incandescent Mantle.	$\frac{\text{Methv.}}{\text{Inct.}}=0{\cdot}28$	$\frac{\text{Methv.}}{\text{Inct.}}=0{\cdot}29$	3

The effect is very noticeable when observed in the following way:—Supposing we are comparing red and green, and have placed the photometer so as to secure balance. Move the photometer until a distinct flicker can just be seen owing to the green being too bright. It will be found that as the eye is removed the flicker gradually disappears. But if the photometer is put out of balance on the red side, the flicker does not disappear but becomes if anything more distinct as the eye is removed.

It appears therefore that this Flicker photometer is affected by the distance of the eye, but, apparently, not to a sufficient extent to be noticeable when the telescope is used, as in ordinary work.

It is difficult to see why these effects should be so much less noticeable than with ordinary photometers. It almost seems as though, when two differently coloured objects are placed side by side, any change in their relative illumination becomes exaggerated, thus creating a different impression to that received when they are viewed alternately (as in a Flicker photometer).

An attempt was also made to discover whether the Flicker photometer was subject to the Purkinje phenomenon. Now it is well known that the speed required to just make the flicker disappear depends upon the illumination, and the writer has found it impossible to judge the point of disappearance of the flicker with any certainty at the extremely weak illuminations necessary to produce the Purkinje effect.

The plan of "colour-reading" referred to above was therefore adopted. It may be objected that in doing so the real question at issue is avoided altogether. But the method, while admittedly not so satisfactory as the "disappearance of flicker" method from this point of view, has been shown to give the same results at ordinary illuminations and, to the

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writer, it seems impossible that the green illumination could be brighter and yet the field of view appear red.

Two 100-volt 8 candle-power lamps screened with red and green glass in the usual manner were used for the experiments. They were first run at 100 volts, and compared against each other at different distances. The two lamps were then run off 50 volts only, so as to produce a very low illumination, and the experiment repeated. The following table shows very clearly the influence of the Purkinje effect.

At the higher illumination the readings differed among themselves considerably (as was only to be expected with such a great colour contrast), but they do not seem to be connected in any way with the illumination.

The readings at the low illlumination can only be regarded as very approximate, but they bring out very clearly the accentuation of the green as the illumination gets weaker.

Lamps run at 100 volts.		Lamps run at 50 volts.	
Distance between Lamps.	Ratio $\frac{\text{Red}}{\text{Green}}$.	Distance between Lamps.	Ratio $\frac{\text{Red}}{\text{Green}}$.
inches.		inches.	
50	2·32	40	2·0
60	2·26	50	1·5
70	2·25	60	1·1.
120	2·15	70	0·7
150	2·25	90	0·6

The conclusion the writer draws from these experiments is that Flicker photometers are affected by the same colour phenomena which affect ordinary photometers.

The interesting assertion was made by Messrs. Simmance and Abady, in a paper before the Physical Society *, that a colour-blind person obtained practically the same results with their Flicker photometer as people with normal sight. On the other hand, Sir Wm. Abney, speaking of colour-blindness, remarks † :—

" We cannot hope for instance that the red-blind, who sees no red in the extreme end of the spectrum, would show any luminosity in that region. . . . One of the most striking experiments in colour-vision is to place a bright-red patch

* Phil. Mag. vii. p. 341 (1904). † 'Colour Vision,' p. 83.

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on the screen and to ask a red-blind to make a match in luminosity with the white. The latter will have to be reduced to almost darkness—a darkness, indeed, that makes the match almost incredible."

It seems incredible that such a person, when comparing red and green with a Flicker photometer, would obtain the same results as if he had normal sight. However, it appears that, according to Dr. Edridge Green*, colour-blindness is of two kinds. A person may be unable to distinguish, say, red-light by colour, but nevertheless a red object may appear as *luminous* to him as to anyone else. On the other hand, the colour-blindness may be due to the fact that the eye is incapable of perceiving red light at all. A colour-blind person of the first variety would presumably make normal photometrical readings. A person of the second class must, surely, obtain abnormal readings with all photometers, Flicker or otherwise.

This is borne out by some of Professor O. N. Rood's experiments on flicker†. He found that those of his students who were colour-blind obtained abnormal results with his Flicker photometer. Indeed, he actually used the Flicker photometer to investigate not only cases of colour-blindness but also the difference in sensibility to light of different colours, of the eyes of persons with normal sight.

In conclusion the writer wishes to express his great indebtedness to Professor Ayrton, and also to Mr. J. M. McEwan, for their assistance and for many valuable suggestions.

XIX. *Retardation of the α Particle from Radium in passing through Matter.* By E. Rutherford, *F.R.S., Professor of Physics, McGill University, Montreal*‡.

[Plate II.]

In previous papers§ I have given an account of some experiments which show that the α particles decrease in velocity in their passage through matter. The source of rays consisted of a fine wire, which had been made active by exposure as the negative electrode in a vessel containing a large quantity of radium emanation.

Fifteen minutes after removal from the emanation, the

* "The Physical Aspects of a Theory of Colour Vision," by F. W. Edridge Green, M.D., British Association, 1902.

† American Journal of Science, 1899, p. 258.

‡ Communicated by the Author.

§ Phil. Mag. July 1905, January and April 1906.

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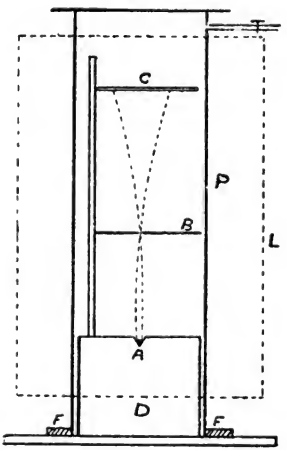
α radiation comes only from the product radium C. I have shown that such an active wire acts as a source of homogeneous radiation; *i. e.*, the α particles which escape from the wire all move at the same speed, and consequently are all deflected to the same extent by a magnetic field.

It is known from the work of Bragg and Kleeman that the α particles from each simple product have a definite range of ionization in air. One of the objects of the present experiments was to determine the velocity of the α particle corresponding to each point of its path in air, in order to determine the law of diminution of velocity of the α particle. This has been accomplished indirectly by observing the velocity of the α particles from radium C after passing through different numbers of layers of aluminium-foil, the stopping power of each layer being separately determined in terms of air by the scintillation or electrical method. The general arrangement of the apparatus is shown in fig. 1.

Fig. 1.



The active wire A was placed in a triangular groove in a solid brass cylinder D which was soldered to a base-plate. A vertical bar was attached to one side of the brass cylinder. By means of a slot in the centre, two movable frames B and C were attached, one of which carried the slit, and the other the

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small piece of photographic plate. By means of side screws, the distance of the slit and photographic plate from the source could be altered at will. Over this apparatus was placed a brass tube P closed at the upper end. To the lower end was attached a flange FF, which was ground accurately to fit the base-plate. By the use of tap-grease the flange formed an air-tight joint, and the vessel could be at once exhausted by a Fleuss pump without the necessity of waxing down the brass tube to the base-plate. This saving of time as well as of trouble was of great importance in experiments to be described later, in which the magnetic deflexion of the α rays from radium A was determined.

The whole apparatus was fixed on a wooden frame and placed between the poles of a large electromagnet so that the source of rays and the slit were parallel to the direction of the magnetic field. In these experiments, the magnetic field was practically uniform over the whole path of the rays. The extent of this magnetic field is shown by the dotted lines LL in the figure. The electromagnet was excited by a constant current of 12 amperes, and was kept from overheating by an electric fan. The duration of exposure of the plate in the magnetic field was usually two hours, the direction of the field being reversed every ten minutes.

In most of the experiments, the region between the active source and the photographic plate was divided into two equal parts by means of vertical mica plates. One half of the active wire was either bare or covered with some known absorbing screen, while the other half of the wire was covered with the number of layers of aluminium-foil under examination. In this way two sets of bands were obtained on the photographic plate, the distance between the centres of the bands of each set representing twice the deflexion in the magnetic field of the pencil of rays from the normal. The distance between the deflected bands is inversely proportional to the velocity of the α particles after emerging from the absorbing screen.

Measurement of the Photographic Plates.

In a previous paper, I pointed out a peculiarity exhibited by the photographic trace of a pencil of α rays from the active wire when a narrow slit was used. The trace of the pencil of rays consisted of two dark lines with sharply defined outside edges, and with a whitish line running symmetrically down the centre of the trace. This effect was shown to be a necessary consequence of the fact that the

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α particles were projected uniformly in all directions from the thin film of active matter distributed over the wire. This sharpness of the edge of the photographic trace with or without the magnetic field materially assisted in making an accurate measurement of the distance between the centres of the deflected bands obtained by reversal of the magnetic field.

In connexion with the experiments described in this and later papers, I have, in all, made more than a hundred photographs showing the effect of a magnetic and an electric field in deflecting a pencil of α rays under various conditions. Since each of these photographic traces, obtained on the plates, had to be accurately measured up, it may be of interest to others engaged in similar observations to describe in some detail the method finally adopted.

A microscopic method was first used, but this proved very trying to the eyes, and could only be used with advantage when the photographic traces were strongly marked. I finally had recourse to the following procedure. The small piece of photographic plate was mounted on a clear glass plate of the same size as a lantern-slide. This was placed as a slide in an ordinary arc-lantern and a magnified image of the photograph thrown on a screen. In most of the experiments, the magnification used was between 20 and 40, and was determined by means of an accurately subdivided millimetre-scale on glass placed in the position of the photographic plate. The image to be measured was thrown on a cardboard screen fastened into a large drawing-board. The observer with a sharp pencil then marked the edges of the photographic traces at intervals along the whole length. The screen was then removed and lines drawn through the points of observation, and the distances between the parallel lines measured by means of a scale and vernier.

With a clear photograph, the measurements obtained by two separate observers in the course of a few minutes did not differ more than a half per cent., while the mean of a number of observations was in very close agreement.

The great advantage of the lantern over the microscopic method lies in two factors. In the first place, the observer sees clearly the whole outline of the photographic traces before him, and is not likely to make a false estimate of the edge of the line on account of local imperfections of the plate. In the second place, even very weak photographic traces show up clearly on the screen, and can be measured with much greater rapidity and certainty and with less labour than by the use of the microscope.

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Retardation of the α Particles in passing through Aluminium.

The experiments given in a preliminary paper have been repeated with greater accuracy, and by using more active wires and with the photographic plate at a smaller distance from the source, I have been able to determine the velocity of the α particle closer to the end of its range of ionization in air.

The method adopted was to determine the deflexion of a pencil of α rays after passing through successive layers of aluminium-foil. Each layer of foil was found by a separate experiment to be equivalent in stopping power to about 0·50 cm. of air.

In these experiments, the slit was 2 cms. from the wire and the photographic plate 2 cms. from the slit. This gave a conveniently large deflexion of the pencil of α rays in a magnetic field of strength about 8000 c.g.s. units. One side of the active wire was bare, and the other covered successively with 2, 4, 6, 8, 10, and 12 layers of aluminium-foil. Although a very strong photographic effect was obtained with the bare wire, the effect through the 12 layers of foil was very weak, but was sufficiently well defined for accurate measurement. The slit was then removed to a distance of 0·5 cm. from the source and the plate to within 1·5 cms. of the slit. The intensity of the photographic impression was consequently increased four times, and the part of the active wire covered with the 12 layers gave a well-marked effect on the plate. The amount of deflexion of the pencil of rays was, however, reduced to ·375 of that observed at the first distance. The other side of the wire was successively covered with 12 and 14 layers of foil, and the deflexions compared with that due to the original 12 layers, used as a standard of comparison.

A weak though measurable impression was obtained through 14 layers. Two layers of Dutch metal, equivalent in stopping power to ·64 of a layer of foil, were then added. A slight darkening of the plate was noted, but was too weak and ill-defined for measurement.

The results obtained are collected in the following table, where the velocity of the α particles is expressed in terms of V_0, the velocity of the α particles from the unscreened wire. It will be shown in a later paper that the value of e/m for the α particle is unchanged by its passage through matter, so that the deflexion of the pencil of rays in a magnetic field is inversely proportional to the velocity of the rays.

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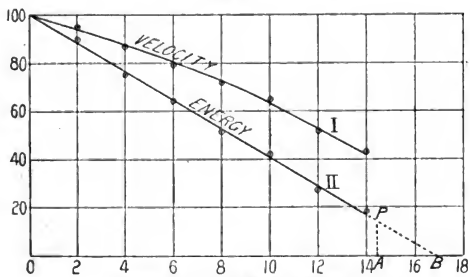
Number of Layers of Aluminium-foil over the Wire.	Velocity.
0	1·00 V_0
2	·95 „
4	·87 „
6	·80 „
8	·72 „
10	·65 „
12	·52 „
14	·43 „
14·6	not measurable.

The lowest velocity observed was ·43 V_0, due to the rays which had passed through 14 layers of foil.

One of the main difficulties in these experiments arose from the slight variations of thickness of the individual layers of foil. The sheet of foil to be used was doubled on itself so as to obtain the requisite number of layers, slightly pressed together, and an area of suitable size cut out. Care has to be taken in making sure of the number of foils in the layers, for it is not an easy matter to count them with certainty when once the foils have been pressed together. This may appear to be a small point; but I have found by experience how easy it is to make a mistake.

The thickness of foil used in the experiment was found by the scintillation method to be equal in stopping power to almost 0·50 cm. of air. 14 layers thus corresponded to a

Fig. 2.



LAYERS OF ALUMINIUM FOIL.

thickness of about 7·0 cms. of air. Since the range in air of the α particles from radium C is 7·06 cms., this thickness of foil was nearly sufficient to absorb completely the α rays.

The results given above are shown graphically in fig. 2,

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Curve I. The velocity of the rays from the bare wire is taken as 100. The abscissæ represent the number of layers of foil over the active wire. Curve II. shows the energy curve, the energy of the α particle escaping from the bare wire being taken as 100. This was obtained by taking the squares of the observed velocities. It will be seen that the points lie nearly on a straight line, which cuts the axis of abscissæ at a distance corresponding to 16·9 layers of foil. Each layer of foil absorbs absorbs about 6·0 per cent. of the maximum energy of the α particle.

The photographic effect of the rays after passing through 14 layers of foil was not more than 5 per cent. of that produced by the bare wire in the same interval. The actual energy of the α particle at this stage is ·18 of the initial energy, so that it is seen that the photographic intensity of the α particles near the end of their course falls off more rapidly than their kinetic energy.

Taking as the most probable value that the α rays are completely absorbed in 14·4 layers of foil, which would correspond to 7·06 cms. of air at atmospheric pressure and temperature, we see that each layer of foil on an average corresponds to slightly less than 0·50 cm. of air.

Now the continuation of the energy curve cuts the axis for a thickness of aluminium corresponding to 16·9 layers of foil, which is equivalent to a distance of air equal to $(16{\cdot}9-14{\cdot}4)\times 0{\cdot}50$, or 1·25 cms. of air beyond the distance 7·06 cms. required for complete absorption. Consequently, the velocity of the α particles which have a range r cms. in air after passing through an absorbing screen is proportional to $\sqrt{r+1{\cdot}25}$. The ratio K of the velocity V of the α particle of range r to the velocity V_0 of the α particle from the bare wire is given by

$$K=\frac{r}{r_0}=\frac{\sqrt{r+1{\cdot}25}}{\sqrt{7{\cdot}06+1{\cdot}25}}={\cdot}348\sqrt{r+1{\cdot}25}.$$

For example, the range r of the α particles from radium C after passing through 8 layers of foil (equal to 4·0 cms. of air) is 3·06 cms. The ratio K is consequently ·72.

We therefore see that the velocity of the α particle of any known range in air can be deduced from a simple formula.

We shall show in a later paper that the value of e/m is the same for the α particles from radium A, radium C, and radium F (polonium). It thus appears probable that the α particles from the various radium products have all the same value of e/m. If this be the case, we are at once able to deduce from the above formula the initial velocity of the

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α particles expelled from a thin film of each of the radium products. The values are collected in the table given below and are expressed in terms of V_0, the initial velocity of the α particles expelled from radium C.

Product.	Range of α particles in air.	Calculated velocity.	Observed velocity.	Value $\frac{mv}{e}$.
Radium	3·50 cms.	·750 V_0	·76 V_0	$3·04 \times 10^5$
Emanation...	4·23 „	·814 „	.....	$3·30 \times 10^5$
Radium A ...	4·83 „	·858 „	·88 V_0	$3·48 \times 10^5$
Radium C ...	7·06 „	1·000 „	1·00 „	$4·06 \times 10^5$
Radium F ...	3·86 „	·787 „	·81 „	$3·20 \times 10^5$

The ranges of the α particles from the radium products are taken from the paper of Bragg and Kleeman. The range of the α particles from radium F (polonium) has been found by Dr. Levin in the laboratory of the writer.

In the fourth column are given the values of $H\rho = \frac{mv}{e}$, where H is the strength of the magnetic field, ρ the radius of curvature of the path of the rays, and e, m, v have the usual signification. I have redetermined accurately the initial value of $H\rho$ for the α particles expelled from radium C and found the value $4·06 \times 10^5$, in place of the value $3·98 \times 10^5$ given in a previous paper. The values of $\frac{mv}{e}$ have been deduced from the calculated velocity and the value given above.

We shall now consider some direct experiments that have been made to determine the velocity of the α particles from some of the radium products in terms of V_0, assuming that e/m has the same value for the α particles from all products.

Magnetic Deflexion of the α Particle from Radium itself.

About one milligram of pure radium bromide, spread uniformly in a narrow groove, served as a source of α rays. The radium was heated to a sufficient temperature to drive off the emanation completely. After three hours, the products radium A and C have nearly disappeared, and the α radiation then mainly comes from radium itself. The magnetic deflexion of the pencil of rays was determined in the standard apparatus (fig. 1), using a narrow slit. The trace of the rays was first obtained without a magnetic field. The deflected traces on each side of the central band were then obtained by

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reversal of the magnetic field. The actual photograph obtained in this way is shown in fig. 3 (Pl. II.)

Since the layer of radium was of sensible thickness, the α particles escaped from the radium over a wide range of velocity, and the rays were unequally deflected by the magnetic field. The inner surface of the deflected bands was, however, produced by the rays of maximum velocity from the radium, which it is our object to determine. The distance, for example, from the right edge of the photographic trace, obtained without a magnetic field, to the inside of the deflected band on the left represents the deflexion of the rays of maximum velocity expelled from the radium. The deflexion was compared with that due to the rays from a bare wire coated with radium C. In this way, it was found that the initial velocity of the α particles expelled from radium is $\cdot 76 V_0$—a value which agrees closely with the value $\cdot 75\, V_0$ calculated from the formula. By observation of the outside limit of visible photographic effect, the lowest velocity of the α rays which just produced visible photographic action was found to be $\cdot 51 V_0$.

Magnetic Deflexion of the α Particles from Radium A.

Since the activity of radium A decreases to half value in 3 minutes, an accurate measurement of the magnetic deflexion of its rays by the photographic method was a matter of some difficulty. The radium A was collected on a thin wire by exposing it for 3 minutes in the presence of a large quantity of emanation. Before the introduction of the wire, the emanation had been exposed to a strong electric field so as to remove as much as possible of the radium C which has a tendency to collect in the emanation vessel. During the time of exposure, 3 minutes, the wire collects one half of the equilibrium amount of radium A upon it and very little radium C. A photographic plate was first placed in position in the apparatus shown in fig. 1. The active wire was then quickly removed, placed in position, and the cover fixed on. Not more than half a minute elapsed after the removal of the wire before the apparatus was in position between the poles of the magnet, and the vessel exhausted to a pressure of a few millimetres of mercury. The field was reversed every half minute for six minutes, when the plate was removed and developed.

After intensification, the deflected bands were sufficiently clearly defined to be measured. The amount of magnetic deflexion was 1·14 times as great as for a bare wire coated with radium C exposed under the same conditions. This

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makes the observed value of the velocity of the α particles of radium A $\cdot 88V_0$, which is in good agreement with the value $\cdot 86V_0$, previously deduced from its known range in air.

Another photograph was taken in which the active wire was covered with a layer of mica equivalent in stopping power to 3·5 cms. of air. The measurements in this case were complicated by the fact that the rays from radium C on the wire produced also a comparable photographic effect. Two broad bands were consequently obtained, the outer parts of which were due to the rays from radium A. Allowing for the known width of the trace of the undeflected pencil, the velocity of the α particles from radium A after passing through 3·5 cms. of air was found to be $\cdot 54V_0$. The value deduced from measurement of the range of the rays in air, viz. 1·3 cm., is $\cdot 56V_0$. Considering the difficulty of measuring such weak photographs, the observed and calculated values are in as good agreement as could be expected.

Radium F.

It has not been thought necessary to determine directly the magnetic deflexion of the α rays from radium F (polonium), for this has already been done with accuracy by Mackenzie (*loc. cit*). Using a rod covered with a thin film of polonium, he found the value of $H\rho$ for the polonium rays to be $3 \cdot 30 \times 10^5$. The value (see previous table) deduced from the range and known value of $H\rho$ for radium C is $3 \cdot 20 \times 10^5$, which is in substantial agreement with the value determined by Mackenzie.

I have not so far separately determined the magnetic deflexion of the rays from the emanation. I think, however, that there can be little doubt that the velocity $\cdot 81V_0$, deduced from the known range of the rays, is close to the true value. Experiments are in progress to obtain if possible on a single photographic plate the separate traces of the rays from each of the four α ray products of radium in equilibrium. This should be theoretically possible using a very thin film of radium in equilibrium as a source of rays. It is doubtful, however, whether it will be practically possible to obtain a complete separation of the traces due to the rays from the emanation and radium A, on account of their velocities of projection being nearly the same.

Scattering of the α Rays.

When Bragg first put forward his theory of the absorption of the α rays, which has received such a remarkable measure

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of verification in the last two years, he pointed out that on general theoretical considerations there should be a marked difference in the character of the absorption of α and β rays by matter. Although the β particle is in most cases projected at a much greater velocity than the α particle, yet, on account of its very small mass, its kinetic energy is much less than that of the α particle. Since the atom is the seat of intense electrical forces, the β particle in passing through matter should be much more easily deflected from its path than the massive α particle. We know experimentally that this is the case. A narrow pencil of β rays, incident on matter, emerges as a diffuse ill-defined cone of rays. This scattering of the β rays, as Bragg pointed out, probably accounts for the exponential law of absorption of homogeneous β rays by matter.

On the other hand, the α particle, on account of its enormous energy of motion, plunges through the atoms of matter without suffering much deflexion from its path. As I pointed out in a previous paper, there is, however, an undoubted slight scattering or deflexion of the path of the α particle in passing through matter. This is clearly seen from the two photographs shown in fig. 3 (Pl. II.). The space between the radiant source and the photographic plate (see fig. 1) was divided into two parts by a mica screen. An active wire coated with radium C was used as a source of rays. One half of the top of the slit was covered with a plate of mica equivalent in stopping power to 3·5 cms. of air. The dark well-defined band in one photograph (fig. 3 A) represents the trace of the rays which have not passed through the mica, the diffuse band the trace of the rays after passing through the mica. If there were no scattering of the rays, the two bands should be equally well defined. A very similar scattering is observed in a strong magnetic field (see fig. 3 B), showing that the effect produced cannot be ascribed to the slow-velocity electrons which are so copiously emitted by matter through which the α particles pass, for these would be bent away from the photographic plate by the strong magnetic field.

This broadening of the photographic trace is naturally not exhibited if the absorbing screen is placed over the source instead of over the slit.

From measurements of the width of the band due to the scattered α rays, it is easy to show that some of the α rays in passing through the mica have been deflected from their course through an angle of about 2°. It is possible that some were deflected through a considerably greater angle; but, if so,

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their photographic action was too weak to detect on the plate.

This scattering of the α particles increases, as we should expect, with the decrease of velocity of the α particle. From the values of e/m and V for the α particle, given in the next paper, it can easily be calculated that the change of direction of 2° in the direction of motion of some of the α particles in passing through the thickness of mica (·003 cm.) would require over that distance an average transverse electric field of about 100 million volts per cm. Such a result brings out clearly the fact that the atoms of matter must be the seat of very intense electrical forces—a deduction in harmony with the electronic theory of matter.

Discussion of Results.

The α particles from a thin layer of one kind of radioactive matter are all expelled with the same velocity, and after passing through an absorbing screen, all emerge with the same, though diminished velocity. We have already drawn attention to the rapid decrease of photographic effect due to a pencil of rays from an active wire when the rays pass through an absorbing screen. The photographic effect of the α particles towards the end of their path in air certainly decreases far more rapidly than the kinetic energy of the α particles themselves.

This rapid decrease of the photographic effect is not to be ascribed to a decrease of the number of α particles falling on the plate, but to a decrease of the photographic effect produced by each α particle. The general results of my experiments have led me to conclude that the photographic effect due to an α particle is approximately proportional to its range in air. On this hypothesis, the α rays from an active wire coated with radium C (range 7 cms.) produce, at a given distance in a vacuum, seven times the photographic effect of the same pencil of rays after passing through an absorbing layer equivalent to 6·0 cms. of air.

The range of the α particles after emerging from the screen is in this case 1 cm. On this view, the α particle from radium itself (range 3·5 cms.) only produces half of the photographic effect of the α particle from radium C.

In a similar way, I have observed that the brilliancy of the scintillations produced by the α rays in zinc sulphide decreases rapidly as the range of the α particles is cut down by the use of absorbing screens. Experiments are in progress to test

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quantitatively whether the intensity of the luminosity produced in a zinc-sulphide screen, as well as of the photographic effect, is directly proportional to the range of the α particles in air. Such a connexion is to be expected if both the photographic and luminous effects are directly or indirectly due to ionization by the α particles of the substances involved.

In regard to the question whether the α particle has a "critical" velocity, below which it is unable to produce its characteristic effects, it is difficult to give a definite answer. The α particle certainly appears to have such a "critical" velocity, but there appears to be no obvious reason why the α particle should suddenly lose its ionizing power when its velocity is still very great. Bronson has made an investigation to see whether the α particle produced any appreciable ionization beyond its normal range of strong ionization in air, but with negative results. We have to conclude either that there is a definite critical velocity (about ·4 of the maximum velocity of the α particle from radium C), or that the velocity of the α particle decreases very rapidly when it falls below that amount. The question is complicated by the slight scattering of the α particles in passing through matter, for this makes definite deductions from the experiments very difficult. I hope to make further experiments to throw light on this important question.

We have seen that it is possible to determine with accuracy the value of $\frac{mv}{e}$ for the α particles emitted by radium, radium A, and radium F by measurements of the retardation of the α particles of the single product radium C in passing through matter. Such a result affords an almost certain proof that the value of e/m is the same for the α particles expelled from each of these products; for while the measurement of the magnetic deflexion of the rays gives the momentum of the α particle, the range of the particle *plus* a constant is proportional to the kinetic energy of the α particle. Unless e/m is the same for each product, we should not expect the observed and calculated values of $\frac{mv}{e}$ to agree.

It will be shown in a future paper, that this conclusion has been confirmed by direct measurement of the value of e/m for the particles expelled from the various radium products.

McGill University, Montreal,
June 14, 1906.

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XX. *Abnormal Polarization and Colour of Light scattered by Small Absorbing Particles. By* R. W. Wood*.

In the course of some experiments on the fluorescence of iodine vapour, I observed on one occasion the precipitation of what appeared to be an iodine fog in one of the glass bulbs. This fog scattered powerfully light of a deep red colour, and on examining it with a nicol prism I was astonished to find that it was plane polarized in a direction at right angles to that which is usually observed in the case of light scattered by small particles. When a powerful beam of light was sent through the bulb in a horizontal direction, the scattered light came off at right angles, with its direction of vibration (electric vector) horizontal instead of vertical. If the light was polarized before it entered the bulb, the light was scattered laterally in the directions of the vibration in the incident light. The phenomenon was first observed in a bulb containing a small flake of iodine, and air at atmospheric pressure, cooled by immersion in a beaker of water to which ice had been added. On attempting to repeat the experiment I met with failure, and had about given up the attempt, when the red fog suddenly appeared again in a bulb which I was exhausting with a mercury pump. In this case the fog was doubtless caused by sudden expansion. This method failed repeatedly in subsequent experiments. I finally found a method which gave the fog with tolerable certainty, namely, to precipitate the iodine upon a smoke-cloud already existing in the bulb. If iodine is vaporized in a bulb and then cooled, it crystallizes either upon the walls, or in the form of floating spiculæ, too large to show the scattering effects. If, however, the bulb is filled with a light smoke and the iodine vaporized, on cooling each smoke particle forms a nucleus upon which the iodine crystallizes. The smoke which appeared best was that from hot sealing-wax. Tobacco-smoke did not work well. At first I obtained the fogs by sucking a little of the vapour given off from a blazing stick of sealing-wax which had just been blown out, introducing a small flake of iodine, and warming it with a flame. Some beautiful red fogs were obtained in this way, but quite as often the scattered light was similar to that obtained from the smoke alone. The best plan appears to be to introduce a bit of sealing-wax the size of a pin-head, and a similar speck of iodine, into a small glass bulb, and then heat them both with a very small

* Communicated by the Author.

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flame. It is a good plan to support the bulb in a clamp-stand in a dark room and focus sun- or arc-light at its centre. The red fog persists for some time, once it is formed. When at its best its scattering power is so great that a reddish colour is seen within the bulb at a distance of a metre from the arc without any concentration. Examined by transmitted light in a well-lighted room, no trace of colour is to be seen, which proves that the red light is selectively scattered, and not produced by absorption. With the concentrated beam from the arc the scattered light is blood-red and of great intensity. A nicol placed with its long diagonal horizontal, nearly, but not quite extinguishes it.

It seemed probable that the polarization might be produced by refraction of the light within the small particles. The light which is given off at right angles to the direction of the incident beam is in part reflected of course. This portion is uncoloured, and polarized with its vibration perpendicular to the direction of the incident rays. Another portion of light may reach the eye after two refractions and one internal reflexion, and this portion will be coloured, and oppositely polarized. We are of course considering the particles as partially transparent, and of such size as to permit them to act as the water-drops in the theory of the rainbow. A small glass bulb filled with a red liquid can be used to show the phenomenon on a large scale; if it is placed in a beam of sunlight and viewed at an angle slightly greater than 90° with the incident rays, two spots of light will be seen—one uncoloured due to reflexion, the other coloured due to refraction; and examination with a nicol shows that they are more or less completely polarized in opposite directions. Attempts were made to obtain fogs of other strongly-absorbing media. Precipitation of the aniline dyes was tried, without much success however. Nitroso-dimethyl aniline, a substance which has proved of interest in connexion with many optical problems, was found to give excellent results. A few grains are placed in the bottom of a large test-tube, and heated with a small flame. A yellow fog at once forms in the tube, which exhibits the same phenomena shown by the iodine fog. Examined with a nicol with its short diagonal horizontal, the scattered light is yellowish green and very bright; if the nicol is turned through a right angle the scattered light is white and much fainter. The nitroso absorbs the blue end of the spectrum. The fog is best examined through the open mouth of the tube, the arc-light being focussed at the centre. Under certain conditions, these nitroso fogs showed brilliant

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colours which were not due to absorption. If the test-tube is fastened at an angle of 45° in a clamp-stand, and the light focussed about an inch below the open mouth, the fog will be found to whirl about in a vortex; and if it is examined with a nicol, the stream lines will often be found to show brilliant colours. One stream will be bright red and the adjacent stream deep green, and the colours appear to change to their complementary tints when the nicol is rotated through 90°; though it is difficult to be sure of this effect, owing to the rapid changes which take place in the distribution of the colours.

These changes reminded me of some curious effects which I have observed with thin films of collodion deposited on silver surfaces*. When a powerful beam of unpolarized white light is concentrated normally upon the film, and the light scattered at grazing emission is examined with a nicol, it was found that one-half of the visible spectrum was polarized vertically, *i. e.*, parallel to the surface, the other half horizontally. The complete explanation of this very singular phenomenon has not yet been found, though it was pretty definitely established that it was due to the frilling of the film, the granular structure being just resolvable with the highest powers of the microscope.

The diameter of the particles in the nitroso fogs averaged about ·003 mm., as was determined by inverting the tube over a glass plate, allowing the particles to settle, and examining the deposit with a microscope. The particles are much larger than the ones treated by Lord Rayleigh, which are too small to regularly refract or reflect the incident light, and which operate in a wholly different manner.

Somewhat similar clouds can be formed by shaking up some of the finely powdered aniline dyes in a test-tube, the finest particles remaining suspended in the air. Soluble prussian-blue makes a good cloud, the scattered light being deep blue when examined with a nicol with its short diagonal parallel to the incident rays, and white under the reversed conditions. The polarization effects are not as marked, however.

* "On some New Cases of Interference and Diffraction," Phil. Mag. vii. p. 376 (1904).

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XXI. *Atomic Conductivities of the Ions.* *By* Philip Blackman *.

THE fact that there exists a definite quantitative relation between molecular conductivities (Phil. Mag. 1906, xi. p. 416; Ostwald's *Lehrbuch der Alg. Chemie*, II. i. p. 672) seems to indicate unmistakably that molecular conductivities are additive properties, *i. e.*, that the molecular conductivity of any substance (in aqueous solution) is equal to the sum of the atomic conductivities of its constituent ions. No such regularity apparently exists between the molecular conductivities of the acids, but if the assumption be made that the atomic conductivity of the $H^{\cdot}$ ion is a function of the relative strengths of acids, then it is possible to calculate its value, and consequently those of other ions.

It was shown (Phil. Mag. 1906, xi. p. 417) that in the general equation

$$\mu v_{HX} + \mu v_{M_1OH} - \mu v_{M_1X} = K,$$

the disappearance of molecular conductivity was equivalent to that required by the equation $H^{\cdot} + OH' = H.OH$ for the particular acid in question. Then, according to the preceding argument, the atomic conductivity (μv_H) of the $H^{\cdot}$ ion may be determined from the equations

$$Rv_{HX} \cdot \mu v_H + \mu v_{OH} = K,$$
$$Rv_{HX_1} \cdot \mu v_H + \mu v_{OH} = K_1,$$
$$Rv_{HX_2} \cdot \mu v_H + \mu v_{OH} = K_2,$$
$$\ldots\ldots\ldots\ldots = \ldots$$

(in which Rv_{HX}, Rv_{HX_1}, Rv_{HX_2}, etc., represent, respectively, the relative strengths of the acids HX, HX_1, HX_2, etc., all measured at the same concentration, v, and at the same temperature).

The calculations effected by means of these equations are so fairly consistent that the assumptions made above may very reasonably be taken as correct.

As an example, the atomic conductivities of the ions $H^{\cdot}$, OH', $K^{\cdot}$, and Cl', at 18° and at concentration $v = 500$, are worked out in full.

$$R_{HCl},\ R_{HNO_3},\ R_{\frac{1}{2}H_2SO_4},\ R_{\frac{1}{2}H_2C_2O_4},\ \text{and}\ R_{CH_3CO_2H}$$

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are respectively 1·0, 0·9946, 0·94, 0·46, and 0·08 ;

$$K_{HCl},\ K_{HNO_3},\ K_{\frac{1}{2}H_2SO_4},\ K_{\frac{1}{2}H_2C_2O_4},\ \text{and}\ K_{CH_3CO_2H}$$

are respectively 474, 472, 456, 285, and 163.

Hence $1{\cdot}0\mu_H + \mu_{OH} = 474$, $0{\cdot}9946\mu_H + \mu_{OH} = 472$,

$$0{\cdot}94\mu_H + \mu_{OH} = 456,\ 0{\cdot}46\mu_H + \mu_{OH} = 285,$$

and $$0{\cdot}08\mu_H + \mu_{OH} = 163.$$

By adding all the equations together and solving the resulting equation $4{\cdot}749\mu_H = 1618$, it is found that $\mu_H = 340$.

$$\mu_{OH} = \mu_{HOH} - \mu_H . R_{HCl} = 474 - 337 = 137\ ;$$

$$\mu_{Cl} = \mu_{HCl} - R_{HCl} . \mu_H = 376 - 1{\cdot}0 \times 337 = 39\ ;$$

$$\mu_K = \mu_{KCl} - \mu_{Cl} = 126 - 39 = 87\ ;$$

or $$\mu_K = \mu_{KOH} - \mu_{OH} = 233 - 137 = 96, \text{ etc.}$$

v (at 18°) =	1000	500	200	100	50	20	10	2	1
H	337	336	333	330	329	323	313	291	270
Rb	90	89	87	85	85	81	77	...	71
Cs	91	89	87	85	...	...	76	...	...
K	89	89	88	86	83	80	78	67	63
NH_4	85	84	82	80	78	75	72	64	63
Tl	85	85	82	77	76	76	76	...	...
½Ba	75	75	69	66	65	57	52	40	36
½Pb	76	73	69	63	59	49	41	19	14
½Sr	74	70	67	62	61	55	54	36	30
Ag	73	72	70	67	...	62	58	43	40
½Ca	69	66	64	60	60	54	44	34	29
Na	66	65	64	61	60	59	58	41	38
½Mg	64	60	56	52	47	42	34	32	28
½Zn	64	60	56	50	45	40	35	29	19
½Cd	...	...	55	46	38	32	26	22	15
½Cu	62	55	48	40	33	27	22	...	13
Li	56	54	53	52	51	48	45	40	34
OH	143	140	140	139	138	135	127	122	109
½CO_3	45	41	36	32	27	21	15	12	7
Cl	40	40	40	40	38	37	37	36	31
NO_3	40	40	40	40	38	37	36	34	28
Br	40	39	38	38	38	37	36	36	...
I	39	38	37	37	37	37	36	36	36
½SO_4	37	37	33	32	29	24	22	21	13
SCN	30	29	28	28	28	28	26	25	24
ClO_3	28	27	26	26	26	24	21	18	...
F	21	20	20	20	20	18	16	16	16
½C_2O_4	12	12	11	11	9	8	7	6	5
⅓PO_4	11	11	11	9	7	6	5	3	2
IO_3	8	8	7	7	7	5	2	...	...
CH_3CO_2 .	7	6	6	6	5	4	2	1	0·8

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The letter μ (strictly speaking denoting "molecular conductivity") has been retained, for the sake of simplicity, to represent also "atomic conductivity." The table has been entirely constructed from the data in Landolt-Börnstein's *Physikalisch-Chemische Tabellen* (latest, 1905, edition). In all cases, the atomic conductivities given are the means of those obtained by all the possible calculations. To calculate the molecular conductivity of any salt, base, or acid, respectively, the equation $\mu v_M + \mu v_X$, $\mu v_M + \mu v_{OH}$, or $Rv_{HX}, \mu v_H + \mu v_X$, should be employed.

East London Technical College,
London, E.

XXII. *Distribution of the Intensity of the Radiation from Radioactive Sources.* By E. Rutherford, *F.R.S., Macdonald Professor of Physics, McGill University, Montreal* *.

[Plate II.]

In the course of my experiments on the magnetic deflexion of the α rays emitted by the active deposit of radium, a very peculiar photographic effect was observed. The experimental arrangement was similar to that described in a previous paper (Phil. Mag. July 1905). The rays from a wire about 1·5 cms. long and 0·5 mm. in diameter, made strongly active by exposure in the radium emanation, passed through a slit and fell on a photographic plate placed some distance above the slit. The whole apparatus was exhausted of air, and a strong magnetic field applied so as to bend the pencil of α rays. With a broad slit, comparable in width to the diameter of the active wire, the photographic trace of the α rays on the plate is shown by a black band with sharply defined edges; and the intensity of the photographic impression is approximately uniform over its cross section. With a very narrow slit, however, the photographic trace of the rays has a very different appearance.

Fig. 1 A, Pl. II., shows a magnified drawing of such a trace. The outside edges of the band are sharply defined, and the photographic impression falls off rapidly from the outside to the centre, but is most intense at the extreme edges. The trace, at first sight, appears as if it consisted of two dark

* Communicated by the Author, having been read before the American Physical Society, April 1906.

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bands with a well-marked white band between them. A photograph taken in a similar way, using an active circular rod 5 mms. in diameter, is reproduced in fig. 8, Pl. II.

This peculiarity in the trace of the rays could not be due to the magnetic field, for it was observed equally clearly when no magnetic field was acting. The same appearance was also observed when the α rays were passed through an absorbing screen. It was at first thought possible that this effect was due to photographic reversal, since it was first observed when the photographic effects were very intense. Later work, however, showed that the effect was equally marked when the photographic effect was too feeble to admit the possibility of photographic reversal.

In another experiment, a small sphere about 1 cm. in diameter, which was made uniformly active by exposure to the radium emanation, was used as the source of rays. A metal plate was placed about 2 cms. above the sphere, and the rays from the sphere passed through a small hole in this plate and fell nearly normally on a photographic plate placed 2 cms. above the hole. The impression on the plate corresponded to a pinhole photograph of the active sphere, taken by the α rays emitted from its surface. A drawing of the photograph is shown in fig. 1 B, Pl. II. The photographic effect is greatest at the extreme edge of the circle, and falls off towards the centre.

This distribution of the intensity of the radiation for the radioactive sphere and cylinder is very different from that which would be observed if the photographs were taken by the luminous rays from a white-hot sphere or cylinder. In a pinhole image of the sun, for example, we know that the intensity of the radiation is practically uniform over its disk.

These striking differences between the distribution of the intensity of radiation for ordinary light sources and for these radioactive sources receive a simple explanation along the following lines.

In the case of a luminous body like the sun, the intensity of the radiation emitted from any point is not uniform in all directions, but is governed by the well-known cosine law, sometimes known as Lambert's law. This asserts that the intensity of the radiation from any point varies as the cosine of the angle between the normal and the direction of the emitted light. This law of distribution for a self-luminous source is, I believe, usually explained by supposing that the light does not come entirely from the surface, but from a sensible depth of an absorbing layer.

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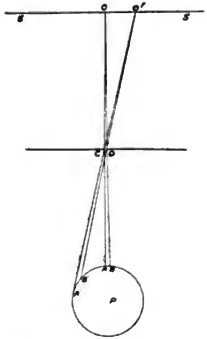
Let the circle P (fig. 2) represent the cross section of a cylindrical uniformly radiating body. The rays pass through a narrow rectangular slit CD, and fall on a screen SS. The radiation which falls on the central point O of the image

Fig. 2.



comes from a length AB of the circle, which is intersected by lines drawn from O touching the edges of the slit. The radiation which reaches O′, a point on the extreme edge of the luminous area, comes from a length A′B′ of the circle, which is seen to be considerably greater than the length AB. In the case of a self-luminous body, this increase of radiating surface which supplies light at a point near the edge of the image is exactly compensated for by the cosine law of distribution, and the intensity of the radiation is consequently uniform over the cross section of the image.

In the radioactive case, however, the radiations come *from an extremely thin film of radioactive matter*, in which the α rays are not appreciably absorbed. There is, in consequence, no cosine law of distribution, but, on an average, the α radiation from any point is equally intense in all directions. We see that, under such conditions, the intensity of the radiations reaching O (disregarding small variations in distance) is proportional to the length AB, and the intensity at O′ to the length A′B′. The intensity of the radiation is consequently greatest at the extreme edge of the image, and falls off towards the centre.

If the distances OC, CP are very large compared with the radius of the cylinder and the slit very narrow, it can readily

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be shown that the intensity at the extreme edge of the image may be very much greater than that in the centre. This is the experimental case already considered (fig. 1 A, Pl II.).

The mathematical theory of the distribution of intensity of the radiation for the case of a radiating sphere or cylinder, when no cosine law applies, can be readily worked out, but it has not been thought necessary to include it here. It suffices to say that the observed distribution of the intensity of the radiation from the radioactive sphere or cylinder receives a simple explanation when we take into account that

(1) the layer of radioactive matter is so thin that the α rays suffer no appreciable absorption in passing through it, *i. e.* no cosine law of distribution applies ;

(2) the α particles are, on an average, projected equally in all directions.

The experimental results obtained afford an indirect proof that the α particles are, on an average, projected equally in all directions from a radioactive source.

Our experience of the distribution of the radiation from luminous sources is so bound up with the cosine law of emission, that we are liable to overlook how different this distribution would be if no cosine law applied. We have seen, for example, that if the radiation from the sun came from a thin non-absorbing superficial layer, the sun would appear to us as a circular disk, the edge of which was marked by a brilliant ring of light, the luminosity falling off rapidly from the edge to the centre.

Consider also another example. If a small plane disk emits light according to the cosine law, the intensity of the radiation at any point some distance from it is greatest when the plane of the disk is normal to the line joining the point and the disk. The intensity of the radiation falls off as the plate is orientated, and is very small when the light is emitted nearly tangentially from the surface. Consider now a similar disk coated with a thin film of radioactive matter, and, in order to avoid the disturbance due to the absorption of the α rays in air, suppose the experiments are made in a vacuum. Since the cosine law does not apply, the intensity of the radiation at some distance from the small plane disk is practically independent of the orientation of the plate, and is nearly the same for normal and tangential emission.

We shall now consider some further experiments which illustrate these points in a striking manner.

A brass rod of square cross section, about 4 cms. long, was exposed for some hours to the radium emanation, in order to

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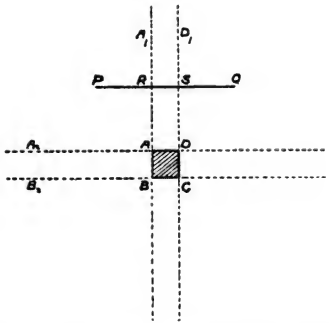
obtain on its surface a thin uniform layer of radioactive matter. After removal, the intensity of the radiation from the rod was examined in various positions, both by optical and photographic methods. In the former, the rod was brought close to a screen of willemite or zinc sulphide.

The results observed are best described by means of a diagram (see fig. 3). ABCD represents the cross section of

Fig. 3.



the square rod. The dotted lines represent continuations of the sides of the square. Suppose the phosphorescent screen is placed in the position PQ, parallel to the upper surface of the rod. The experiment at once showed that the part RS within the dotted lines was far less illuminated than the parts RP, SQ immediately beyond. This was confirmed by placing a photographic plate in the same position. The intensity of the impression is much less within the region RS than on either side of it. The photographic negative, consequently, has the appearance of a whitish band of width equal to the side of the square on a dark background. The edges of the band are very sharply defined, and there is no gradual transition of intensity from one region to another.

It is, at first sight, somewhat startling that the intensity of the photographic effect is least immediately above and closest to the active surface, but a little consideration shows that such an effect is to be expected. The region RS is acted on only by the rays from the upper surface AD of the rod, while the regions RP, SQ are, in addition, acted on by the rays from the sides of the square. If the distance AR is large compared with the length AB, the intensity of the radiation

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just outside the point R, due to the side AB, is nearly equal to that due to the side AD. Consequently, the intensity of the radiation to the left of the point R is about twice that to the right of R in the region RS.

This conclusion was confirmed by a slight alteration of the experiment. Two thin plates of metal, which completely absorbed the α rays, were placed in the positions AR and DS. The radiation reaching a point in the region RP is then due entirely to the rays from the side AB. Under such conditions, it was observed that the intensity of the photographic impression in the region RS was very nearly the same as that in the region RP close to R.

Some very beautiful optical effects can be produced by a different arrangement of the experiment. The active rod rests on the phosphorescent screen with its axis normal to the surface. The illuminated area is seen to be divided by four dark bands at right angles to each other, forming a well-marked cross. The width of these bands is the same throughout their length and equal to the side of the square. The bands correspond to the regions included in fig. 3 between the lines AA_1 and DD_1, AA_2 and BB_2, &c. The effects observed are best illustrated by means of the reproduction of an actual photograph shown in fig. 4, Pl. II. The active rod rested normally on the photographic plate, as in the case of the experiment with the screen.

The explanation of these bands, where the luminosity is less than in the region immediately outside them, is obvious from what has already been stated.

In most of the experiments, the rod was charged negatively, then exposed in the emanation vessel, in order to concentrate the active deposit upon it. The active deposit was consequently not uniformly distributed, being in somewhat greater amount at the edges of the rod. This lack of uniformity in the distribution had, however, no sensible effect on the main features of the photographs, for equally clear effects were obtained when no electric field was acting during the exposure of the rod to the emanation. With an active rod clear photographs of this character can be obtained in a few minutes.

It was thought of interest to examine the distribution of the radiation in two other cases. In one experiment, the cross section of the rod was an equilateral triangle of 6·5 mms. side, and in the other a regular hexagon of 2·75 mms. side. The rods were placed normally on the plate, and the photographs obtained are shown in figs. 5 and 6, Pl. II.

The explanation of the results obtained is clearly seen from

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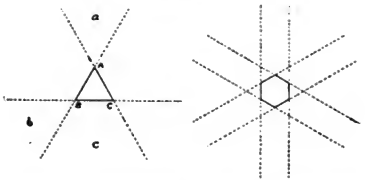
the diagrams shown in fig. 7. ABC is the cross section of the triangular rod. The regions *a*, *b*, and *c*, bounded by the continuations of the sides of the triangles, are clearly shown in the photograph, since the radiation outside these triangular areas is due to one side alone.

Fig. 7.



The photograph obtained with the hexagonal rod presents several points of interest. A six-armed cross is clearly seen as well as the outline of the six small equilateral triangles obtained by the continuation of alternate sides of the hexagon. The explanation of this six-armed cross and the smaller equilateral triangles is clear from the diagram in fig. 7, where the full lines represent the hexagon, and the dotted lines the boundaries of the arms of the cross and of the triangles.

A sudden change of intensity occurs at every point where the tangential radiation from another side of the rod adds its effect.

Experimental examples of this kind can easily be multiplied, but sufficient have been given to illustrate the striking variations of the intensity of the radiation round a radiating body when no cosine law of distribution applies. There is no doubt that similar effects could be obtained if the β instead of the α rays were used as a source of rays. The lines of demarcation of the shadows would not, however, be so marked as in the case of the α rays, on account of the considerable penetrating power of the β rays.

Theoretically, the distribution should be quite different if the radiation comes from a thick instead of a thin layer of radioactive matter. It is difficult to obtain such radioactive sources in definite geometric form, but we should expect that the distribution for a thick layer should approximate closely to that observed for light sources where the cosine law of emission applies.

I am indebted to my colleague, Professor Cox, for his assistance in explaining these effects.

McGill University, Montreal,
June 5, 1906.

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XXIII. *Fish-Eye Views, and Vision under Water. By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University*.*

[Plate III.]

In discussing the peculiar type of refraction which occurs when light from the sky enters the surface of still water, it seems of interest to ascertain how the external world appears to the fish. As is well known, a submerged eye directed towards the surface of still water sees the sky compressed into a comparatively small circle of light, the centre of which is always immediately above the observer, the appearance being as if the pond were covered with an opaque roof provided with a circular aperture or window. If our eyes were adapted to distinct vision under water, it is clear that the objects surrounding the pond, such as trees, horses, fishermen, &c., would appear round the edge of this circle of light. Objects not much elevated above the plane of the water would be seen somewhat compressed and distorted, but the circular picture would contain everything embraced within an angle of 180° in every direction, *i. e.* a complete hemisphere. Our eyes behave so abominably under water, however, that we can see nothing of this curious picture; the focus being so poor that it is difficult even to distinguish between a piece of blank paper and paper with black letters an inch high and a quarter of an inch wide, the letters being almost invisible. A lens of half an inch focal length, placed in front of the eye, helps matters, but even with this, vision is far from distinct. A great many years ago I attempted to get an idea of how this circular picture appeared to the fish, by constructing a diving-bell out of an old butter firkin, weighted with lead, and provided with a plate-glass window, forgetting that the refraction of the rays from water to air again would banish the illusion of the round window.

It occurred to me last autumn during a lecture, that an excellent notion of how we appear to the fishes could be obtained by immersing a camera in water, and photographing the circle of light, for it is easy enough to arrange a lens and plate so as to obtain a sharp image when both are immersed in water. The apparatus was constructed out of a lard pail, a short focus lens, provided with a very small diaphragm, being cemented over a hole perforated in a metal disk which rested on a rim soldered around the inside of the pail. The plate was placed on the bottom of the pail, and the whole filled with clean water in a dark room. The lens

* Communicated by the Author.

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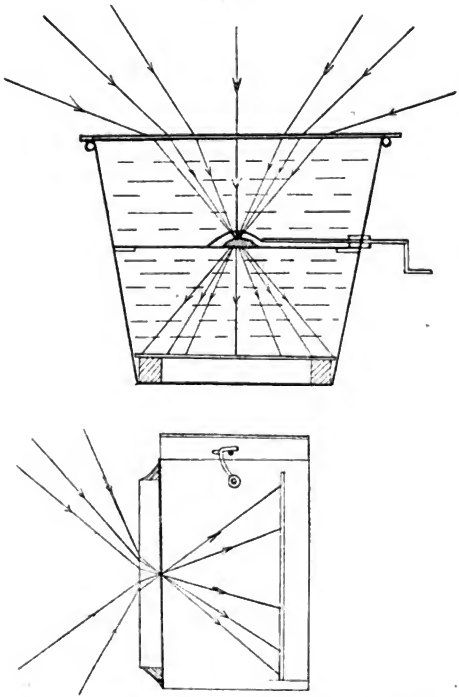
was covered with a metal cap operated by a handle on the outside of the pail. The apparatus was placed on the ground, and the surface of the water covered with a sheet of glass to prevent ripples, the pail being so full that the glass was in contact with the water. This arrangement obviated the necessity of immersing the affair in a pond, since the function of the latter was performed by the water in the pail above the lens. A diagram of the arrangement is shown in fig. 1,

Fig. 1.



together with the paths of the incident and refracted rays. A number of extremely interesting pictures were obtained with the device, which proved to be the equivalent of a lens

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having a working angle of 180°. One of these is reproduced in Pl. III. fig. 2, and represents the appearance of a circle of men standing around the edge of a small pond, to a fish stationed at the centre. The appearance of the buildings, telegraph-poles, and small trees is also shown.

The water camera was then modified so that it could be pointed in a horizontal direction. The lens was dispensed with and a pin-hole substituted, since it was found that as good, if not better, definition could be obtained in this way. The hole was made in the opaque film on the back of a piece of modern mirror-glass, which was cemented, glass side out, over a hole in the end of a water-tight box (fig. 1, lower diagram). The plate was inserted in a dark room, and the whole filled full of water, after which the cover was closed, and a little additional water added through a small hole to displace the enclosed air. The hole was then closed with a stopper and the exposure made as desired. It will be seen that the views obtained with the apparatus in the horizontal position, represent things as seen by a fish looking out through the glass sides of an aquarium. The cone of light entering the fish's eye has an aperture of about 96°, but the rays within it came originally from a cone of 180°. Very curious pictures were obtained with this apparatus. It will photograph all three sides and the complete ceiling and floor of a room, or when placed at a point where two streets cross at a right angle, will give a view looking down any three streets, the view including the ground quite up to the base of the tripod, and the sky from the horizon to the zenith. Suspended from a balloon it would photograph the entire surface of the earth out to the horizon in all directions.

Some very curious pictures were obtained with the apparatus in this form. A straight row of nine men, standing side by side in a garden path, was photographed with the camera standing not more than 18 inches in front of the central figure. The straight path appears as if bent into a semicircle, and the end figures appear distorted, as if seen by reflexion in a cylindrical mirror. This view (fig. 3, Pl. III.) gives us a good idea of how the visitors at an aquarium appear to the fishes. Fig. 4 is a view taken at the intersection of two narrow streets, and shows Monument Street, looking west, with the dome of the physical laboratory, and Eutaw Street looking north and south; the foreground is recorded up to the base of the camera's tripod, and the sky to the zenith, while an overhead wire appears bent into an arc.

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A kite or balloon picture being out of the question, on account of the long exposure required, the camera was carried to the top of the monument located at the centre of Mount Vernon Place, lashed to the underside of a twelve-foot piece of timber, and run out over the edge of the parapet, which surrounds the top of the cylindrical shaft. The view obtained in this way is shown in fig. 5, a portion of the city being of course hidden by the monument.

An improved form of fish-eye camera was subsequently constructed. The box was made of sheet-brass, and measured $4 \times 5 \times 2$ inches (inside measurement). The plate was inserted through a slot provided with a hinged metal cover and rubber washer, and the optical part made by cementing a piece of tinfoil, perforated with a needle-hole, between two pieces of plate-glass, with Canada balsam. The plates were then cemented over a circular hole 2 inches in diameter, cut in the top of the metal box. This arrangement was so compact that it could be carried in the coat-pocket, and was absolutely water-tight. It seems quite possible that the peculiar type of refraction described in the present paper, which makes a working angle of 180° possible, may be made use of in certain cases. Since the device will photograph the entire sky, a sunshine recorder could be made on this principle, which would require no adjustment for latitude or month. While the views used for the illustration of this paper savour somewhat of the "freak" pictures of the magazines, it is believed that the fact that they illustrate how one half of the world appears to "the other half" is sufficient excuse for their publication.

Baltimore, June 1906.

XXIV. *Notices respecting New Books.*

Physical Optics. By Robert W. Wood, Professor of Experimental Physics in the Johns Hopkins University. New York: The Macmillan Co. 1905.

If we have not been so forward with the review of this book as we might have been, it is in part to be ascribed to the merits of the book itself. We have had, midst the pressure of other work, to read a great deal more than the reviewer is wont to read before committing his opinions to paper.

Professor Wood's 'Physical Optics' is at once a teaching book of value, a compendium of practical details for laboratory work, and a brilliant exposition of recent problems of Physical Optics. We

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are carried along by the clear and pleasant style and the manner in which, step by step, the subject is unfolded.

A detailed account of the book chapter by chapter would serve no purpose, for it is certain that before long the book will be in every laboratory and will be included in the curriculum of many teaching institutions. A few special points of excellency we will notice.

Some of the most interesting recent matter arises out of the advances made by Professor Wood himself. While the place given to this work is no more than it deserves, the collection of so much of it together, for the first time, is a feature of no inconsiderable value.

The general plan of the work is to treat in the earlier pages the more elementary aspect of the subject. In the later chapters a fuller and more advanced study of the several leading topics is offered. A good sample of the author's manner of handling his subject will be got by reading, for instance, the elementary chapter on Dispersion and then turning to the chapter on the Theory of Dispersion. The latter is one of the most interesting in the book. It would be difficult to make it better. The author's beautiful experiments on the anomalous dispersion of sodium vapour are used with telling effect in conveying an insight into the manner in which experiment has been found to substantiate the dispersion formula derived from electromagnetic considerations for a medium with a single absorption-band, accepting the D lines as forming a single absorption-band in the dense vapour of sodium studied in the experiments. A careful examination of the Ketteler-Helmholtz dispersion formula applied to the dispersive properties of quartz is previously given.

But it is invidious to pick out any one section of Professor Wood's book for special commendation. The fresh and original treatment which is the prevailing charm of the work is found throughout. The chapter on Refraction containing the experiments on artificial mirage; that on Interference of Light with its many practical details and ingenious experiments; on Diffraction; Rotatory Polarization; and Magneto-Optics, wherein the Zeeman effect is excellently dealt with; the Scattering by Small Particles, wherein much is new to textbooks; the interesting treatment of the topic of the Nature of White Light—a topic which Lord Rayleigh has made of special interest to readers of the Philosophical Magazine: all are excellent. Nothing can be more beautiful than Professor Wood's shadow or refraction photograph of a sound-wave reflected by a flight of steps as an illustration of Lord Rayleigh's treatment of grating problems. All these chapters are, singly, valuable expositions of the subjects dealt with. They close with a review of results and theories of the Relative Motion of Æther and Matter.

There are in our opinion a very few sins of omission. We think photo-electric effects and such experiments as those of Elster and

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Geitel on the dependence of the photo-electric effect on the orientation of the plane of polarization of the light, are sufficiently in the subject of physical optics and of a teaching order and experimental beauty to justify their inclusion here. A little might well have been said in reference to the experiments of FitzGerald and Trouton on the plane of polarization of electromagnetic waves, seeing that the results of these and similar investigations are, of course, assumed in the book. There appears to be some confusion about the statement on page 90 as to what is given in tables of the optical properties of different kinds of glass. Not the relative dispersion, but the reciprocal of the dispersive power is given in such tables as we have seen; and this is what the formula on p. 90 should be called. J. J.

The Nucleation of the Uncontaminated Atmosphere. By CARL BARUS. Hazard Professor of Physics, Brown University. Washington, D.C.: Published by the Carnegie Institution of Washington, 1906. Pp. xii + 152.

THIS monograph contains an account of experiments and observations carried out with the object of obtaining a continuous record of the nucleation of the atmosphere in localities relatively free from the habitations of man, and therefore free from nucleations of local and artificial origin. The observations were made at two stations, Providence and Block Island, simultaneously and with similar apparatus. In addition to the tabulated results and the conclusions to be drawn from them, the monograph contains an account of various experiments on the distribution of the nuclei, the numbers and gradations of size of the efficient nuclei in dust-free air, and the critical conditions in the formation of ions and of nuclei.

Derniers Progrès du Telphérage Electrique. Par EMILE GUARINI, Professeur à l'Ecole d'Arts et Métiers de Lima (Pérou). Paris: H. Dunod et E. Pinat. 1906. Pp. 27.

THE telpherage system of transport is one well adapted to new countries, where the cost of constructing a railway for the same purpose would be prohibitive. The author of the small pamphlet under review gives a very useful account of some of the more recent examples of the telpherage system as practised in the United States, and the booklet should prove valuable to those interested in the subject.

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XXV. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 96.]

February 21st, 1906.—Sir Archibald Geikie, D.C.L., Sc.D., Sec.R.S., President, in the Chair.

THE following communications were read:—

1. 'The Constitution of the Interior of the Earth, as revealed by Earthquakes.' By Richard Dixon Oldham, F.G.S.

This paper is not devoted to a fresh theory of the earth, but is intended to set forth some of the information which can be obtained from the study of the records of distant earthquakes. The modern seismograph has given to geology a new instrument of research, and extended its scope much in the same way as the spectroscope extended the scope of astronomy, by enabling us to see into, and determine the physical constitution of, the interior of the earth at depths removed from any other possible means of research.

The distant record of a great earthquake exhibits three distinct phases, of which the third represents wave-motion which has travelled along the surface of the earth and is not dealt with in this paper, as it can give no information regarding the interior of the earth. The other two phases form the preliminary tremors, and it is shown that they represent the emergence of two distinct forms of wave-motion, which have been propagated through the earth.

A study of the intervals taken by these waves to reach remote points shows that, up to a distance of 120° of arc from the origin, they are propagated at a rate which increases with the depth of the wave-path, and reaches an average of over 10 km. sec. in the case of the first-phase, and over 6 km. sec. in the case of the second-phase waves. The increase, being gradual and continuous, may be attributed to the effect of increased pressure and temperature, and there is no indication of any change in physical constitution of the material traversed by waves which emerge at 120° or less from the origin.

Beyond this limit the first-phase waves show a reduction in the mean rate of transmission, while the second-phase waves are not to be found where they would be expected, but at about 11′ later. The interpretation is that the wave-paths emerging at these greater distances have entered a central core, in which the rate of transmission of the first-phase waves is reduced to about nine-tenths, and of the second-phase waves to about one half, of the rate in the outer shell. The great reduction of rate in the case of the second-phase waves means great refraction, and the wave-paths which emerge at distances of over 130° from the origin must have

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reached their emergence after passing on the farther side of the centre of the earth; the increase of length of wave-path means a longer interval between origin and emergence, and the sudden increase of interval at about 130° is explicable in this way. An alternative explanation is that the second-phase waves are extinguished at these distances, but this explanation is not regarded as probable by the author.

Either alternative leads to the conclusion that, after the outermost crust of the earth is passed, there is no indication of any material or rapid change of physical condition, nor probably of chemical composition, until a depth of about six-tenths of the radius is reached; but that, below this, there is a rapid passage to matter which has very different physical properties, if not also differing in chemical constitution. Without advancing any hypothesis as to the nature of this difference, the author points out that it will have to be reckoned with in any theory of the earth.

2. 'The Tarannon Series of Tarannon.' By Miss Ethel M. R. Wood, D.Sc.

A general historical review is given of the existing state of knowledge respecting the character and fossils of the strata included under the general title of 'Tarannon' on the maps of the Geological Survey in Wales. The Tarannon strata are most fully developed in the Llanbrynmair-Tarannon district, after which they are named; and the present paper gives the results of a detailed survey of the entire Tarannon Series as there exhibited. Various sections—typical and confirmatory—are described, showing the local sequence of the strata. Lists of the contained graptolites are given, and the species are paralleled with those from the corresponding beds of the South of Scotland, the Lake District, North Wales, Central Wales, and Sweden, demonstrating the similarity of the graptolitic succession in all these districts.

The authoress establishes the following local sequence :—

C. Wenlock Series.

Cb. Fynyddog Group.

Ca. Nant-ysgollon Group.

Zones of *Monograptus riccartonensis* and *Cyrtograptus Murchisoni*.

B. Tarannon Series.

Bd. Dolgau Group. (Zone of *M. crenulatus*.)

Bc. Talerddig Group. (Zone of *M. griestonensis*.)

Bb. Gelli Group. (Zone of *M. crispus*.)

Ba. Brynmair Group. (Zone of *M. turriculatus*.)

A. Llandovery Series.

Ac. Twymyn Group.

Zone of *M. Sedgwickii*.

Zone of *Cephalograptus cometa*.

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Ab. Dolgadfau Group.
Zone of *M. convolutus.*
Zone of *M. fimbriatus.*

Aa. Fachdre Group.
Zone of *Dimorphograptus Swanstoni.*

The Tarannon Series in this district has a maximum thickness of 3500 feet, but thins somewhat as it is traced north-westward. It rests conformably on Llandovery rocks below, and passes up without a break into Wenlock beds above. This rock-series is stratigraphically continuous from base to summit, and includes the four divisions of the Brynmair, Gelli, Talerddig, and Dolgau Groups, which, while they possess distinctive features of their own, are bound together by common palæontological characters. The lowest two, namely the Brynmair and Gelli Groups, consist mainly of grey shales and mudstones with beds of thin flags, which increase in number and thickness as one ascends the sequence. The Talerddig Group is distinctly an arenaceous one, and contains numerous bands of thick grit which are generally massed together at four or five distinct horizons. The highest member of the Series, the Dolgau Group, answering to the local 'Tarannon Shales' of the Geological Survey, consists of pale-grey and purple mudstones, the latter being inconstant in number and thickness in different parts of the district.

The strata of the overlying Wenlock Series present all the characters of the Denbigh Grits and Flags of North Wales. Some 2000 feet are developed in this district, the upper beds consisting of grits and flags, while the lower are mainly shales and mudstones.

The Llandovery Series, which underlies the Tarannon Series, has, at present, been recognized only in the western part of the district, namely in the valley of the Twymyn, and its rocks are brought to the surface by an anticlinal fold. Representatives of nearly the whole of the Llandovery beds have been met with at different localities, and five distinct graptolitic zones have been recognized. The rocks, which consist almost entirely of soft shales and mudstones, are probably not more than 400 feet thick.

A comparison of the graptolitic lists shows that the Tarannon Series, as here defined, corresponds almost exactly with the Gala or Queensberry Group of the South of Scotland, includes all the palæontological zones hitherto assigned to the Tarannon, and fills up the whole period intervening between the Llandovery below and the Wenlock above. It includes the extreme beds which have been mapped as Tarannon by the Geological Survey in Wales; and in the Tarannon District, at all events, the thickness of the Series is equivalent to its maximum development elsewhere.

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March 7th.—Sir Archibald Geikie, D.C.L., Sc.D., Sec.R.S., President, in the Chair.

The following communications were read:—

1. 'On the Occurrence of Limestone of the Lower Carboniferous Series in the Cannock-Chase Portion of the South-Staffordshire Coalfield.' By George Marmaduke Cockin, F.G.S.

Silurian limestone underlies the Coal-Measures in the southern part of the South-Staffordshire Coalfield, and a rock, probably similar, was found in a borehole at No. 2 Cannock-Chase Colliery. A shaft was sunk some 30 years ago, about 5 miles north of the latter locality, at No. 1 Fair Oak, but was abandoned, as no workable coal-seam was found. Before the undertaking was abandoned, an exploration-heading was driven for 44 yards in the direction of the dip, and from it heads along the strike for 150 yards. In the waste-heaps, which have remained undisturbed since 1875, a number of fossils belonging to the Lower Carboniferous Limestone have been found. A fault must be presumed to bring Carboniferous Limestone into such a position as to be reached by the headings. About 1½ miles north-west of Fair Oak, rocks (determined by Mr. Walcot Gibson as Millstone Grit) were reached by a boring at 396 yards. An account of the strata pierced by the Fair-Oak boring is appended.

2. 'Liassic Dentaliidæ.' By Linsdall Richardson, F.G.S.

March 21st.—Aubrey Strahan, M.A., F.R.S., Vice-President, in the Chair.

The following communications were read:—

1. 'The Chalk and Drift in Möen.' By the Rev. Edwin Hill, M.A., F.G.S.

In 1899 Prof. Bonney and the writer published a paper on Möen and Rügen: the present contribution contains results of further studies.

The problem of Möen is to account for portions of Drift, isolated, and seemingly included, in cliffs of Chalk. It has been generally assumed that these portions occupy dislocations, and that the dislocations were either simultaneous with, or subsequent to, the deposition of the Drift. But, in this paper, cases are described where Drift is seen to occupy cavities in dislocations, which had been waterworn, and consequently had been produced, before the advent of the Drift. The assumption hitherto generally made is, therefore, incorrect: the Chalk had been disturbed in pre-Glacial times. A probable assumption that there were pre-Glacial hills and cliffs similar to

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the present, with similar clefts and furrows in the cliffs, which were covered in Glacial times with a mantle of Drift now in course of removal by denudation, explains every variety of Drift-inclusion. These varieties are described, and proofs are given of such an overlying mantle of Drift, even now over 100 feet thick on the flanks of the hills, and rising to their summits.

Differences from the Rügen phenomena are noted and explained. An apparent upward succession of Boulder-Clay, shingle-beds, sand, and scattered boulders is discussed.

Slopes of uniform inclination which rise from the beach, often 60 feet and more, to the bases of the vertical cliffs, appear to be talus-slopes. In reality they are everywhere solid Chalk, with only a skin of débris: this suggests post-Glacial changes in sea-level.

Certain weathered-out hollows in cliff-faces indicate a way in which isolated portions of Drift might, when denudation had commenced, be long protected from destruction.

One instance of a Chalk-boulder included in Drift emphasizes the general contrast, not likeness, between Möen and Cromer. The position of the Drift in Möen is similar to that of the Boulder-Clays at Flamborough and along the Yorkshire coast. Had the Yorkshire Chalk been dislocated as that at Swanage or Lulworth, probably we should have in England similarities to Möen.

2. 'On the Relations of the Chalk and Boulder-Clay near Royston (Hertfordshire).' By Prof. T. G. Bonney, Sc.D., LL.D., F.R.S., F.G.S.

On the uplands south of Royston, Mr. H. B. Woodward, F.R.S., has described three sections, which in his opinion indicate that a great ice-sheet, as it advanced from the north, sheared off large masses of Chalk and mixed them up with its ground- or englacial moraine (the Chalky Boulder-Clay).

The author points out that this interpretation rests on an hypothesis—namely, that the latter deposit is the direct product of land-ice—which, as it involves some serious difficulties, cannot yet be taken for granted. For instance, this clay in many parts of England contains chalk-pebbles, more or less well-rounded, and often striated. But it is improbable that fragments of rock in either a ground- or an englacial moraine would be shaped into ordinary pebbles; and they would be brought into contact so seldom, and for so brief a time, that they would be but little scratched. But these chalk-pebbles resemble those formed by water, either in a river-bed or (more probably) on a sea-beach. How, in the latter case, they could be striated Col. H. W. Feilden showed twenty-eight years ago,

That ice is capable of shearing off and thrusting before it large masses of rock, is also an hypothesis, for which the author, after doing his best to study ice-work in the field, can find no valid evidence. He maintains that these sections do not suggest the

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above explanation. Passing over that 'north of Reed' as unimportant, we come to the Pinner's Cross Pit. Here the Boulder-Clay is not, strictly speaking, 'banked-up' against the Chalk, as stated by Mr. Woodward, but occupies a hollow in the Chalk, as described by the late Mr. Penning. The Chalk has a fairly-high dip, but there is little other sign of mechanical disturbance. In the pit south-west of Newsell's Park, a shear-plane can indeed be seen in one face, which, however, is explicable by ordinary faulting; and on the same face there are (or were) some small clayey patches. A few yards farther to the south-east, Boulder-Clay appears above the floor of the pit, filling an arched cavity. This is, no doubt, a singular position, but there is nothing to show that the Chalk has been thrust over the Clay. The author suggests that, as in Möen and occasionally in Rügen, the Clay has been carried down from above into cavities already formed in the Chalk, and quotes a case from the latter island of a clay-filled cavity, which was connected with the surface and might have yielded a section like that in the above-named pit. Penning's diagram shows (probably nearly over this spot) Boulder-Clay resting upon the Chalk. So the author maintains that, even if the fundamental hypotheses be true, they are not applicable to these sections.

3. 'Brachiopod Homœomorphy: *Pygope*, *Antinomia*, *Pygites*.' By S. S. Buckman, F.G.S.

April 4th.—R. S. Herries, M.A., Vice-President, in the Chair.

The following communications were read:—

1. 'On a Case of Unconformity and Thrust in the Coal-Measures of Northumberland.' By Prof. G. A. L. Lebour, M.A., M.Sc., F.G.S., and J. A. Smythe, M.Sc., Ph.D.

The sections described occur on the coast north of the Tyne, near Whitley Sands, between Table Rocks and Briar-Dene Burn. The base of the 'Table-Rocks Sandstone' is found to rest unconformably upon a series of alternating shales and sandstones, among which is a well-marked band of clay-ironstone crowded with *Carbonicola acuta*, one of those 'mussel-bands' which are found to be perhaps the most remarkably-persistent strata in the North-of-England Carboniferous rocks. According to the correlation of the Geological Survey, this particular band is the one that is well known as occurring a little way above the Low-Main coal-seam. The entire junction, so far as it can be seen at the base of the cliffs and on the foreshore, many parts of which are only swept clear during exceptional weather, has been studied as opportunity offered during

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a series of years. The unconformity is shown by discordance in dip, by overlap of the Table-Rocks Sandstone, and by the existence of a pebble-bed, containing fragments of the mussel-band and other parts of the underlying series, in the lower part of this sandstone. But the upper, more massive, beds in the section have been thrust in a northerly direction over the lower and more yielding beds, the plane of gliding corresponding accurately along parts of the section with the plane of erosion. Towards the north of the section the beds of the upper series are weakened by intercalated bands of shale, and then differential action has been set up. The result is that the thrust-plane is no longer a simple one coinciding with the unconformity, but extends some way above it. The effects of the thrust are seen in the ploughing-up, folding, and faulting of the lower series, in the penetration of tongues of sandstone from the upper series into the lower, in the curling-up and shattering of the pebble-bed, in the puckering and hardening of the shale, and in the blending of fragments of the various rocks subjected to its influence. The marks of intense action are practically confined to the surface of the beds of the lower series. The action of the thrust is markedly rhythmical, and its effect is to mask the unconformity; and the great discordance in dip which is at times produced is no criterion of the unconformity. Although the amount of rock removed by intro-Carboniferous denudation from the lower series is unknown, chemical and other evidence is given to show that the lower series has undergone weathering and some leaching-out of constituents in the interval between the two series.

2. 'The Carboniferous Succession below the Coal-Measures in North Shropshire, Denbighshire, and Flintshire.' By Wheelton Hind, M.D., B.S., F.R.C.S., F.G.S., and John T. Stobbs, F.G.S.

April 25th.—J. E. Marr, Sc.D., F.R.S., Vice-President, in the Chair.

The following communications were read:—

1. 'Trilobites from Bolivia, collected by Dr. J. W. Evans in 1901–1902.' By Philip Lake, M.A., F.G.S.

2. 'Graptolites from Bolivia, collected by Dr. J. W. Evans in 1901–1902.' By Ethel M. R. Wood, D.Sc.

3. 'The Phosphatic Chalks of Winterbourne and Boxford (Berkshire).' By Harold J. Osborne White, F.G.S., and Llewellyn Treacher, F.G.S.

Data collected in the district dealt with in this paper suffice to show that the more or less Phosphatic Chalks above the *Uintacrinus*-band lie in a trough or basin, the formation of which antedates the

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deposition of the Reading Beds. When the area of observation is extended, it is found that the *Uintacrinus*-Chalk of that tract itself lies in a structural depression. In view of the development of phosphatic and of hard, rocky beds, indicative of slow and interrupted sedimentation, in the underlying *cor-anguinum*-zone, it seems not unlikely that this basin is an original or inherent feature of the Chalk directly attributable to a local attenuation of that zone; but the authors believe the depression to be due mainly to differential earth-movements, of which there are many indications. The exposures are found in the eastern (Winterbourne) and western (Boxford) sides of a spur lying in the angle between the converging valleys of the Lambourn and the Winterbourne. On the eastern side the following succession is recognized:—

1. Reading Beds of the Borough-Hill outlier.		
2. Chalk with some flints.		Zone of *Actinocamax quadratus.*
3. Phosphatic Chalk.		
4. Phosphatic Chalk.	*Marsupites*-band.	Zone of *Marsupites.*
5. Very feebly-phosphatic Chalk.	*Uintacrinus*-band.	
6. Chalk with tabular flints.		Zone of *Micraster cor-anguinum.*
7. Chalk with flints.		

and on the western side:—

1. Reading Beds.		
2. Phosphatic Chalk.		Zone of *A. quadratus.*
3. Phosphatic Chalk.	*Marsupites*-band.	
4. Feebly-phosphatic to normal Chalk.	*Uintacrinus*-band.	Zone of *Marsupites.*
5. Normal flinty Chalk.		
6. Phosphatic Chalk.		Zone of *Micraster cor-anguinum.*
7. Normal flinty Chalk.		
8. Phosphatic Chalk.		
9. Normal flinty Chalk.		

The maximum thickness of the Phosphatic Chalk Series is about 130 feet. Detailed accounts of the various exposures are given, as also lists of fossils from the various bands, and an account of the microscopic features of washed residue from the softer chalks of the pit a quarter of a mile north-west of Winterbourne Church. This yields: (1) pieces of *Inoceramus*-shell, the majority unphosphatized; (2) foraminifera, frequently phosphatized; (3) angular chips of scales, bones, and teeth of fishes; (4) irregular, angular or subangular lumps and platy pieces of calcite, probably in the main fragments of oysters; (5) rod-like objects, smooth, cylindrical, or tapered — probably spines of echinoids; (6) coprolites of small fishes, etc.; (7) tests of entomostraca; (8) quartz in subangular and well-rounded grains; (9) black granules, apparently of iron-oxides; (10) rich-brown, polished phosphatic concretions; and (11) dull-green grains, imperfectly rounded. These constituents are enumerated as nearly as possible in order of abundance.

The Winterbourne-Boxford phosphates have a known range in time considerably greater than those of Taplow. Their advent far down in the *cor-anguinum*-zone is especially interesting; for in

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England, as Mr. Jukes-Browne has remarked, that subdivision of the Chalk almost everywhere 'presents the appearance of having been quietly and continuously accumulated in water that was seldom disturbed by bottom currents,' albeit a tendency to develop hard bands at one horizon, at least, is apparent in the western part of the London Basin. The Phosphatic Chalks of Winterbourne and Taplow evidently mark places on the sea-floor particularly liable to the impingement of strong currents, and may mark places above which the water commonly had a gyratory motion. In any case, their zonal range argues a marked degree of stability in the current-system of the body of water in which they were laid down.

May 9th.—Aubrey Strahan, M A., F.R.S., Vice-President, in the Chair.

The following communications were read :—

1. 'The Eruption of Vesuvius in April 1906.' By Prof. Giuseppe de Lorenzo, For.Corr.G.S.

After the great eruption of 1872 Vesuvius lapsed into repose, marked by merely solfataric phenomena, for three years. Strombolian activity followed, varied by lateral outpourings of lava in 1885, 1889, 1891, 1895, etc., and by outbursts from the principal crater in 1900 and 1904. Fissuring of the cone and slight outpourings of lava began in May 1905, and continued until April 4th, 1906, when the first great outburst from the principal crater occurred, accompanied by the formation of deeper and larger fissures in the southern wall of the cone, from which a great mass of fluid and scoriaceous lava was erupted. After a pause the maximum outburst took place during the night of April 7th & 8th, and blew 3000 feet into the air scoriæ and lapilli of lava, as well as fragments derived from the wreckuge of the cone. The south-westerly wind carried this ash to Ottajano and San Giuseppe, which were buried under 3 feet of it, and even swept it on to the Adriatic and Montenegro. At this time the lava which reached Torre Annunziata was erupted. The decrescent phase began on April 8th, but the collapse of the cone of the principal crater was accompanied by the ejection of steam and dust to a height of from 22,000 to 26,000 feet. On April 9th & 10th the wind was north-easterly, and the dust was carried over Torre del Greco and as far as Spain ; but on April 11th the cloud was again impelled northward. The ash in the earlier eruptions was dark in colour, and made of materials derived directly from the usual type of leucotephritic magma; but later it became greyer, and mixed with weathered clastic material from the cone. The great cone had an almost horizontal rim on April 13th, very little higher than Monte Somma, and with a crater which possibly

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exceeds 1000 feet in diameter; this cone was almost snow-white from the deposit of sublimates. Many deaths were due to asphyxia, but the collapse of roofs weighted with dust was a source of much danger, as was the case at Pompeii in A.D. 79. The lava-streams surrounded trees, many of which still stood in the hot lava with their leaves and blossoms apparently uninjured. The sea-level during the 7th and 8th of April was lowered 6 inches near Pozzuoli and as much as 12 inches near Portici, and had not returned to its previous level on April 18th. The maximum activity coincided almost exactly with the full moon, and at the time the volcanoes of the Phlegræan Fields and of the Islands remained in their normal condition. The author believes that this eruption of Vesuvius is greater than any of those recorded in history, with two exceptions—those of A.D. 79 and of 1631.

2. 'The Ordovician Rocks of Western Caermarthenshire.' By David Cledlyn Evans, F.G.S.

The ground dealt with is practically identical with that examined by the late Thomas Roberts, whose notes were published in 1893. It extends from the River Cywyn on the east to the Tave on the west, and from the base of the Old Red Sandstone on the south to the top of the *Dicranograptus*-Shales on the north. The area is, in the main, a denuded anticlinal fold with an east-and-west axis, complicated by minor folds, faulting, thrusting, and crushing. The succession on the northern limb of the anticline is as follows:—

BALA-CARADOC.	
LLANDEILO.	*Dicranograptus*-Beds. *Asaphus-tyrannus* Beds.
LLANVIRN.	*Didymograptus-Murchisoni* Beds. *Didymograptus-bifidus* Beds.
ARENIG.	*Tetragraptus*-Beds.

The succession on the southern limb is similar, but passes up into higher beds, as shown below:—

LOWER LLANDOVERY.	
BALA-CARADOC.	Slade Beds. Redhill Beds. Sholeshook Beds. Robeston-Wathen Limestone.
LLANDEILO, LLANVIRN, and ARENIG as above.	

The crushing and fracturing of the rocks has brought together into a small area facies of rocks differing lithologically and in faunas, the difference being well shown in corresponding beds on

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opposite sides of the anticline. The majority of faults are strike-faults, many of them being thrusts; these are sometimes so close together that the structure cannot be fully displayed on the 1-inch scale. Cleavage affects the rocks on the northern limb, but is almost absent on the south; it was induced at a later date than the faulting. The chief igneous rocks are intrusions of diabase; and there are ashes in the Llanvirn and Llandeilo beds and an ashy shale in the Arenig.

A description of the entire succession of rocks is given, accompanied by lists of fossils, and each division of importance is followed through its whole extent as shown in the area. In correlating the rocks, it is shown that the Middle and Upper Arenig rocks compare with those of St. David's; the Llanvirn rocks are very similar, lithologically, to the beds of Llanvirn and Abereiddy, and the faunas are practically identical; the Llandeilo rocks are in the main like those of Pembrokeshire, but the *Leptograptus*-division of the *Dicranograptus*-Beds closely resembles the Rorrington Flags of Shropshire; the Bala-Caradoc rocks fall into the divisions already established by Marr & Roberts in an adjoining area; and the rocks assigned to the Lower Llandovery are barren of organic remains, as they are elsewhere.

May 23rd.—R. S. Herries, M.A., Vice-President, in the Chair.

The following communications were read:—

1. 'On the Importance of *Halimeda* as a Reef-forming Organism; with a Description of the *Halimeda*-Limestones of the New Hebrides.' By Frederick Chapman, A.L.S., F.R.M.S., and Douglas Mawson, B.E., B.Sc.

2. 'Notes on the Genera *Omospira, Lophospira*, and *Turritoma*; with Descriptions of New Species.' By Miss Jane Donald.

June 13th.—Sir Archibald Geikie, D.C.L., Sc.D., Sec.R.S., President, in the Chair.

The following communications were read:—

1. 'Recumbent Folds produced as a Result of Flow.' By Prof. William Johnson Sollas, Sc.D., LL.D., F.R.S., F.G.S.

Prof. Lugeon, in his treatise on the pre-Alps of Chablais, has described a series of recumbent folds so greatly exceeding in horizontal extension their vertical thickness, that they are commonly spoken of as sheets rather than folds; they lie with remarkable flatness one on the other, and, as a rule, those higher in the series extend

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farther to the front than those below, a feature referred to as 'déferlement.' The roots of several of the lower of these folds are visible in the high Alps adjacent, but the roots of the higher folds, which form the pre-Alps, must be sought in the zone of Mont Blanc and the Briançonnais. Thus some of the uppermost folds may have surmounted the obstacle presented by Mont Blanc, on their way to the front in the pre-Alps. Many of the features presented by recumbent folds are more suggestive of flowing than bending. Experiments have been made with pitch-glaciers (poissiers) in which an obstruction had been placed. In this way folds were produced, one of which was not unlike the Morcles fold behind the Diablerets, another was like the Pilatus, and yet another like the Sentis, and the fourth compared with the overslide of the Bavarian front; all four exhibit déferlement. In this experiment the lower limb of each fold is adjacent to the similar limb of its neighbours; but, in another experiment, in which two obstacles were used, the results were nearer to those seen in the mountains where the lower limb of a superior fold reposes on the upper limb of the fold immediately beneath it. Movement of this character may possibly explain the want of continuity of certain beds at the conclusion of the movement.

2. 'The Crag of Iceland—an Intercalation in the Basalt-Formation.' By Dr. Helgi Pjetursson.

The existence of fossiliferous deposits on the west coast of Tjörnes, Northern Iceland, has been known for nearly 160 years. Mörch enumerates 61 species of mollusca, and concludes that the temperature must at that period have been much milder than at present. Gwyn Jeffreys and Searles Wood, from the shells, considered that the deposit could not be younger than Middle Red Crag, but Mr. Starkie Gardner was inclined to assign a greater age to it. Dr. Thoroddsen thinks that these Crags are younger than the 'Old Basalts' of Tjörnes. The author finds, however, that, at a height of about 500 feet above the sea, they are overlain by the 'Eastern Basalts,' and are indurated and altered by them. Thus there is a fossiliferous intercalation, over 500 feet thick, occupying part of the great gap between the Tertiary and the Pleistocene rocks, the latter containing indurated ground-moraines. The basal layer of the Pleistocene Series is fossiliferous, and has yielded 22 species of mollusca, 20 of which represent a highly-arctic fauna (with *Yoldia arctica*), such as is, at the present day, found living along the coasts of Spitsbergen. Certain of the larger basalt-dykes are cut off at the base of the Crag. The absence of the Crag-deposits from other localities is explained by the erosion of the coast-line.

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THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

SEPTEMBER 1906.

XXVI. *On the Origin of the β Rays emitted by Thorium and Actinium.* By M. LEVIN, *Ph.D.* (*Göttingen*)*.

THE following research was undertaken to determine which of the products of thorium and actinium emitted β rays. The radiations from the radium products have been completely examined, and it has been shown that penetrating β rays are emitted only from the last of the rapidly changing products of radium, namely radium C†. H. W. Schmidt‡ has, however, recently obtained evidence that the supposed rayless product radium B does emit β rays, but of much less penetrating power than those from radium C. We are not considering here the emission of slow-velocity electrons from the α rays, products first observed by J. J. Thomson§, and later examined by Miss Slater‖. These electrons are emitted at a much slower speed, and have an insignificant power of penetrating matter compared with the ordinary β rays. Ewers¶ has recently determined that the velocity of the slow moving electrons emitted from a plate coated with radiotellurium is about 3×10^8 cms. per second, or less than one-fiftieth of the average velocity of the β rays from radium C. In this paper we are considering only the

* Communicated by Professor E. Rutherford, F.R.S.
† Rutherford, Phil. Trans. A. 204. p. 169 (1904).
‡ *Phys. Zeitschr.* vi. p. 897 (1905).
§ Proc. Cambr. Phil. Soc. Nov. 14, 1904.
‖ Phil. Mag. Oct. 1905.
¶ *Phys. Zeitschr.* vii. no. 5 (1906).

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emission of β rays of considerable penetrating power; for example, those which are able to traverse a thickness of 0·1 mm. of aluminium with an absorption of only a few per cent.

Thorium X.

In its radioactive transformations thorium gives rise successively to radiothorium*, thorium X, and the thorium emanation†. The latter forms the active deposit, which contains three distinct products—thorium A, thorium B‡, and thorium C§. Rutherford showed that the emanation of thorium emitted only α rays, and that the active deposit emits α, β, and γ rays. Since thorium A is a rayless product, the β rays from the active deposit come only from thorium B or thorium C. It was not known with certainty whether or not thorium, radiothorium, or thorium X emitted β as well as α rays‖.

In order to settle definitely these points it was necessary to obtain thorium and thorium X as far as possible free from all other products, and to examine whether or not β rays were emitted.

Thorium X was separated from thorium in the usual way by precipitating the thorium with ammonia. For the separation of the active deposit from thorium X several methods can be employed. As the active deposit is formed from the emanation, a solution of thorium X can be partly freed from the active deposit by removing the emanation either by rapid boiling or by aspirating air through the solution, until the thorium A, B, and C, initially present in the solution, have been transformed. Another possible way is to separate chemically or electrochemically thorium A, B, and C from the thorium-X solution by one of the methods employed in the investigations of Schlundt and Moore¶ and of v. Lerch**. It was found, however, simplest to volatilize the active deposit by heating the thorium X to a high temperature††. The following procedure was adopted. The ammonia-filtrate was freed from traces of thorium remaining in solution by precipitating several times with ammonia from a concentrated solution; the solution of thorium X was then dried on a

* O. Hahn, *Jahrb. d. Radioaktivität*, ii. (1905).
† Rutherford & Soddy, Phil. Mag. May 1903.
‡ Rutherford, Phil. Trans. A. 204. p. 169 (1904).
§ O. Hahn, Phil. Mag. June 1906.
‖ See Rutherford, 'Radioactivity,' 1905, p. 363.
¶ Journal of Phys. Chemistry, ix. (1905).
** *Wien. Ber.* March 1905.
†† Miss Gates, Phys. Rev. May 1903.

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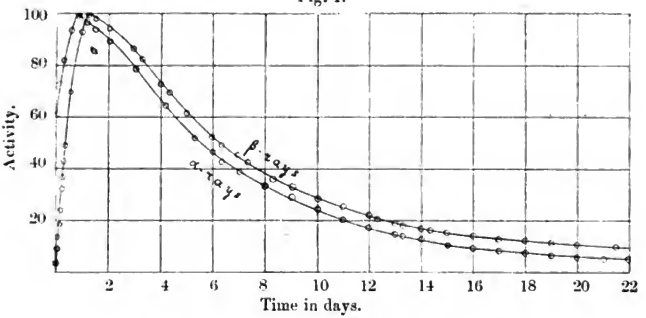
platinum dish and heated with a blowpipe. As soon as the platinum plate was cold the β ray activity was measured. The thorium X was placed beneath a Wilson electroscope, the bottom of which was covered with an aluminium plate of 0·06 mm. thickness. This is sufficient to absorb all α rays. The α activity was measured with a Dolezalek electrometer.

The results of one experiment are shown in fig. 1. In

Fig. 1.



the figure the abscissæ denote the time in days reckoned from the end of the heating, the ordinates the activities, the maximum activities being taken as 100. The shape of the α and β curves is very similar. The β ray activity is initially only 3·5 per cent. of the maximum value reached one day later. The α ray activity, on the other hand, has an initial value of 60 per cent. The rapid increase of the α and β ray activity is to be expected since the thorium X produces the emanation, which in turn is transformed into the active deposit. A maximum is reached when the active deposit is in approximate equilibrium with the thorium X. The α and β ray activity then decay exponentially according to the period of thorium X, *i. e.* the activity falls to half value in about 4 days. The character of these curves is in close agreement with that to be expected from the theory of successive changes.

The above experiment was made with a preparation of thorium X from radiothorium, kindly provided by Dr. Hahn. Other experiments made with thorium X separated from commercial thorium behaved in a similar manner. The experiments are, however, more simply made with the thorium X separated from radiothorium. The thorium X separated

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from commercial thorium always contains impurities, which are difficult to remove, and the complete volatilization of the active deposit was not easy under such conditions. Thorium A was almost completely driven off by heating the platinum dish for 10 minutes with a blowpipe, but some thorium B still remained behind after heating to the melting-point of iron by means of a blowpipe-furnace, kindly provided by Dr. Stansfield. Consequently, both the α and β ray activities were initially higher than those shown in fig. 1. As thorium B decayed, the α and β ray activities at first decreased, passed through a minimum and gradually increased again in exactly the same way as in the curves shown in fig. 1.

In a recent investigation, Dr. Hahn has shown that the active deposit of thorium contains two distinct α ray products, called thorium B and thorium C*. If each of these four α ray products present with thorium X supplies an equal proportion of the total equilibrium activity, the activity due to thorium X and the emanation together should be 50 per cent. of the total. Now when the active deposit is completely removed, the initial α activity observed is due to thorium X and the emanation together; for, in consequence of its rapid rate of change, the latter reaches its equilibrium value in a few minutes after the platinum dish is removed from the furnace. The maximum activity is reached about one day later, when the active deposit is in equilibrium with the thorium X. During this time the activity of the thorium X has been diminishing, and consequently the maximum activity observed is less than the theoretical value, which would have been obtained if the active deposit had initially been in equilibrium with the thorium X. This theoretical value can be determined in a simple way by extrapolation from the curve of decay of thorium X. In this way it was deduced that the theoretical initial value was 126, showing that the activity of thorium X and the emanation together is 48 per cent. of this maximum, thus agreeing with the assumption made above.

In a similar manner, the initial percentage of the β activity is reduced to 2·61 per cent. In other experiments the values found were 3·06, 2·75, and 2·92 per cent. If thorium X gave out β rays, one would expect about 50 per cent. for the initial value of the β activity—an amount of another order of magnitude from that observed. Therefore we conclude that the transformation of thorium X into the emanation is accompanied only by the emission of α rays.

It will be seen from fig. 1 that the α ray activity reaches

* *Loc. cit.*

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a maximum earlier than the β ray activity. The reason of this can be simply explained when we take into consideration the fact that thorium X does not emit β rays*. If the exponential decay curve of thorium X is drawn through the point of initial α ray activity, this decay curve represents the loss of activity of thorium X and the emanation together. If the ordinates of the β ray curve are reduced in the ratio of 1 : 2, such a curve represents the variation of the α ray activity due to the active deposit alone. If these two curves are superimposed, we obtain the observed curve of the variation of the α ray activity. It is easy to see without calculation that the maximum of this curve is reached earlier than that of the β ray curve.

A few remarks may be added regarding the period of decay of thorium X. In the original experiments made by Rutherford and Soddy, the time required for the activity of thorium X to decay to half value was found to be about four days. v. Lerch finds this period to be 3·637 days. In the above experiments, the period deduced was 3·67 days for the α ray activity, and 3·66 days for the β ray activity. In these experiments with radiothorium, a residual activity was observed after the thorium X had completely lost its activity. Subtracting this residual activity, the logarithms of the activity due to thorium X were found to form a straight line. This residual activity was found to be due to radiothorium and a trace of radium, and in accounting for this the values given above should be slightly diminished. The average of some other experiments, including a recovery curve of thorium, gave 3·65 days as the period. These numbers are thus in good agreement with the value given by v. Lerch.

Thorium.

In order to obtain thorium free from thorium X and its later products, the following procedure was adopted. The thorium was precipitated 8 to 10 times by ammonia in order to completely remove the thorium X. The precipitated thorium was then heated to remove the active deposit, which is not soluble in ammonia. For this, the temperature of the blowpipe was not sufficient, so recourse was had to a blast furnace. The α and β ray activities of the thorium were then examined. Both the α and β ray activities initially diminished a few per cent., showing that the active deposit had not been completely volatilized. After passing through a minimum, the activities steadily increased, reaching a

* For the following, *cf.* Rutherford, 'Radioactivity,' 1905, chap. ix.

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maximum about a month later. The lowest value of the α ray activity obtained in this way was about 23 per cent. of the maximum. This value lies between the lowest values observed by Rutherford and Soddy. For the minimum β ray activity, two separate experiments gave 4·0 and 2·8 per cent. It is thus clear that neither thorium nor radiothorium emits β rays. The small value obtained is undoubtedly due to the presence of some of the active deposit, which was not volatilized at the highest temperatures employed.

It might be thought possible to remove the active deposit from the thorium in equilibrium by heating, without first separating the thorium X. An experiment in this direction was tried but was unsuccessful; it is impossible to volatilize all the active deposit when it is distributed throughout the mass of the thorium.

Actinium X.

From a study of the actinium products it is known that the actinium-emanation emits α rays, and it will be seen later that it emits no β rays. The active deposit consists of two products, actinium A and actinium B*; the former is a rayless product, whilst the latter emits α, β, and γ rays. Godlewski†, in his work on actinium, pointed out that actinium X probably emitted β as well as α rays. We have seen that the product thorium X in thorium, which corresponds to the product actinium X in the actinium, emits only α rays. The close analogy between thorium and actinium both in the character and number of their products and the radiations emitted by them, suggested a closer examination to see whether actinium X emitted β rays.

In these experiments, the emanating substance of Giesel was used. The actinium X was separated by ammonia. Since the active deposit of actinium is more readily volatilized‡ than that of thorium, the method of heating was employed to get rid of the active deposit, which is separated with actinium X. But the elimination of the active deposit by this method was more difficult than in the case of thorium, since the residues remaining in the filtrate with ammonia prevent the complete escape of the volatile products. It was

* Miss Brooks, Phil. Mag. Sept. 1904.

† Godlewski, Phil. Mag. July 1905.

‡ According to measurements of the writer, which will be published later, the temperatures at which actinium A and actinium B commence to volatilize are 400° C. and 700° C. respectively.

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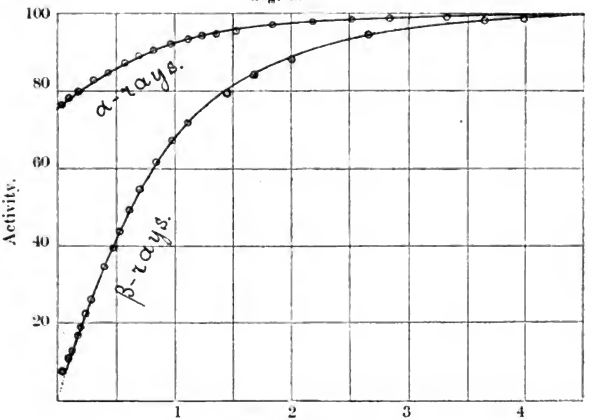
found necessary to remove these residues by successive precipitations before it was possible to drive off most of the active deposit by heat.

The results are shown graphically in fig. 2, the ordinates

Fig. 2.



giving the activities, and the abscissæ the time in hours measured from the end of the heating.

The initial decrease of the α and β activities found in some experiments with thorium X was not observed with actinium X. The curves in some experiments were nearly horizontal for the first few minutes, indicating that the active deposit had not been completely removed. The activities rose rapidly to a maximum in the course of about three hours in consequence of the formation of the active deposit by the emanation. By producing the rise curve backwards in order to obtain the activity at the moment the heating ceased, it was found that the initial β activity in different experiments was 2·5, 3·5, and 5·0 per cent. of the maximum β activity. The initial α ray activity, on the other hand, was betwen 72 and 76 per cent. of the maximum. These values would be only slightly lowered by taking into account, as in the case of the experiments with thorium X, the fact that the equilibrium activity in the beginning should be higher than the maximum

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which is finally reached. In this case, the time required to reach the maximum is only three to four hours, and in this short time actinium X has not lost 1 per cent. of its initial activity. When actinium X is in equilibrium with its products three α products are present, namely, actinium X, the emanation, and actinium B. No product analogous to thorium C exists in actinium. This point has been examined by Dr. Hahn working in the same laboratory as the writer. Assuming that each of these products supplies an equal fraction of the total activity, the initial value of the α activity would be about two-thirds of the maximum, whilst no lower value than 72 per cent. has been experimentally observed.

From the measurements of the β ray activity we must conclude that actinium X does not emit β rays, but only α rays, and in this respect is quite analogous to thorium X.

Since the actinium emanation is in equilibrium with actinium X from the beginning of these observations, this experiment also shows clearly that the emanation does not emit β rays.

Godlewski concluded that actinium X emitted β rays, since he was unable to reduce the β activity to less than half its maximum value by heating to a high temperature. It appears probable that the residues present with his actinium X prevented the escape of the active deposit. Under more favourable conditions, we have seen that all but a few per cent. of the β rays can be got rid of.

Since the active deposit of actinium consists of the rayless product actinium A of period of about 36 minutes, and of actinium B period 2·15 minutes, it is to be expected that the β ray recovery curve should be complementary to the decay curve of the active deposit corresponding to a long exposure to the emanation. This was found to be the case. From an examination of a number of recovery curves, the mean value of the period of actinium A was found to be 36·0 minutes—a value agreeing closely with that found by Bronson, and by Meyer and Schweidler, and by Hahn and Sackur.

Actinium.

By means of successive precipitations of actinium by ammonia in order to remove actinium X, Godlewski was able to obtain actinium which was almost free from α and β rays, and consequently concluded that actinium was a rayless substance. In the course of this research, the experiments of Godlewski were repeated in the manner described by him. The recovery curves of actinium measured by the α and β

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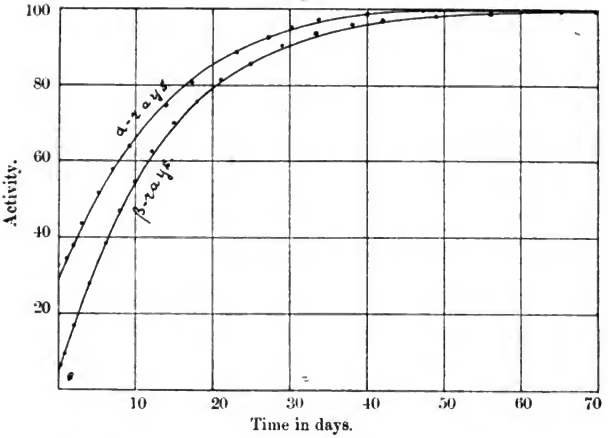
rays, after separation of actinium X, are shown in fig. 3*. The period of actinium X deduced from the recovery curves was about 10 days; a value closely agreeing with that originally found by Godlewski, and later by Meyer and Schweidler.

Fig. 3.



From the small initial value of the β-ray curve, it is seen that actinium does not emit β rays. In other experiments the initial β activity was between 3 and 5 per cent. of the final maximum.

There is, however, one marked point of difference between my results and those obtained by Godlewski. We have seen that the latter found that actinium freed from actinium X showed very little α ray activity, while in my experiments the α ray activity could not be reduced to less than about 28 per cent. of its maximum value. The cause of this discrepancy was at first very puzzling, and an account of the experiments may be given that have been made to clear up

* The measurements of the activity were made twice a day at the beginning of the recovery curve and daily towards the end. Only a few points are reproduced in the figure to show the general accuracy of the measurements. There was, on several occasions, a sudden irregularity in the activity of the preparation, which could not be accounted for by supposing it to be due to errors of measurement, for the value found for the activity of a standard preparation of uranium oxide tested under the same conditions showed no variation. Similar alterations have been noted by Godlewski.

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this point. In the first place, it can be clearly shown, by several distinct methods, that the residual α ray activity of actinium cannot be ascribed to the presence of actinium X. The very small initial value of the β ray activity shows that very little active deposit was present. If an appreciable amount of actinium X had been present, the α and β ray activity should have rapidly increased in the first three hours (see fig. 2). But no such effect was observed. This was corroborated by another distinct method. A current of air was passed through the actinium solution and the emanation rapidly carried into an electroscope. The rate of discharge of the electroscope was very small compared with that observed when an equal current of air was passed through the solution of actinium X previously separated from the actinium. Such a result conclusively shows that the α activity of actinium itself cannot possibly be ascribed to the presence of actinium X.

An unexpected result was observed too with another preparation of actinium. Both α ray and β ray activities rose rapidly and reached a maximum after 20 days instead of the usual 60 days. Half of the maximum value was reached in about 4 days instead of the 10 days, which is to be expected if the recovery of actinium is due to the production of actinium X. In this case, too, the initial β activity was very small, but the initial value of the α activity was about 40 per cent., a value which could not be explained by the known radioactive changes of actinium. The above preparation of actinium, which behaved so irregularly, was obtained by filtrating the precipitate immediately after adding the ammonia instead of boiling for some time on a water-bath, as recommended by Godlewski.

While this experiment was being repeated, in order to examine more closely this anomalous recovery curve, Dr. Hahn, working in the same laboratory on experiments he had begun several months earlier, came independently to the conclusion that there existed a new α ray product of actinium intermediate between actinium and actinium X, which he has called radioactinium. This product emits α rays and has a period of transformation of about 20 days. A preliminary account of this new product has been given by Dr. Hahn in 'Nature' (April 1906).

Hahn separated the radioactinium from actinium by means of a sulphur precipitate, which carries down the radioactinium, and he has shown that the actinium made inactive in this way recovers its activity with a period of about 20 days.

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He has further shown that the radioactinium gives rise to actinium X. The activity of radioactinium rises to a maximum in about 20 days, and after passing the maximum finally decays, according to an exponential law, with a period of about 20 days. The rise is due to the formation of actinium X and its successive products, as Hahn has clearly shown, by separating actinium X from a radioactinium preparation several weeks old.

In the light of these experiments the anomalous results, which I had observed, at once receive a simple explanation. The residual α ray activity observed was not due to actinium itself but to radioactinium. The recovery curve, which rose to a maximum in 20 days, is the recovery curve not of actinium itself, but of radioactinium. This was confirmed by the fact that the curve decreased again after passing the maximum. The unexpected initial value of the α activity can be explained in this way too. The radioactinium is separated free from actinium by immediately filtering after precipitation by ammonia. When the solution so treated stands for some time, it becomes turbid and, after boiling, the precipitate then consists of the actinium itself. The α ray activity of this precipitate is very small at first, but gradually rises for three months in consequence of the gradual production of radioactinium and its successive products.

It may be of interest to record some experiments which show conclusively that actinium X can be almost completely separated from actinium by successive precipitations with ammonia. After ten rapid precipitations, a current of air was aspirated through the actinium solution and passed into a suitable electroscope. The rate of discharge observed was less than 1/1000 of that observed, when the same current of air was passed through the solution of actinium X separated from it. Such a result shows clearly that hardly a trace of actinium X remained with the actinium.

Godlewski states that he found some difficulty in completely freeing actinium of α ray activity by successive precipitations with ammonia. This difficulty does not arise from the presence of actinium X, but of radioactinium. There is no doubt that in the course of the successive precipitations of actinium, Godlewski had in some way also separated radioactinium. The recovery curve of actinium given by him, which starts from a small initial value and rises with a ten-day period, would at once be explained if the β rays, and not the α rays, were used as a means of measurement.

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Conclusion.

The above investigations have shown that actinium freed from actinium X still possesses considerable α ray activity, and that this cannot be due to actinium itself, but must be ascribed to the presence of the product radioactinium, earlier discovered by Hahn. A method for separating radioactinium from actinium was found. Actinium itself is a rayless substance.

It is shown that the products thorium X, actinium X, and the actinium emanation emit only α rays. Thus among the numerous products of thorium and actinium, only the last of the series of products emit β rays. All the other products either emit only α rays or are rayless. The β rays emitted by thorium in equilibrium arise only from the product thorium B or thorium C, and most probably the latter; the β rays from actinium come only from actinium B. Such a result brings out clearly the remarkable similarity that exists between the modes of transformation of thorium and actinium, not only in reference to the number and chemical properties of their products, but also in regard to the rays emitted by them.

In the Bakerian Lecture, 1904, Rutherford pointed out that the emission of β rays only in the final rapid transformations of the radio-elements was probably more than a coincidence, and marked a definite stage in the career of disintegration of the atoms. In the case of thorium, or of actinium, the expulsion of a β particle, as well as of an α particle, occurs only in the last rapid change, and the atoms of the resulting product are either permanently stable or are transformed very slowly. In the case of radium, the expulsion of a β particle together with an α particle occurs only in the product radium C, and the resulting product radium D is comparatively stable. This peculiarity in the mode of transformation of the radio-elements is certainly striking, and may prove to be of some theoretical importance.

I sincerely thank Professor Rutherford for the help he has given to me during this work, which I undertook at his suggestion, and for his guidance in the study of radioactivity.

McDonald Physics Building, McGill University,
Montreal, May 15, 1906.

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XXVII. *On the Radioactive Matter in the Earth and the Atmosphere.* By A. S. Eve, *M.A., McGill University, Montreal**.

NUMEROUS observers have found that part of the ionization occurring within closed vessels is due to penetrating radiation. These rays are attributed to the presence of radium in the earth and of radium emanation in the atmosphere. Experiments are described in this paper which lead to an estimate of some of the magnitudes of the quantities involved.

Let K denote the number of ions, due to the γ rays only, generated per cubic centimetre per second at a distance of one centimetre from one gram of pure radium bromide, supposed concentrated at a point, and so placed that all the rays are absorbed in air. Then Q grams at a point will generate at a distance r, by the γ rays alone, a number of ions per c.c. per second denoted by N and given by

$$N = K\frac{Q}{r^2}e^{-\lambda r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ is the coefficient of absorption of the γ rays by air.

The value of K has been approximately determined by finding the number of ions produced per c.c. per second in closed vessels made of various metals. These vessels were placed at a distance of 303 cms. from 14·3 mg. of radium bromide sealed in test-tubes, and placed behind lead 7 mm. thick. The radium bromide used was that which Professor Rutherford found gave per gram a heating effect of 110 gram-calories per hour. The experiments were made in the Chemistry Building, where radium emanation has not been artificially introduced. Vessels were made of clean commercial sheets of lead, copper, zinc, iron, aluminium, and tinned iron. These vessels were 51 cms. high and 23 cms. in diameter (fig. 1). Two tubes were soldered into the sides, so that dry filtered air could be drawn into the vessels when required. Above each vessel was a small electroscope, 8 cms. high and 6·5 cms. in diameter, made of clean tinned iron. A rod within the electroscope carried a Dutch-metal leaf observed by a microscope with a graduated eyepiece. This rod was supported by sulphur, originally poured when in a molten state into a larger cylinder of ebonite. This arrangement gives excellent insulation and firm support. The rod terminated within the lower vessel in a hook to which a rod

* Communicated by Professor E. Rutherford, F.R.S.

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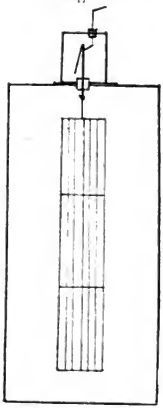
or wire cage was suspended. A rod, one centimetre in diameter, was first used, but difficulties arose in obtaining a saturation current. A cylindrical cage of clean copper wire about 40 cms. long and 7 cms. in diameter, was therefore substituted for the rod. Each vessel had then a capacity of about 11 electrostatic units.

Fig. 1.



The electroscopes could be interchanged when required, and insulated from their respective vessels by mica sheets. This most convenient method was recommended to me by Professor Rutherford. Bragg has shown the difficulty of securing complete saturation in the case of weak currents due to feeble radiation. But, by the arrangement described, the lower vessels could be earthed, the case of the electroscope maintained at a high potential by a constant battery, and the wire cage and gold-leaf charged to a yet higher potential. In this way saturation may be ensured between the large vessel and wire cage, whilst the gold-leaf is deflected to a convenient amount due to a moderate difference of potential between the gold-leaf and the case of the electroscope.

After deducting the corrections for natural leaks the results were as follows :—

TABLE I.

Substances.	Thickness in mm.	Ions per c.c. per sec.
Lead	1·6	550
Lead within lead	2·7	482
Tinned iron	·4	448
Zinc	·57	351
Copper	·57	345
Iron	·6	320
Aluminium	·4	297

There is clear evidence of much secondary radiation from the heavier metals. In the lead vessel 45 per cent. of the ions must be attributed to this cause, for there is no other reason why the ions should be more numerous in the lead than in the aluminium. It will be noted that tinned iron takes the place due to tin rather than iron. The order of ionization need not necessarily be that of atomic weight,

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because the secondary radiation due to transmitted rays is a function of the thickness traversed*.

If the penetrating radiation, discovered by McLennan, and by Rutherford and Cooke, consists of γ rays, it follows that the weak ionizations resulting from it within closed vessels made of different metals should have the same characteristics as those determined in these experiments.

It is now possible to calculate the value of K in formula (1). The value of N in the thin aluminium vessel was 297, and this is probably not more than 10 to 20 per cent. in excess of the value in free air †

Hence
$$K = 297 \times 303^2/\cdot 014$$
$$= 1\cdot 95 \times 10^9;$$

or adding a small correction for the absorption by 3 metres of air,

$$K = 2 \times 10^9.$$

But the γ rays have been absorbed to a considerable extent during their passage through 7 mms. of lead, and if we take the coefficient of absorption of lead to be initially ·64 ‡, the intensity has been diminished by the factor $e^{-\cdot 64 \times \cdot 7}$, so that the value of K is about

$$2 \times 10^9 \times 1\cdot 56$$

or

$$3\cdot 1 \times 10^9. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This result has been calculated from the ionization within a thin aluminium vessel, and it is, therefore, owing to secondary radiation, probably about 10 to 20 per cent. in excess of the value of K in the open air. The value of K within a zinc or copper vessel is $3\cdot 6 \times 10^9$.

On the Total Number of Ions generated per Second by the γ Rays from a Gram of Pure Radium Bromide.

It is now possible to determine the total number of ions which would be generated every second by the γ rays from Q grams of pure radium bromide, surrounded entirely to a great distance by air, so that the rays are all absorbed by it.

If λ is the coefficient of absorption of the γ rays in air, then at a distance r the number of ions produced per second per c.c. by the γ rays from Q grams is $\frac{KQ}{R^2}e^{-\lambda r}$, and in a

* Eve, Phil. Mag. Dec. 1904.
† Eve, *ibid.*
‡ McClelland, Phil. Mag. July 1904.

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spherical shell of thickness dr the total number of ions per second would be $4\pi r^2 dr$ times as great. Hence N, the total number of ions produced per second in all the surrounding air, is given by

$$N = \int_0^\infty \frac{KQ}{r^2} e^{-\lambda r} 4\pi r^2 dr = \frac{4\pi QK}{\lambda} \quad . \quad . \quad . \quad . \quad (3)$$

But McClelland has shown that the coefficients of absorption of the very penetrating rays vary approximately as the densities of the absorbing substances. The value of λ for water is ·034, hence λ for air is about ·000044. Thus the γ rays are reduced to half value after penetrating 157 metres of air, and to one per cent. after passing 1000 metres of air.

From (3) we find the total number of ions per second, due to the γ rays from 1 gram of pure radium bromide, when the rays are wholly absorbed in air, equal to

$$4\pi \times 3{\cdot}1 \times 10^9 / {\cdot}000044$$

or about

$$8{\cdot}9 \times 10^{14}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and this is probably an overestimate of the true value.

A comparison can therefore be made between the total number of ions, due to the α and γ rays respectively, produced per second by a gram of radium bromide, when all the rays are absorbed in air. Professor Rutherford has found that the α rays from one gram of radium bromide in radioactive equilibrium produce about $1{\cdot}24 \times 10^{16}$ ions per second if wholly absorbed in air. By an independent method the writer found about $1{\cdot}67 \times 10^{16}$ ions per second due to the same cause. Hence the γ rays appear to produce less than one-sixteenth part of the total number produced by the α rays. Without insisting on the exact value of this ratio, it is clearly seen that all the ions due to the α rays of a given mass of radium are more numerous to a considerable extent than all the ions due to the γ rays. And this might be expected, inasmuch as the heating effect of the γ rays is only a small percentage of that of the α rays.

On the Ions due to the γ Rays from the Emanation in the Atmosphere.

In a recent paper * in this Magazine it has been shown that the equivalent amount of radium bromide which would be required to provide the emanation actually present in one cubic kilometre of the atmosphere near the earth's surface is

* Eve, Phil. Mag. July 1905.

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between ·14 and ·5 gram. This quantity, which may be denoted by R, has been determined by a method which is quite independent of any theory of the radioactivity of the atmosphere. It was found by measuring the radiation from the active deposit on a negatively charged wire placed in a large vessel. The lower value, ·14, was that obtained by observations in the open air, and is probably the more correct. But since it is desired at the present moment to find a higher limit, let the larger value of R, namely ·5 gram, be taken. Then from the result (4) it is easy to calculate the total number of ions produced per second in a cubic kilometre of the atmosphere due to the γ rays from the active matter in it. The calculation will be made on the supposition that an infinite volume of air contains emanation, to the amount per c. k. above stated, uniformly distributed throughout it. A little consideration will show that a volume distribution gives results identical with distribution at points.

The number of ions produced per c. k. per second is

$$\cdot 5 \times 8\cdot 9 \times 10^{14}$$

or ·44 ions per c.c. per second.

It is clear that at the earth's surface, which is a boundary of the atmosphere, one-half of the above value should be taken, or ·22 ions per c.c. per second.

A more probable value is 14/50 of this, or ·06 ions per c.c. per second. In any case we must conclude that the ionization due to the γ rays of the active matter in the atmosphere is almost a negligible quantity and quite inadequate to account for the relatively large effects due to penetrating radiation observed by Cooke, Campbell, and others. For example, in a well-cleaned brass vessel, H. L. Cooke found $q = 13\cdot 6$ *, and by a screen of about 5 cms. of lead he was able to reduce this value to 9·1. No further reduction could be effected by thicker screens, so that about 4·5 ions per c.c. are produced in a brass vessel every second by the penetrating radiation near the surface of the earth. Campbell appears to find even larger results from this cause. Since it has been shown that the γ rays from the active matter in the atmosphere are quite insufficient to account for such large effects, let us consider the penetrating rays due to the active matter in the crust of the earth. If, for instance, radium is mixed with solid matter such as sand, or found in an ore, the penetrating radiation from the radium in the body would be larger than

* The amount originally stated was 7·5, but on the assumption that the ionic charge was $6\cdot 8 \times 10^{-10}$.

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the radiation from the limited quantity of emanation which escapes from it. It is not unreasonable to expect a much larger effect from the γ rays of the radium (and its successive products) in the earth than from the γ rays due to the emanation (or its quick transformation products) found in the atmosphere. The question may be investigated quantitatively.

Before doing so it is well to note the rate of absorption of the γ rays under different conditions.

TABLE II.

Substance.	Density.	λ.	Thickness in cms. to half value.
Lead	11·6	·5	1·4
Earth.....	2·7	·092	7·5
Water	1·0	·034	20·4
Air.....	·0013	·000044	15700

In this table the value of λ for water is taken from McClelland's results*, and the values of λ for air and for the constituents of the upper crusts of the earth are deduced from the density law which he established. It is assumed that 2·7 is a fair average value to take for the density of the constituents of the upper crust of the earth. It follows that the γ rays are cut down to one per cent. after traversing half a metre of the earth's crust, or 1000 metres of the atmosphere.

The Amount of Radium in the Earth estimated by the Penetrating Radiation.

We have seen that the number of ions produced by the penetrating radiation near the earth's surface is found from H. L. Cooke's experiments with a brass vessel to be about 4·5 per c.c. per second. If a small hollow cavity were taken many metres below the earth's surface, we should expect the ionization due to the penetrating radiation to be twice as great, because rays would come from above and below, not from below only as on the earth's surface. If $N'(=9)$ be the number of ions which would be produced per c.c. per second in a brass vessel, supposed to be within such a cavity, by the γ rays from the active matter in the earth surrounding it; if Q' be the equivalent number of grams of radium bromide which is a measure of the active matter per c.c. in the earth's crust; then, by a calculation precisely similar to

* Phil. Mag. July 1904.

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that made for the atmosphere (3), we have

$$N' = \frac{4\pi Q' K'}{\lambda'}$$

$$\therefore \qquad Q' = \frac{\lambda' N'}{4\pi K'} = \frac{\cdot 092 \times 9}{4 \,.\, \pi \,.\, 3\cdot 6 \times 10^9} = 1\cdot 8 \times 10^{-11}. \quad . \quad (5)$$

In this case K′ has been taken as for a zinc or copper vessel (Table I.), because these would involve some secondary radiation to an extent approximately the same as brass.

It is interesting to compare this value of the equivalent amount of radium bromide per c.c. of the earth's crust near the surface, found by measurement of the penetrating radiation, with that deduced by Professor Rutherford from consideration of the earth's temperature gradients. His result is about seventy times as small, or $2\cdot 6 \times 10^{-13}$, also expressed in terms of radium bromide. But Strutt has just published a paper *, stating that he finds by direct observation of rock-specimens that the radium present in the upper 45 miles of the earth's crust is alone sufficient to account for the existing temperature gradients in the earth. This indicates that he finds actually present thirty times as much radium as Rutherford showed was necessary to account for the earth's temperature gradients on the assumption that radium was distributed uniformly throughout the whole earth. In fact Strutt finds about $8\cdot 5 \times 10^{-12}$ grams of radium bromide, in equilibrium, as the average equivalent of the active matter per c.c. of the earth's crust. But this result needs a further correction, because Rutherford and Boltwood state † that 1 gram of uranium is associated with $3\cdot 8 \times 10^{-7}$ grams of radium, not with $7\cdot 4 \times 10^{-7}$ as previously stated. Therefore, from Strutt's investigations we may conclude that $4\cdot 25 \times 10^{-12}$ grams of radium bromide is a fair average measure of the active matter per c.c. of the earth's crust, and this result is four times as small as the amount which I have calculated as sufficient to account for the penetrating radiation. The discrepancy is not large considering the uncertainty of the distribution of active matter in the earth's crust ‡.

It will be seen, therefore, that whilst the active matter in the earth is of the right order to account for the observed magnitude of the effects of penetrating radiation, that in the atmosphere is not sufficient to do so. This conclusion is capable

* Proc. Roy. Soc., A. vol. lxxvii.

† American Journal of Science, July 1906.

‡ Measurements of the penetrating radiation in various mines would lead to a valuation and survey of the active matter in the earth's crust.

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of further verification by experiments, some of which I hope to undertake. For example, an electroscope well screened from the earth, vertically and obliquely, should show a greater decrease of ionization than one equally well screened from the atmosphere. Again, the penetrating radiation in a deep mine or well should be stronger than on the surface of the earth. Such experiments on penetrating radiation should be made out of doors, because the walls of a building made of brick or stone may contribute almost as much radiation as they absorb*.

As the matter stands at present the active matter in the earth seems to be the probable and sufficient cause of penetrating radiation. It should be added that experiments have been made by the writer with Ebert's apparatus at Montreal, and the average number of ions measured in the atmosphere indicate that the conditions at Montreal are normal and resemble those found over a large area in Europe.

Activity of Metals.

In the course of these investigations the "natural leak" of each vessel was determined before any radioactive matter was brought near them. The value of q was found from the relation

$$qSe = C\frac{V}{t},$$

where q is the number of ions produced per c.c. per second in the volume S, due to whatever causes; e is the ionic charge ($3{\cdot}4 \times 10^{-10}$): C is the capacity and V the fall of potential in time t.

In most cases it was necessary to let the vessels remain for one or two days after filling them with fresh air, before a minimum value could be obtained. The cause of this initial decrease has not been determined, but it has been previously observed by McLennan and others. It cannot well be attributed to fine dust, for I have found that tobacco smoke introduced into an electroscope produces an opposite effect.

The electroscopes were frequently interchanged between one vessel and another, so as to compare the natural leak due to one metal with that due to another, simply by measurement of the rate of fall of the gold-leaf of the electroscope, and apart from all calculations. The results were confirmed by repeated observations lasting for some months, but they are

* Observations of the penetrating radiation at the top of the Eiffel Tower and in a deep mine would throw much light on this subject.

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not altogether in good agreement with those found by several other observers.

In the lead vessel there were 95 ions generated per c.c. per second. In the case of all the other metals examined, there were 24 ions per c.c. produced per second. No difference could be detected between clean zinc, copper, aluminium, iron, tinned iron, and aluminium in lead. It was natural to suspect that the lead was coated with radioactive matter, but after cleaning it with emery-paper and washing it with hydrochloric acid, there was no diminution in the value of q. On the other hand, when the lead was lined with aluminium ·4 mm. thick, q was equal to 24. My results appear to agree with the work of other observers in the case of lead, for which all agree in finding a high value of q; and the effect is certainly not due to a surface-deposit merely. It was also found impossible to lower the value of q for zinc by a cleansing treatment of the surface. It appeared possible, from Campbell's work, that the results obtained in these experiments might be due to an accidental relationship between volume and surface. Cylindrical rolls both of aluminium and copper were therefore compared, 51 cms. long, having diameters 23, 17, 9·8 cms. successively, and the central wires or rods were charged to potentials varying from 150 to 420 volts. The results obtained for copper and aluminium, under any similar conditions, were identical in each case.

If the ionization is due to intrinsic radiations from the metals, it is extraordinary that so many different metals should give equal values. If the ionization is due to radioactive impurities in the metals, it is no less strange that the impurities should be distributed in so uniform a manner.

Corrections amounting to 7 or 8 per cent. were made for the current across the small electroscopes, after deducting which the results were those given in column 4.

TABLE III.

Metal.	Density.	Thickness in mm.	Natural Ionization. q.	Artificial Ionization. Q.
Lead	11·4	1·6	95	550
Lead in lead...	11·4	2·7	95	482
Tinned iron ...	...	·4	24	448
Copper	8·8	·67	24	345
Iron	7·2	·6	24	320
Zinc	7·0	·57	24	351
Aluminium ...	2·6	·4	24	297
Al. in lead.....	2·6	·4 in 1·6	24	244

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In this table the values of Q have for convenience been repeated from an earlier part of this paper. Q denotes the number of ions produced per c.c. per second in the vessels when 14·3 mg. of radium bromide were placed at 303 cms. from the centres of the vessels.

The six equal results in the third column are the mean of very many observations. Their equality is based on the fact that by comparative measurements no difference could be detected between the metals observed under identical conditions. Further experiments are in progress on the effect of screening the vessels and on the values of q for various diameters.

In several cases a gradual increase of ionization occurred due to an emanation proceeding from the sides of the vessel. Sometimes the rise curves had the character of those of radium emanation. The presence of an emanation has been noted by McLennan, Burton, and others. Further investigation of this emanation may throw light on the activity of metals. The effects were irregular, but they could always be eliminated by pulling dry clean air through the vessels for a few hours.

Ionization in the Atmosphere.

In a previous paper* the writer gave an account of the measurement of the number of ions produced per c.c. per second in a large iron tank, 2·5 cms. thick, $8 \times 1\frac{1}{2} \times 1\frac{1}{2}$ metres in dimensions. The value obtained was $q = 9{\cdot}6$; and as the active matter collected in the tank was sufficient to account for such ionization, the greater part of it was attributed to the emanation in the air in the tank. Mr. N. R. Campbell dissents from this view, writing thus:—"Surely if the quantity of emanation in the small volume of air inside the vessel causes an effect so large, the far larger quantity outside the vessel must have an appreciable effect even though the walls are so thick as to cut out all α rays."

This criticism arises from a misconception. The α rays are much more effective in producing ionization than the γ rays. The experiments and calculations given in this paper show that the α radiation of the emanation in the tank, despite its limited range, produces a much greater effect in the tank than the γ radiation from the emanation in all the air both inside and outside the tank to any distance, even supposing the walls of the tank were thin; and they were 2·5 cms. thick. And since Cooke found that 5 cms. of

* Eve, Phil. Mag. July 1905.

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lead cut off the penetrating rays entirely, 2·5 cms. of iron would reduce the penetrating radiation to about half value.

It is, however, possible that I underestimated the radiation from the sides of the tank and the penetrating radiation passing through them. On the whole, the tank experiments seem to prove that the emanation in the atmosphere is an important ionizing agent in virtue of its α radiation; and the excited activity actually collected in the tank bears out the results of direct ionization experiments.

It is certain that emanation exists in the atmosphere and that it produces ionization, but more experimental work must be done before any exact value can be assigned to the number of ions thus produced. My work and calculations point to the conclusion that the γ rays from the emanation in the atmosphere may be disregarded as a negligible quantity. The β rays may probably be ignored in the same manner. On the other hand, the α rays from the emanation do produce several ions per c.c. per second, whilst the penetrating rays from the active matter in the earth probably contribute the balance of those observed near the earth's surface.

Summary.

1. The ionization produced within closed vessels at a given distance from a given quantity of radium, and due to the γ rays alone, is dependent on the substance and thickness of the metal employed, owing to the secondary radiation (see Table I.).

2. From the results obtained within a vessel of aluminium an approximate value of K was obtained equal to $3{\cdot}1 \times 10^7$, where K is the number of ions produced per c.c. per second by the γ rays from 1 gram of pure radium bromide at 1 cm. from the source, supposing the rays are all absorbed in air.

3. Hence the total number of ions produced by the γ rays from 1 gram of pure radium bromide completely surrounded by air would be about 9×10^9 per c.c. per second.

4. Concluding from a former paper that the equivalent radium bromide required to supply the observed quantity of emanation per cubic kilometre of the atmosphere near the earth's surface is between ·14 and ·5 gram; it follows that the number of ions per c.c. per second due to the γ rays from the active matter in the air is between about ·06 and ·22 near the earth's surface.

5. The α rays from the emanation in the atmosphere produce much more ionization per c.c. than do the γ rays. The

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ratio may be estimated at 16 to 1, so that the α rays of the emanation produce about 2 to 7 ions per c.c. per second.

6. The penetrating radiation, observed by H. L. Cooke in a closed brass vessel, produced about 4·5 ions per c.c. per second. This cannot be attributed to the active matter in the atmosphere, but the radium present in the earth's crust appears to be of the right order to account for it.

7. About $1{\cdot}8 \times 10^{-11}$ grams of radium bromide is the estimated equivalent of the active matter per c.c. present in the earth's crust sufficient to account for the penetrating radiation. This appears to be about four times as large as the average amount found by Strutt by direct observation of rock specimens.

8. The ionization in the atmosphere is due partly to penetrating radiation from active matter in the earth, partly to α radiation from the emanation in the atmosphere.

I am most grateful to Professor Rutherford for his assistance in difficulties and for valuable suggestions in experimental work and theoretical considerations.

Montreal, June 1906.

XXVIII. *Supplementary Note on the Deflexions caused by a Break in an Overhead Wire carried on Poles.* By W. Hawthorne, *B.A., B.E., and* W. B. Morton, *M.A.**

In a paper published in the May number of this Magazine we gave an investigation of this question in which the wires were treated as inextensible. It is not difficult to modify the work so as to take into account the elastic stretch accompanying increase of tension. This is the object of the present note, in which, as in the former case, numerical values are worked out, so as to get an idea of the importance of this element in actual cases.

The notation of the former paper is retained, and in addition we use λ to indicate the elastic modulus of the wire, *i. e.* increase of stretching force divided by fractional increase of length. Thus we have

$$dT = \lambda \frac{ds}{s},$$

and, in order to take account of the change of w caused by extension, we write

$$T = wc = \frac{W}{s} c,$$

* Communicated by the Authors.

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W being the weight of the half span. Differentiating the last equation and using the first,

$$\frac{dT}{T} = \frac{dc}{c} - \frac{ds}{s} = \frac{dc}{c} - \frac{dT}{\lambda},$$

$$dT\left(\frac{1}{T} + \frac{1}{\lambda}\right) = \frac{dc}{c}.$$

Differentiate the equation

$$s = c \sinh \frac{x}{c},$$

and put for ds and dc their values in terms of dT. We arrive at the equation

$$dx = dT\left[\frac{x}{\lambda} + \frac{c}{T}\left(\frac{x}{c} - \tanh\frac{x}{c}\right)\right]$$

$$= dT\left[\frac{x}{\lambda} + \frac{w^2x^3}{3T^3}\right].$$

Writing f as before for $\frac{3T^3}{2w^2x^3}$ we have

$$\frac{\text{increase of tension}}{\text{increase of horizontal span}} = \frac{f}{1 + \frac{2fx}{\lambda}} = f' \text{ say.}$$

If the constant f is replaced by this decreased value, the remainder of the analysis holds.

Taking the actual numerical values used before, viz., six wires each weighing ·42 lb. to a foot, tension of each 1000 lbs. and span 100 yards, we found for f for the whole set, the value 15120. Assume for λ the value $2{\cdot}8 \times 10^6$ (for one wire) and we find

$$f' = \frac{f}{1{\cdot}27} = 11900.$$

In arriving at this value of λ for 000 wire we use the figures given for 00 wire by Mr. A. P. Trotter ('Electrical Times,' 10th May, 1906).

The modified values of the constants a and b are

$$a = 2{\cdot}27 \quad \text{instead of } 2{\cdot}21,$$

$$b = {\cdot}50 \quad \text{,, } 0{\cdot}40.$$

The maximum deflexion caused by breakage of all the wires in a bay works out to ·752 feet instead of the former value ·696 feet; *i. e.*, a difference of about $\frac{2}{3}$ inch in the permissible deflexion of the top of the pole.

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The following table gives the modified values in feet for the deflexions when the break occurs after the 1st, 2nd, . . . 7th pole, counting from an anchor-pole :—

Deflexion of..	1st	2nd	3rd	4th	5th	6th	7th	Pole.
When $n=1$	·40							
2	·27	·61						
3	·17	·38	·70					
4	·10	·23	·42	·73				
5	·06	·14	·26	·44	·74			
6	·04	·08	·15	·26	·45	·75		
7	·02	·05	·09	·16	·27	·45	·75	

Also we give below the *maximum* deflexion liable to be caused by breakage of 1, 2, . . . 6 wires, the point of break being distant from both fixed ends.

No. of Wires broken.	Deflexion.
1	·036 feet
2	·084
3	·151
4	·251
5	·419
6	·752

XXIX. *The Dead Points of a Galvanometer Needle for Transient Currents. By* ALEXANDER RUSSELL, *M.A., M.I.E.E.**

THE effects produced by an alternating magnetic field on a magnetic needle suspended in it have been studied by several physicists†. The phenomena have been shown to be amenable to calculation in certain cases, and have been employed by Rayleigh‡ to measure the power factor in alternating-current circuits.

* Communicated by the Physical Society: read May 11, 1906.

† Lord Rayleigh, 'Brit. Assoc. Reports, 1868; see also 'Scientific Papers,' vol. i. p. 310, and vol. ii. pp. 401 and 579: A. Schuster, "Experiments on Electrical Vibrations," Phil. Mag. [4] vol. xlviii. (1874); G. Chrystal, "On Bi- and Unilateral Galvanometer Deflexion," Phil. Mag. [5] vol. ii. p. 401 (1876).

‡ Lord Rayleigh, "On the Measurement of Alternate Currents by means of an Obliquely Situated Galvanometer-needle, with a Method of Determining the Angle of Lag," Phil. Mag. [5] vol. xliii. p 343 (1897); or 'Scientific Papers,' vol. iv. p. 299.

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As a knowledge of these effects is a great help in understanding the action of transient currents, whether direct or oscillatory, on a galvanometer-needle or coil, we shall give a brief *résumé* of the main phenomena and indicate the theory.

Effect of Alternating Currents on a Galvanometer-Needle or Coil.

Let us suppose that we have an ordinary Kelvin mirror-galvanometer, and that we connect it with the alternating-current mains through a high resistance. Let us first suppose that the mirror, and consequently the needle, is at right angles to the axis of the galvanometer coil, and that it is in stable equilibrium in this position before we close the switch. If the effective value of the alternating current be less than a certain critical value, the needle will still be in stable equilibrium after closing the switch, but the time of swing will be longer. As the current approaches the critical value the time of swing gets longer and longer, and when it equals the critical value the equilibrium is neutral. For greater values of the alternating current the spot of light moves off the scale to one side or the other. This is what Professor Chrystal calls bilateral galvanometer deflexion. If the needle be in stable equilibrium, the effect of the alternating field is to make it more sensitive to magnetic impulses.

If we now twist the fibre or move the controlling magnet so that the spot of light is no longer in the centre of the scale initially, then, when we increase the alternating current, the spot of light moves steadily away from the centre of the scale to a new position of equilibrium. This is called unilateral deflexion.

Similar effects are produced by alternating currents in the coil of a d'Arsonval galvanometer. In this case, when the spot of light is in the centre of the scale the coil is in stable equilibrium. The unilateral deflexion, however, is generally towards the centre of the scale. If the moving coil be enclosed in a damping metallic cylinder, a small unilateral deflexion away from the centre of the scale can sometimes be observed.

To explain these effects let us first consider the case of a Kelvin mirror-galvanometer. Let Mk^2 be the moment of inertia of the mirror and needle about an axis through the suspending fibre, let θ_0 be the initial angular deflexion of the mirror and θ its deflexion at the time t. The equation determining the motion is

$$Mk^2\ddot{\theta}+2b\dot{\theta}+\mu H \sin(\theta-\theta_0)=\mu Gi\cos\theta+\gamma i^2\sin\theta\cos\theta. \quad (1)$$

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In this equation $2b\dot{\theta}$ represents the damping torque due to air friction, μ the magnetic moment of the needle, H the strength of the controlling field, G the coefficient* of the galvanometer, that is, the strength of the field at the magnetic poles of the needle due to unit current in the galvanometer coil, and i the instantaneous value of the alternating current flowing in the coil. Following Rayleigh, we have assumed that the magnetism of the needle is made up of a constant part and a part which is proportional to the applied magnetic force. If we suppose that the eddy currents in the needle are negligible, the torque produced by the variable component of its magnetism is easily shown to be equal to $\gamma i^2 \sin\theta \cos\theta$, where γ is a constant.

As the frequency of the alternating current is very high compared with the free period of the galvanometer-needle, and the amplitude of the forced vibration is generally very small, we see that the apparent position of equilibrium of the spot of light is given by

$$\mu \mathrm{H} \sin(\theta-\theta_0)=\gamma \mathrm{A}^2 \sin\theta \cos\theta,$$

where A is the effective value of the alternating current. When θ and θ_0 are small we have

$$(\theta-\theta_0)/\theta=\gamma \mathrm{A}^2/(\mu \mathrm{H}).$$

The author has verified this equation experimentally, and finds that for a given value of A the expression $(\theta-\theta_0)/\theta$ is practically constant. If, however, A be varied between wide limits, the agreement between this formula and experiment is not so satisfactory.

If we assume that the deflexion of moving coil galvanometers by alternating currents is due to the eddy currents in the magnets, we find that

$$(\theta_0-\theta)/\theta=k\mathrm{A}^2,$$

where k is a constant. For a given value of A the author found experimentally that $(\theta_0-\theta)/\theta$ was practically constant, but when A was varied this ratio was only approximately constant.

The Neutral Position of the Needle for Steady Currents.

Since the moment of the applied forces acting on the needle is measured by $Mk^2\ddot{\theta}$, we get, as in (1),

$$Mk^2\ddot{\theta}=\mu \mathrm{G} i \cos\theta+\gamma i^2 \sin\theta \cos\theta-\mu \mathrm{H} \sin(\theta-\theta_0)$$
$$-\text{ retarding torque due to air friction}$$
$$+\text{ torque due to eddy currents in the needle.}$$

* Maxwell, 'Electricity and Magnetism,' vol. ii. § 748.

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The deflexion θ is found by solving the equation

$$\mu G i \cos\theta + \gamma i^2 \sin\theta \cos\theta = \mu H \sin(\theta - \theta_0).$$

Hence, if the left-hand side of this equation be zero, $\theta = \theta_0$ will be a position of equilibrium and there will therefore be no deflexion. In this case

$$\mu G/\gamma = -i \sin\theta_0.$$

In many types of galvanometer the neutral position for a given current can be found very easily, and hence this equation can be used to give us the ratio of μG to γ.

The Dead Points of the Needle for Transient Currents.

Lord Rayleigh, in a paper to the British Association in 1883*, points out that a galvanometer is a very imperfect instrument for indicating whether the integral sum of the transient currents through it is zero or not, and he also mentions the limitations that this imposes on Maxwell's method of comparing mutual inductances. The author finds that with many needle-galvanometers it is easy to arrange so that a, relatively speaking, gigantic charge can be passed through the coil without producing any throw at all. He also finds that all the galvanometers he has tested, whether needle or moving coil, will produce throws with certain transient currents, although their integral values are zero. A simple explanation of the effects produced by a condenser discharge can be given as follows.

Let us consider the case of a condenser K connected with the galvanometer coil in the usual manner by a charge and discharge key. Let q_0 be the initial charge in the condenser, then we have

$$\int_0^\infty i\,dt = q_0$$

and

$$R\int_0^\infty i^2 dt = q_0^2/(2K) - W;$$

where R is the *effective resistance* of the discharging circuit and W the total energy given to the needle. The latter equation is very approximately true whether the discharge be oscillatory or not. If we assume that the current has become negligibly small before the needle has moved appreciably, we

* Lord Rayleigh, Brit. Assoc. Reports, p. 444 (1883), "On the Imperfection of the Galvanometer as a test of the Evanescence of a Transient Current"; or 'Scientific Papers,' vol. ii. p. 228.

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find, by multiplying both sides of equation (1) by dt and integrating, that

$$Mk^2\Omega = \mu G q_0 \cos\theta_0 + \gamma \sin\theta_0 \cos\theta_0 \{q_0^2/(2KR) - W/R\}, \quad (2)$$

very approximately, where Ω is the initial angular velocity of the needle. Thus Ω will be zero if the right-hand side of equation (2) vanishes. In general the heat generated by eddy currents in the needle is negligibly small, and so W may be put equal to zero when Ω is zero. Hence, in this case

$$2\mu G \cos\theta_0 + \gamma \sin\theta_0 \cos\theta_0 \{q_0/(KR)\} = 0,$$

and therefore

$$\sin\theta_0 = -2\frac{\mu G}{\gamma} \cdot \frac{R}{V}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where V, the initial P.D. between the terminals of the condenser, may be positive or negative.

At the points determined by equation (3) there will be no throw produced on charge and discharge respectively. These points, which the author calls dead-points, can be determined rapidly and accurately with low-resistance galvanometers.

If we vary the resistance x between the terminals of the condenser and find the position y of the dead-point corresponding to charge, then plot a curve of y and x, we get a straight line. This line, however, does not pass through the origin. We have to suppose, therefore, that the "effective internal resistance" of the condenser is R_1 and that R in formula (3) is given by $R = x + R_1$.

Drawing a line through the point $(-R_1, 0)$, making the same angle with the axis of X as the line-locus of the dead-points for charge, but on the opposite side from it, we get the locus of the dead-points for discharge. The distances of the dead-points also from the centre of the scale were found experimentally to be inversely proportional to the applied voltages and all the lines passed approximately through the point $(-R_1, 0)$. In these experiments, a standard 1-microfarad condenser by a well-known maker was used, and its effective internal resistance was 4 ohms. The resistance of a 0·05-microfarad condenser was found to be 32 ohms, and of a 3-microfarad condenser for alternating-current work 1·6 ohms.

In practice the energy given to the needle is negligibly small compared with that expended in heating the path of the electric current. We may consider therefore that the internal resistance R_1 of the condenser is defined by the

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equation

$$(R_1+G)\int_0^\infty i^2 dt = q_0^2/(2K),$$

where G is the resistance of the galvanometer-coil and connecting leads.

The internal resistance R_1 of a condenser can be found easily as follows. Let us suppose that when only the galvanometer coil is in circuit the distance of the dead point on discharge from the symmetrical point is D_1. Let us now suppose that a resistance R is put in series with the galvanometer and that the new reading for the dead point is D_2. Then, by (3), we have

$$\frac{G+R_1+R}{G+R_1}=\frac{D_2}{D_1},$$

and thus

$$R_1=RD_1/(D_2-D_1)-G.$$

The author has found experimentally that this formula gives consistent values for R_1, both when R and the applied pressure are varied between wide limits.

Equation (3) above gives an accurate method of determining the ratio of μG to γ. It also shows that the circumstances favourable to the production of dead-points near the symmetrical position are:

(1) low resistance, (2) small galvanometer coefficient, (3) small magnetic moment of needle, (4) a high charging voltage, and (5) a large value of the Rayleigh coefficient γ.

We shall now consider the throws produced when the needle is initially in a position θ_0. Equating the change in the kinetic energy of the needle to the work done in stopping the swing, we get

$$(1/2)Mk^2(\Omega^2-\omega^2)=\mu H\{1-\cos(\theta-\theta_0)\},$$

approximately, where ω is the angular velocity of the needle when the deflexion is θ. If θ_1 be the value of θ at the extremity of its swing and if θ_1 be small, we have

$$Mk^2\Omega^2=\mu H(\theta_1-\theta_0)^2,$$

and hence

$$\theta_1-\theta_0=(Mk^2/\mu H)^{1/2}\Omega.$$

Therefore by (2)

$$\theta_1-\theta_0=(1/Mk^2\mu H)^{1/2}\{\mu G q_0+\gamma\theta_0(q_0^2/2KR)\}, \quad (4)$$

since in most cases W is a negligibly small part of the total energy.

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Hence $\theta_1 - \theta_0 = \theta' + \lambda\theta_0$,

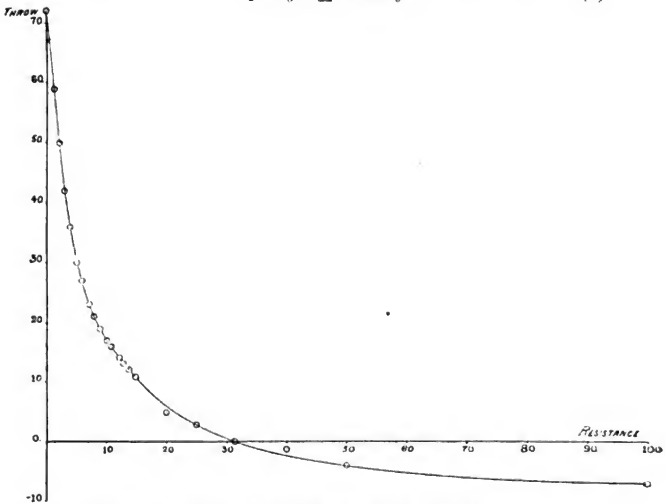
where $\theta' = \mu G q_0/(M k^2 \mu H)^{1/2}$

$=$ the throw in the symmetrical position,

and $\lambda = \gamma q_0^2/\{2KR(M k^2 \mu H)^{1/2}\}$.

We see that whether q_0 be positive or negative λ must be positive. Hence we may write

$$\theta_1 - \theta_0 = \pm\theta' + \lambda\theta_0. \quad . \quad . \quad . \quad . \quad . \quad (5)$$



Variation of the throw with the resistance in the discharging circuit. Capacity one microfarad. Charging voltage 105. The equilibrium position of the needle is one degree from the symmetrical position.

In the figure the throws produced when the needle was initially one degree from its symmetrical position are shown for various values of the resistance in the discharging circuit. The experimental curve in the figure is very nearly coincident with the hyperbola whose equation is

$$y = -12 + 432/(x + 5).$$

The resistance of the galvanometer and leads was one ohm, the inductance of the galvanometer-coil 0·0021 henry, and

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the internal resistance of the condenser found experimentally by means of the dead-point line was 4 ohms. In addition to the experimental points shown the following were also found :—

Resistance in ohms ...	200	500	1000	10000	100000
Throw	−8	−10	−10	−12	−12

The throws on charge were given by

$$y = 12 + 432/(x + 5),$$

very approximately.

Equation (4) can also be written in the form

$$\theta_1 - \theta_0 = (1/Mk^2\mu H)^{1/2} KV\{\pm\mu G + (\gamma V/2R)\theta_0\}.$$

Hence if we vary V keeping everything else constant and plot a curve showing the relation between the throw and the voltage, we get a parabola. Similarly, if we vary K (keeping R constant) we get a straight line.

The above tests could not be taken with the greatest accuracy, as the needle-galvanometers were purposely left unshielded so as not to complicate the problem, and the magnetic effects on the needle of the currents required for a neighbouring tramway were very pronounced. They, however, amply prove that the error made by the assumption that γ is constant is small.

The following simple experiment shows the effects produced by a transient current, the integral value of which is zero, on the throw of a neeedle galvanometer. A coil of No. 16 insulated wire, weighing about one pound is placed on the top of a similar coil, the two coils thus forming an air-core transformer. The terminals of one coil are connected with the galvanometer terminals. The terminals of the other are connected by a Morse key with the terminals of a one-microfarad condenser in such a way that the transient currents obtained either by charging the condenser from the 100 volt mains or by discharging it pass through the coil. The integral value of the current induced in the first coil in either case will obviously be zero. When the needle is in the symmetrical position the throws produced on charge and discharge are negligibly small, but if the needle be on one side or the other of the symmetrical position the throws will be large and will always be away from the symmetrical

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position. Putting resistance in series with the condenser diminishes the magnitude of these throws very considerably.

With d'Arsonval galvanometers having unshielded coils similar effects are produced. The throws, however, are now towards the symmetrical position, and thus if the circuit be made and broken rapidly the spot of light moves to the centre of the scale whatever may be its initial position.

In conclusion the author wishes to emphasize the importance of the Rayleigh correction for the galvanometer equation. It explains many of the troublesome phenomena often noticed by those engaged in practical tests. When this correction is taken into account, the usefulness of the galvanometer is considerably increased, and the ease with which the dead-points can be accurately determined enables this correcting factor to be readily found. The positions of the dead-points also enable us to find the effective internal resistance of condensers. In several of the ordinary tests used in practice a knowledge of this resistance is necessary.

XXX. *An Investigation of the Potential required to maintain a Current between Parallel Plates in a Gas at Low Pressures. By* J. A. Brown, *M.A.**

The Potential required to maintain a Current in a Spark-Discharge.

In some recent papers Professor J. S. Townsend† called attention to the fact that the potential necessary to maintain a current in a gas is in general less than the potential required to cause a spark to pass, and proceeded to explain this phenomenon on the theory of collision or the production of fresh ions by the impact of positive and negative ions on the molecules of the gas. Several determinations of the current and the potential necessary to maintain it were made, using a high-resistance voltmeter and a known external resistance, which clearly showed the reality of this fact and suggested the necessity of further and more accurate measurements along these lines. Further observations were also needed with parallel plate electrodes to show definitely how the potential required to maintain a current varies with the intensity of the current, in addition to the ordinary dependence on spark-length and pressure which the law for the sparking

* Communicated by Prof. J. S. Townsend, F.R.S.

† J. S. Townsend, Phil. Mag. viii. p. 750 (Dec. 1904); Phil. Mag. ix. p. 289 (March 1905).

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potential seems to require. It was thought worth while accordingly to extend the determination of the variation of the current with the potential at various pressures, and particularly below the critical pressure.

The general phenomena of the spark-discharge are now well known. The so-called Paschen's* Law, confirmed primarily by the determinations of Peace† and the careful measurements of Carr‡, is now well established within certain limits. The sparking potential for parallel plate electrodes may confidently be said to depend simply on the amount of gas between the electrodes, and not to vary with the material of the plates.

A number of the phenomena in connexion with these potentials, and other effects noticed in the spark-discharge, have been explained by the theory of ionization by collision. There are peculiarities, however, which still need investigation. Under certain conditions the electrodes produce effects which cannot be explained by the simple theory mentioned above, which has so successfully accounted for the general action. When precautions are not taken to eliminate these effects, the peculiarities in the character of the discharge are very complicated; and some account of these is given in the second part of this paper.

Part I.

The direct object of the investigation, then, was the measurement of the current and at the same time the potential required to maintain it between parallel plate electrodes at comparatively small pressures.

Several similar determinations of this have been made, notably by Riecke §. All this work has, however, been undertaken with ordinary vacuum-tubes, with electrodes projecting into the gas and with no attempt to secure a uniform field. In Riecke's work, for instance, the tube was a sphere, with electrodes near the ends of diameters at right angles to each other, their distance apart being approximately 6·5 cms. Sparking accordingly frequently started from the back of the cathode when the pressure was decreased below a certain value. So the statement he makes, that the value of the current and the potential to maintain it is a function depending on the tube, cannot be wondered at.

Carr has shown the necessity of trying to secure uniformity

* Paschen, Wied. *Ann.* xxxvii. p. 79 (1889).
† Peace, Proc. Roy. Soc. lii. p. 99 (1892).
‡ Carr, Proc. Roy. Soc. lxxi. p. 374 (1903).
§ Riecke, *Ann. der Phys.* iv. p. 592 (1901).

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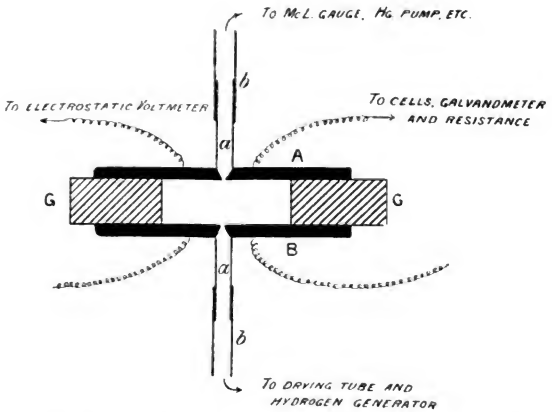
of the field and definiteness of the spark-length in his discussion of Peace's results of the sparking potential. The latter for pressures below the critical value could discover no indications that Paschen's law held true; but the precautions taken by Carr to eliminate the possibility of a brush-discharge around the edges of the plates at low pressures, and other disturbing influences, proved that the law extended to the lower pressures as well. Such precautions were accordingly also taken in the present work, in the hope that results would be more definite than any other obtained previously.

Apparatus.

The spark-gap used was of the form suggested by Carr * in his determination of sparking potentials. The only important difference was that the walls were of glass instead of ebonite, to enable one to watch the spark so as to determine whether it filled the space entirely, and whether there was any flickering or other unsteadiness.

Fig. 1.



The diagram needs no lengthy explanation. The electrodes A and B were accurately turned, three pairs in all being used, of zinc, aluminium, and silver respectively, the latter two being set into brass to enable the tubes *a a* to be more easily soldered on and to ensure a solid and air-tight plate.

The spark-chambers were connected to one another in

* Carr, *l. c.* p. 406.

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parallel, the gas being able to flow freely from one to the other; but no difference in the results obtained from the different electrodes was eventually noticed, although at first there was a decided effect due to the material. (See Part II.)

The walls G G were formed by a ring of plate-glass accurately ground and polished, approximately 1 cm. thick, the hole which formed the spark-chamber being 3 cms. in diameter. The plates were stuck to the glass with ordinary commercial elastic glue, as was also the fine brass tubing *a a* into the glass tubes *b b*, the joints being carefully tested to see whether they were air-tight. These were the only joints made in this way, all the rest of the apparatus being glass, and the whole being frequently tested for leaks. The current was measured by an ordinary d'Arsonval galvanometer, of resistance 5·5 ohms, and giving 1 mm. deflexion on a scale 1 metre distant for 10^{-6} ampere. The potential was read off on a multicellular electrostatic voltmeter, graduated up to 600 volts and connected directly to the electrodes. A battery of small storage-cells furnished the current, a large liquid resistance being always in the circuit. It was at first decided to use air in the spark-gap, but it was found that one could not be certain of steady conditions with this gas, as the spark would often flicker and suddenly become unsteady, and would not always fill the whole space. This was most probably due to an effect on the electrodes, so it was decided to use hydrogen throughout the investigation as it is so easily obtained in a pure state. The gas was prepared electrolytically from a solution of very pure $Ba(OH)_2$ in a generator specially constructed to prevent any possibility of the hydrogen being contaminated by impurities. The gas, after being evolved, was carefully dried by allowing it to stand at least twenty-four hours in close contact with dry phosphoric pentoxide before being admitted to the spark-chamber. In connexion with this, too, there was a drying-tube; so it would seem that sufficient precautions were taken to ensure the gas being dry and pure.

For about four months, while the final readings were being taken, no air sufficient to be detected by the McLeod gauge came in contact with the plates at any time. To make sure of the continued purity of the gas between the plates, it was entirely cleaned out every few days, the pumping being carried down to at least $\frac{1}{80}$ mm., and fresh gas was admitted. During the running of the current there was sometimes a slight absorption of gas by the electrodes, this and similar effects being noted below; but the method of taking the readings prevented any serious inaccuracy due to this cause.

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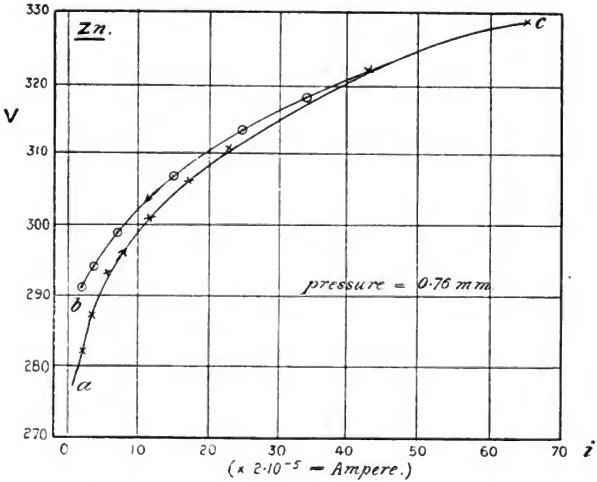
Procedure.

The method of procedure was briefly this:—The pressure having been adjusted and carefully read on the gauge, the current was put on and readings of the galvanometer and voltmeter taken as soon as possible. Then the resistance was decreased and a new value taken. The largest current having been attained, the reverse process was followed, the resistance increased and various new points on the curve for the current determined. In this way the effect of any permanent change in the pressure could be at once detected. The curves of fig. 2 show the method of procedure.

Fig. 2.



The first reading obtained was the one marked *a* on the diagram. The resistance was then gradually decreased until *c* was reached, and then the reverse process carried on down to *b*. The difference in the potentials of *a* and *b* was so small that taking the average of these two curves would introduce no great error; and this was done, the pressure being assumed to be the mean of the readings on the gauge before and after the series. (But see later, the explanation given for the form of the curves below the critical pressure.)

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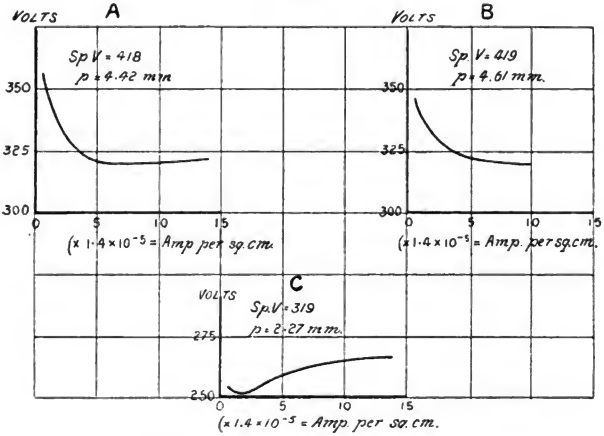
The potential of the battery was in all cases considerably in excess of the potential at the electrodes, and a large resistance was used to adjust the current, for reasons which are explained below. (See Part II. A.)

Results.

The curves for the pressures above the critical pressure (fig. 3) show clearly that the potential required to maintain a current is less than the sparking potential which is indicated on each curve (Sp. V.). It was found that the curves are the same for zinc, aluminium, or silver electrodes, those given being the average of a number obtained with zinc. The currents in the figure are expressed as amperes per square centimetre of the electrode, the distance between the plates being 1 cm.

Fig. 3.



At Pressures above the Critical Pressure.

The galvanometer used was not sensitive enough to detect smaller currents than those indicated; but the tendency of the curve is shown in each case, and the sparking potential is given. This drop soon reaches a limit, and for current densities larger than $\frac{1}{10{\cdot}000}$ of an ampere per sq. cm., the potential may be taken as practically constant as the current

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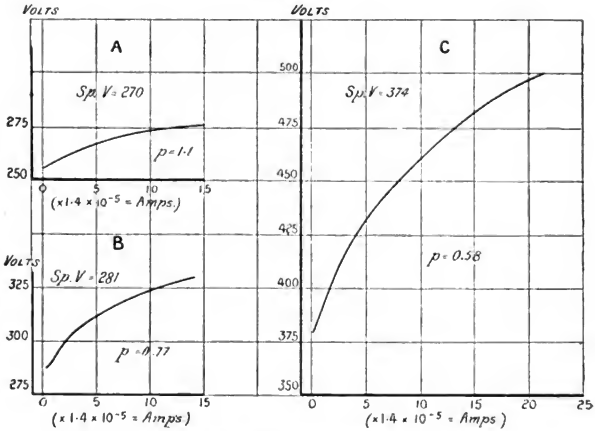
is increased as far as the values were taken. In general, the curves were not pushed on beyond the values given, but Curve 5, fig. 10 (see below), with aluminium plates shows that carrying the current to greater values would not introduce any change in the above generalization.

For pressures below the critical pressure, however, the case is quite different. Curve A, fig. 4, is practically at the

Fig. 4.



Below the Critical Pressure.

minimum sparking potential. Here there is a slight drop from the sparking potential to the potential required to maintain the current; but it soon rises until it passes the sparking potential and remains above it as far as the investigation was carried. At lower pressures this is still more easily seen. Curves B and C (fig. 4) show absolutely no drop; the curve at first increases very rapidly from the direction of the sparking potential, and gradually seems to approach a limit.

These curves are all the average of a large number of sets, which sometimes differ considerably in their values, so that undue emphasis must not be put on the absolute values. A sufficient number were taken, however, to ensure a fair degree

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of accuracy. Curve C, for instance, being an average of eight separate curves, the greatest variation from the mean being 4 per cent., the error for the most part being very much less, and in none of the other curves does it rise to more than 2 per cent. In all cases the general form for the same pressure was the same, the only difference being in the steepness of the curve. Fig. 2 (see above) shows the average number of readings taken for each set, and may serve to indicate the regular nature of all curves obtained.

Thus we clearly see that there are two distinct cases depending on the pressure. Above the critical pressure, the potential to maintain the current drops below the sparking potential, and tends towards a limiting value which may perhaps be independent of the current. Below the critical pressure, the maintaining potential rises more or less rapidly above the sparking potential.

Discussion of Results.

The first condition of affairs observed, that above the critical pressure, falls in exactly with Professor Townsend's theories previously advanced. Sparking will take place when

$$\alpha - \beta e^{(\alpha-\beta)d} = 0;$$

where α is the number of the new ions produced by a negative ion in going across a centimetre of the gas, β is the corresponding number produced by a positive ion, and d is the distance apart of the electrodes.

Knowing α and β, the sparking potential Xd can then be calculated for a given pressure. This has been found to be in accurate agreement compared with the sparking potential determined experimentally *. The sparking potential determined experimentally may consequently be used in the determination of α and β. The quantity α has been already determined over a large range of pressures and forces. The same has not yet, however, been done for β. It has been shown that the value of $\frac{\alpha}{p}$ bears a constant relation to $\frac{X}{p}$, where X is the electric force and p is the pressure; consequently from the curve connecting these two ratios the value

* J. S. Townsend, Phil. Mag. vi. p. 598 (1903).

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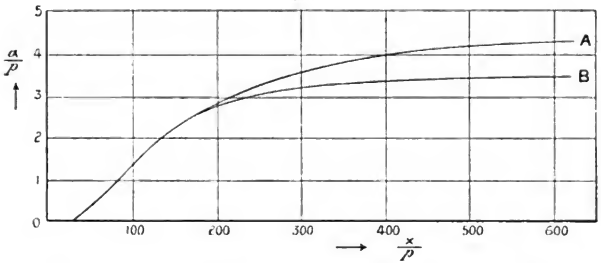
of α can easily be read. Such a curve is given in fig. 5 A, this being based on the first determination of α made by Professor Townsend*. More recently, for low pressures, when the effect of β must be more strongly felt, it was found that Curve B more nearly expresses the true state of affairs.

Fig. 5.



Consequently for the larger values of $\frac{\alpha}{p}$, the quantity read off from this may be taken to be more nearly correct, based as it is on results from the simultaneous determination of α and β, which Professor Townsend points out as being obviously the most accurate. But it must be remembered that the curve is only an approximation, better determinations being necessary before it can be definitely given.

Taking α then as given by this curve, β can be calculated from the equation

$$\alpha - \beta e^{(\alpha-\beta)d} = 0,$$

and the effects of the positive ions at high pressures and small forces can be determined and compared with the curves previously found, connecting $\frac{\beta}{p}$ and $\frac{X}{p}$. This has been done with two of the curves given in fig. 4, the values of α being taken as 2·54 and 2·09 respectively for the two pressures 0·77 mm. and 0·58 mm.

A very simple graphical solution gives an easy way of making this calculation. The above equation may be written

$$\alpha d e^{-\alpha d} = \beta d e^{-\beta d}.$$

Constructing the graphs of each of the members of this equation to solve for values of αd and βd respectively, they

* J. S. Townsend, Phil. Mag. v. p. 389 (1903).

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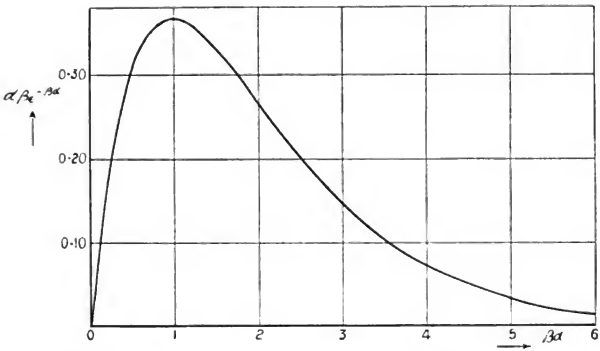
will absolutely coincide, giving two solutions for each value of the ordinate. The quantity β is known to be small in comparison with α from other work; consequently for each large value of αd the corresponding value of βd can be read off directly. Such a curve is given in fig. 6.

Fig. 6.



It should be noted, however, that a small error in the determination of the sparking potential will make a very large variation in the value of β, since the part of the curve where $\frac{\beta}{p}$ is read off is so much steeper than the corresponding part for $\frac{\alpha}{p}$. So this is not a good way to determine β. Notwithstanding this fact, the results obtained seem to fall in extremely well with the theory. In fig. 7 (p. 220) the heavy line gives the curves showing the relation of $\frac{\beta}{p}$ and $\frac{X}{p}$ for hydrogen, as determined by Professor Townsend. The two marked points are the values determined for the two curves under discussion, B and C, fig. 4; and the dotted line shows how closely they coincide with a continuation of previous observation. The calculation is given in Table I. But it should be noted that in the case of the larger values for β, and probably at the higher pressure too, it is very doubtful if the equation from which these values have been derived can be expected to hold very accurately. The assumption upon which it is based is that there is a large number of collisions between the ions and the neutral molecules of the gas. But at

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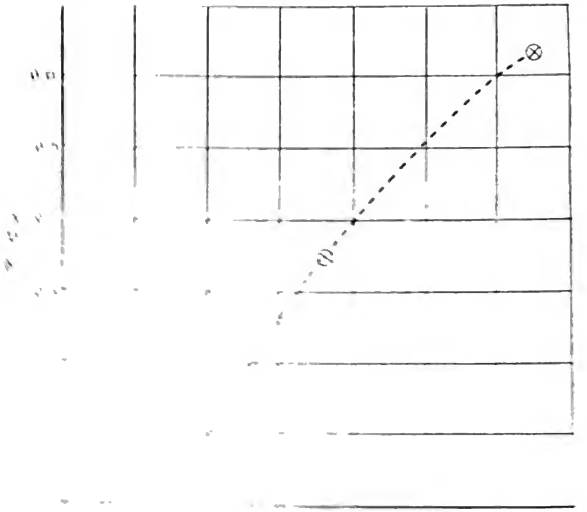
Fig. 7.



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the pressure of only 0·5 millimetre, the mean free path of the positive ions on the kinetic theory, assuming that they are atomic in size, is nearly ·4 millimetre, and that of the negative ion about 1·6 millimetre, which is not very small in comparison with the total distance between the plates. For higher pressures when the value of β is very small, the determination of β in this way must necessarily be more accurate; but in the range under investigation it seems very doubtful whether absolute confidence can be placed in this equation as expressing the true state of affairs, even though it appears to fit in remarkably well with the facts. It may be that the assumed curve for $\frac{\alpha}{p}$ was too low, and consequently the results are more favourable than theoretical grounds would warrant.

Leaving the sparking potential, and coming to the values of the potential required to maintain a current, in the first place, above the critical pressure, a complete discussion of results similar to those obtained in the present work has been given by Professor Townsend *. He clearly shows that the difference in the velocities of the positive and the negative ions, tending to cause a surplus positive charge in the gas near the electrode, will fully account for the observed difference between the sparking potential (which is really the potential required to maintain a minute current) and the corresponding potential to maintain larger currents. The latter should be lower, at least for pressures that are above the critical value. Thus the curves obtained (fig. 3) fit in exactly with the theory, and lend further confirmation to the previous work.

Below the critical pressure, however, some other explanation of the effect observed must be given. It seems probable that it is masked by another and more marked effect which was not got rid of in the course of the experiments. At high pressures the mass of the carriers is fairly large, the ions being composed of groups of molecules travelling together with a comparatively small velocity. At low pressures and high forces, on the other hand, such as are obtained below the critical pressure, the mass may be very much smaller, in all probability of the order of the mass of a molecule. Hence the velocity is probably very much greater than the velocity at the higher pressures. The charge in the gas for a given current would then be very much less than is obtained with the same current above the critical pressure. We should therefore expect that in this case the drop from the sparking

* J. S. Townsend, Phil. Mag. ix. p. 289 (March 1905).

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potential to the potential required to maintain the current would be very small even if nothing else entered to affect the results. The curves found show, however, that there is such a disturbing effect, and the cause is not difficult to see.

The heat energy generated by the current must be considerable, and the presumption is that it will in some way show itself in the action of the discharge. The cause for the noticeable rise of the potential with the current, must then be sought in temperature changes. A direct calculation of the amount of heat energy given to the ions by the passage of the current, in itself is difficult to make on account of the many disturbing influences which tend to lower the temperature. But the rise must be considerable, and, in the course of the present experiments, must have had a modifying influence on the conditions of the gas. Some of the gas must have been driven out through the openings to the spark-chamber, so decreasing the amount of gas between the plates, and consequently raising the potential required to maintain the current. Owing to the small mass of the carriers, they must have given up their heat immediately on striking the massive metal electrodes, and the temperature of the whole mass must have adjusted itself almost at once to the original conditions, the change probably not taking more than a fraction of a second. It will be interesting to test this hypothesis further, and an experiment is being planned to throw more light on the subject.

On this view, then, the current should heat the gas, driving some of it out of the chamber—the greater the current, the greater the amount of heating, and consequently the smaller the amount of gas left between the plates. But below the critical pressure the smaller the amount of gas, the higher is the potential required to spark, and hence to maintain a constant current. Therefore just as the curves (fig. 4) show the actual state of affairs to be, the potential would rise as the current was increased.

At the higher pressures this effect would not be so noticeable, the driving out of the gas in this case tending to lower the sparking potential, and consequently also the potential required to maintain the current, the effect being in the same direction as that caused by the polarization or charge in the gas spoken of before.

Mention must be made of the explanation given by Stark * in the new edition of Winkelmann's *Handbuch der Physik*, of similar results obtained by Riecke and others under much

* J. Stark, Winkelmann's *Handbuch der Physik*, iv. 1. p. 518.

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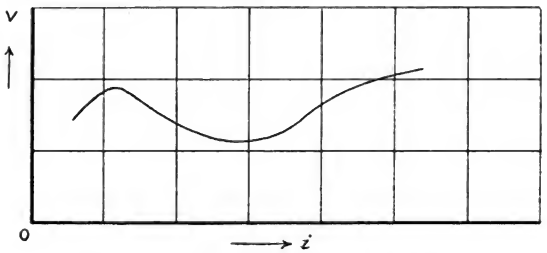
less definite or determined conditions. He explains the observed phenomena by saying that the nature of the curve is determined by the predominance in the tube of one or another of the characteristic parts that are associated with the discharge through gases at low pressures. In the "dark space," when the current increases, the potential rises, the same effect being observed in the space of the "cathode fall of potential," provided that this is no longer normal, *i. e.* that the glow covers the whole surface of the cathode, so that it cannot expand any further. Before that, the cathode fall is independent of the current as well as of the pressure. In the "positive column," on the other hand, the potential decreases with a growing current. A perfectly general curve, connecting the current and the potential required to maintain it, would be given by fig. 8. Starting at small

Fig. 8.



currents, the potential would at first rise with the current, the dark space having the preponderating influence. Reaching a maximum, the positive column would begin to encroach upon the dark space until it would overbalance it, and the potential would begin to fall. Hitherto the cathode fall has been constant, but it now begins to rise, so that the potential goes up again. Not all of these conditions, he declares, will be observed at a given pressure. For high pressures the dark space will predominate; for smaller pressures, but above one millimetre, the second characteristic part of the curve will be observed, while finally, for very low pressures, the cathode fall would alone determine the nature of the curve.

This representation, which is framed to account for the observed facts, cannot be said to be entirely satisfactory. The cathode fall has been shown to equal the minimum sparking potential, so that at the critical pressure the cathode

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fall is exactly the same as the difference of potential between the electrodes. So when the pressure is decreased below this value, the actual cathode fall as such has ceased to exist. In the present investigation with a sparking distance of one centimetre, the critical pressure was about 1·1 mm.; so that in all the cases noted, when the potential rose with the current (fig. 4), the actual cathode fall of potential was no longer anything definite. On the other hand, it is most improbable that there should not be a considerable heating effect under the circumstances, which would of necessity cause the gas to expand and drive some of it out from between the plates. That being so in any case, it is much simpler to find in this the explanation for the phenomenon without calling in further hypotheses.

Thus, to sum up the explanations given to the observed conditions:—In Case I., where there was a drop from the sparking potential tending toward a constant value, a polarization effect is set up in the gas, which tends to maintain the current at a lower potential than is needed to cause the current to pass originally. In Case II., below the minimum sparking potential, when the potential to maintain the current rises with the current, the polarization is small, and the rise in the potential is to a large extent due to a heating effect which drives out some of the gas.

Part II.

Some Peculiarities of the Spark-Discharge.

It has seemed worth while to tabulate a few of the peculiarities noticed at various times during the course of the investigations, some of which entered as sources of error which had to be eliminated or which disappeared of themselves, and others which did not seem to affect the results given, and to which in consequence less attention was paid.

A. *Effect of the E.M.F. of Battery on Value of Potential required to maintain a given Current.*

As was mentioned above, the curves at first obtained were often very complicated, especially at high pressures, when one would expect, from previous work, that the curve would be practically straight. The variations in the current were produced in this case by changing both the resistance and the number of the cells in the battery. On going over the Tables, it was found that whenever there was a small resistance

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in the main circuit, the potential-difference at the electrodes in the gas would be high. Series of readings were accordingly taken for values of the potential at the electrodes when

Fig. 9.

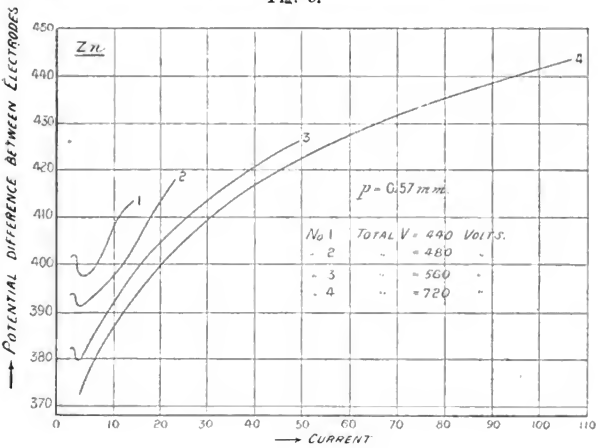
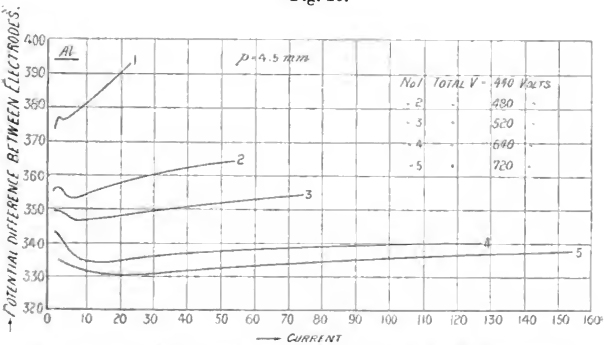


Fig. 10.



the number of the cells was kept constant and the current altered by changing the resistance of the circuit, a number of curves for different forces being obtained.

Two sets are given, figs. 9 and 10, the E.M.F. of the cells

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used being indicated in each case. One set was taken above and one below the critical pressure, which, with the sparking distance used, through the whole course of the investigation was always approximately at 1·1 millimetre. In both cases the potential to maintain a definite current will be seen to be lower as the number of cells is increased. The drop is not a constant one however, as the lines tend to come closer with fairly large total potentials, very little decrease being noticed when the number of cells is increased beyond that indicated by Curve 5 (fig. 10). From *a priori* reasoning it would seem to follow that when external resistance and electromotive force are large, the potential of the electrodes should attain a definite value. This has accordingly been taken as being nearest the true value for a steady current, as it is obvious that a current is most stable when both R and E are large.

This explains the reason why the sets of curves in Part I. of this investigation were taken in the way that they were, *i. e.* keeping a large force on (1040 to 1200 volts), and varying the current by changing the ohmic resistance. It should be mentioned, in passing, that the sparking distance of the zinc and the aluminium was not exactly the same length, a fact that was only to be expected from the way in which the gaps were put together. This will explain any little discrepancy between tables taken with different metals during the latter course of the experiment (but see below).

B. *Effect of Material of Electrodes.*

When the apparatus was first planned it was assumed, on the evidence of previous investigators, that the material of the electrodes would have no effect on the sparking potential, and consequently on the potential required to maintain the current. The electrodes were accordingly made of zinc, as it happened to be the most convenient metal on hand. It was found, however, that the zinc tended to form a film on the glass walls of the sparking-chamber, the spark-length in this case being only 0·5 centimetre, and so to give a metallic path for the current. It did not, however, make a stable connexion, so that definite determinations for permanent correction could not be taken as at first hoped. It was in trying to obviate this that the effect on the sparking potential due to the electrodes was first noticed.

Another gap with aluminium electrodes was put in to do away with the disintegration observed with the zinc. It was soon noticed, however, that the two sets of electrodes did not behave in the same way. After a fairly large current

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($\frac{1}{260}$ ampere) had been running for some time, it was found that the sparking potential was considerably changed. It was at first supposed that the gas was being heated up and driven out from between the electrodes. A thermometer, however, placed in close contact with the plates showed not the slightest increase in temperature, and further, all the gas being pumped out to $\frac{1}{80}$ of a millimetre, and fresh gas admitted and adjusted to the same pressure as before, seemed to have no effect on bringing back the sparking potential to its original value. If the plates were left to themselves the sparking potential would gradually return, after about twenty-four hours, to what it was before the current passed. But the most peculiar thing was that while for zinc, running the current raised the sparking potential, for aluminium it lowered it. The following Table shows this effect:—

TABLE II.

Effect of Current on Sparking Potential.

p=45/80 mm.			Dec. 19, 1905.
Sp. V. . .	Al=436.	Zn=380.	Ag=458.
Current (1/4000 amp.) run for 15 min. between Al electrodes.			
p=47/80, adjusted to 45/80.			
Sp. V. . .	Al=392.	Zn=362.	Ag=442.
Allowed to stand 3 hrs. p then =45/80.			
Sp. V. . .	Al=422.	Zn=358.	Ag=438.
Current run for 15 min. between Zn electrodes.			
p=47/80 adjusted to 45/80.			
Sp. V. . .	Al=446.	Zn=472.	Ag=448.
Allowed to stand 20 min.			
Sp. V. . .	Al=448.	Zn=458.	Ag=454.
Allowed to stand 20 min.			
Sp. V. . .	Al=446.	Zn=448.	Ag=454.

It will be noticed that the values of the sparking potentials for all the spark-gaps seems to change in the same direction, an effect undoubtedly due to a change in pressure in all the gaps; as it is difficult to readjust the pressure instantly, and small variations make large changes in the sparking potentials, it cannot be expected that the other spark potentials should return exactly to their previous values. It will be seen, however, that after running the current between the aluminium plates, the decrease in the sparking potential for these is much greater than for the other two sets, when the current had not been flowing; while the same is true for the marked increase

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in value for the zinc, nearly six times larger than the increase of the other two.

The holes in the plates were quite large enough to ensure an immediate adjustment of pressure in all these gaps, which were joined in parallel. The fact that the gauge showed an increase in both cases, shows that we are not dealing with an absorption of gas in the one case and an emission in the other, as one might *a priori* be led to believe. That this cannot be an effect in the gas is thus definitely shown from the fact that the two metals had an opposite effect on the sparking potential, and that pumping out and admitting fresh gas (not shown in Table) did not influence the process of return to its original value. Many sets of this kind were taken, with several different spark-gaps, the last ones being always about one centimetre in length, and all with consistent results; but it should be noted that at higher pressures, above the critical pressure, this effect was much less noticeable. Two plates of absolutely pure silver were obtained from Messrs. Johnson, Matthey & Co., of London, and were found to behave exactly as the zinc, *i. e.* running the current raised the sparking potential.

It was not absolutely certain, however, that the apparatus was entirely air-tight, and it was feared that some oxygen or other gas might be having a temporary chemical effect on the electrodes. Great care was accordingly taken to obviate any possibility of leak, and the apparatus was left standing with only hydrogen in contact with the plates for about a month. On resuming work again, it was found that this effect had entirely disappeared; nor after two months' constant use has it made its appearance again. It should be said that absolutely no air has been allowed to come in contact with the plates since that time. Hydrogen has been the only gas having access to them. It seems probable that occluded gas has something to do with this effect, which is undoubtedly on the electrodes and not in the gas of the sparking-chamber.

Whatever its cause or seat, it has certainly disappeared with the assurance of the presence of no gas but hydrogen. Since then, the only difference between the results obtained from the aluminium and the zinc electrodes has been the slight and consistent one already mentioned, due probably to a small difference in the distance apart of the electrodes.

C. *Effect of Reversing the Current.*

Another proof that the effect noted above was the result of something closely connected with the plates, and not a phe-

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nomenon due to a change in the gas, was shown by the behaviour on reversing the current. Table III. shows a typical case.

TABLE III.

Effect on Sparking Potential of Reversing the Current.

$p=45$. Jan. 25, 1906.

Zn.

Sp. V. with A+ = 391 volts.
" " A− = 403 "

After running current 1/4000 amp. 11 mins.

Sp. V. with A+ = 431 volts.
" " A− = 405 "

The sparking potential to start with was in general the same whether the top plate (A) was positive or negative. After running the current with A positive, the sparking potential taken in the same way showed an increased value in this direction, if the plates were zinc; but if reversed, *i. e.* A negative, the sparking potential was still at its original value. This phenomenon must be distinguished from the previous as it is a very temporary one, the effect disappearing almost immediately; and a further difference is that the effect is in the same direction, whatever the metal. It has been found that the current running only for a few seconds will have this effect, while the effect due to the material of the electrodes, spoken of before, required a large current for a number of minutes, and lasted for hours after the current had been stopped. Unfortunately, it was not thought of in time to take the sparking potential in the reverse way from which the current had been running, in investigating the previous effect; so this having disappeared, there is no criterion to decide whether the present phenomenon could also have been distinguished then, when much larger changes in the potential due to running the current were available. In general, the determination of the sparking potential, especially with fresh gas or some that had been left standing overnight, did not immediately give its permanent and correct value. Quite a large force could be put on before the spark would pass, but as soon as this had happened, the proper value for the sparking potential could be obtained; and in the new state of affairs, further sparking would have no appreciable effect on the value. This effect was the same whatever the material or the original direction of the current. On reversing the electrodes, however, the value for the sparking potential was always lower, gradually rising to the same value as for

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the original direction. In other words, if, for example, at first the sparking potential was taken with the top plate (A) positive, and then the poles reversed, and without running the current further the sparking potential determined with A negative, its value now would be considerably lower than it was at first with A positive. Running the current with A negative, however, would bring the sparking potential up to the same value as that obtained originally for A positive. If now the poles were again reversed without running the current, A positive would in turn be at first lower, gradually rising to its original value. Table IV. shows a typical set of observations which will make this effect clear.

It looks as if there might perhaps be some kind of polarization-effect on the glass walls of the spark-gap, caused possibly by ions adhering to the sides which would tend to affect the uniformity of the field between the plates. Whatever the cause, it is an effect that only enters into the determination of the sparking potential, as no similar phenomenon was noticed in the value for the current on reversing the poles.

D. *Other Effects.*

(*a*) The effect of capacity in the circuit already noticed by previous investigators was also confirmed. When the electrostatic voltmeter was thrown in, the current in general increased, sometimes as much as 10 per cent. This effect was entirely got rid of by having a large resistance in the main circuit. With the series given in Part 1. of this investigation, there was not the slightest indication of any error due to this cause.

(*b*) Another result observed by previous experiment was that the current is not always steady, even though not the slightest flickering in the glow of the spark could be detected, and the galvanometer deflexion remained absolutely stationary. The effect was noticed when a telephone-receiver was put in the circuit. Frequently there would be a shrill whistling noise in the instrument, which would change with the capacity, as when the metallic conductor was touched with the hand. It also varied with the resistance, and seemed quite erratic, as one could never be certain what the pitch of the note would be, nor even if it would be present in a given case. With the large resistance, however, the value of the current seemed entirely independent of the presence or absence of the noise; so in general, except for noting when the phenomenon was present, no attention was paid to it.

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TABLE IV.

Zn. $p=45$. Jan. 29, 1906.

	Sp. V. A− =443.	Immediately after=435.	Then 431.
After running current 1/5000 amp. A− for 30 secs.	„ A− =429.	Then 435.	
	Sp. V. A− =447.	Then 439.	
„ „ „ „ „ A+ „ 10 secs.	„ A− =416.	After 1 min.=429.	
	Sp. V. A− =441.	Immediately after=433.	
„ „ „ „ „ A+ „ „ „	„ A− =410.	„ „ =417.	
	Sp. V. A− =435.	„ „ =429.	
„ „ „ „ „ A− „ „ „	„ A− =433.		
	Sp. V. A+ =447.	Immediately after =441.	
„ „ „ „ „ A+ „ „ „	„ A+ =437.		
	Sp. V. A+ =451.	„ „ =445.	
„ „ „ „ „ A− „ „ „	„ A+ =418.	„ „ =424.	
	Sp. V. A+ =441.	„ „ =426.	
„ „ „ „ „ A+ „ „ „	„ A+ =426.		
	Sp. V. A+ =447.	„ „ =445.	
„ „ „ „ „ A+ „ „ „	„ A+ =443.		
	Sp. V. A+ =453.	„ „ =449.	
„ „ „ „ „ A− „ „ „	„ A+ =424.	„ „ =428.	

What significance all these effects have, or what modifications not yet eliminated they introduce, it is as yet impossible to say. It is hoped that some further investigation may clear them up. At any rate, with so many peculiarities that are constantly met with in investigations of the spark discharge, it has seemed worth while to make some sort of

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tabulation, as above, to indicate some of the difficulties to be met with in work of this kind.

Work is now going on to determine some of these things more definitely, as well as possibly to repeat all the observations with another gas, perhaps nitrogen.

Conclusion.

To sum up:—

Various peculiarities in the behaviour of a spark-discharge must be noted, which tend to complicate an apparently simple investigation.

(*a*) A large number of cells is necessary to secure stable conditions.

(*b*) Occluded gases, or other causes, may tend to introduce subsidiary results depending on the material of the electrodes.

(*c*) The capacity of the electrodes affects the current when the total external ohmic resistance is small.

(*d*) The sparking potential is slightly affected by the previous direction of the current, this not being a gas effect, and due either to the sides of the chamber or the electrodes.

(*e*) The current may be intermittent, even if the appearance of the spark or the galvanometer (not sensitive) shows no unsteadiness.

None of these things, however, seemed to vitiate the following main results of the investigation so far established.

(1) The potential required to maintain a current in general varies as the current is increased.

(2) At pressures above the critical pressure this value is lower than the sparking potential, tending toward a limiting value.

(3) At pressures below the critical pressure it at first rises very rapidly, as the current is increased, the smaller the pressure the more rapid being the rise.

In conclusion, I must express my deep appreciation of the unfailing kindness of Professor J. S. Townsend, under whose direction and constant guidance the present work was undertaken and carried out.

University Museum, Oxford.

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XXXI. *On the Lateral Vibration of Bars subjected to Forces in the Direction of their Axes.* By JOHN MORROW, *M.Sc.(Vict.), D.Eng.(L'pool), Lecturer in Engineering, University College, Bristol* *.

CONTENTS.

Section I. Statement of the General Problem.
" II. Supported Massive Bar, Axial Tension.
" III. Clamped-Clamped Massive Bar, Axial Tension.
" IV. Clamped-Supported " " " "
" V. Supported Loaded Bar of Negligible Mass, Axial Thrust.
" VI. Clamped " " " " " "
" VII. Deduction of Solutions to some Static Problems.

Section I. *Statement of the General Problem.*

§ 1. The general differential equation for a rod of uniform or varying sectional area subjected to an axial tensile force may be written, neglecting the rotatory inertia,

$$\frac{d^2}{dx^2}\left(\mathrm{EI}\frac{d^2y}{dx^2}\right)-\mathrm{P}\frac{d^2y}{dx^2}+\rho\omega\frac{d^2y}{dt^2}=0. \quad . \quad . \quad . \quad (1)$$

In which

y = lateral displacement at a distance x from the origin at time t, the axis of x being parallel to that of the bar, and vibration occurring in the xy plane;

ρ = density of the material;

ω = sectional area at x;

P = total axial pull;

I = Moment of Inertia of section at x about a line through its centre of gravity at right-angles to the plane of vibration;

$\mathrm{E}=\frac{\mathrm{P}}{\omega}$ + Young's Modulus (sensibly equal to Young's Modulus for the material, which is assumed homogeneous and isotropic).

§ 2. If the bar carries loads concentrated at different points in its length, equation (1) holds between every pair of singular points, that is between points of support and load or between any pair of concentrated loads.

If undashed symbols refer to values of y on the left, and dashed symbols to those on the right of any singular point, we have, in addition to the end conditions, that at all singular points

$$y=y' \quad \text{and} \quad \frac{dy}{dx}=\frac{dy'}{dx}.$$

These serve to determine the constants in the solution of (1).

* Communicated by the Physical Society: read April 27, 1906.

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When the bar is of uniform sectional area the complete integral of (1) is well known to be

$$y = A \cosh \alpha x + B \sinh \alpha x + C \cos \beta x + D \sin \beta x. \quad . \quad (2)$$

In the cases dealt with in this paper the cross-section is assumed to be constant.

Section II. *Unloaded Massive Bar, Supported at each End and subjected to an Axial Pull* P.

§ 3. In the consideration of a special problem we may take, instead of equation (1), a particular differential equation expressing the conditions of that problem only. This method will be adopted as it brings out the physical significance of each term very prominently. The solution for a bar supported at each end has been otherwise obtained. (*Cf.* Rayleigh's 'Sound,' vol. i. article 189, in which rotatory inertia is taken into account.) Taking the origin at one end, the particular form of the differential equation is

$$-EI\frac{d^2y}{dx^2} = -\frac{x}{2}\int_0^l \rho\omega\ddot{y}\,dx + \int_0^x \rho\omega\ddot{y}_z(x-z)dz - Py$$

where y_z = deflexion at a distance z from the origin,
l = length of bar.

If y_1 = displacement at centre of span, the solution is

$$y = y_1 \sin\frac{\pi x}{l},$$

and

$$-\frac{\ddot{y}_1}{y_1} = \frac{EI\pi^4 + Pl^2\pi^2}{\rho\omega l^4},$$

from which the frequency is obtained.

When the force P is compressive vibration is impossible if $\frac{\ddot{y}_1}{y_1} = 0$, that is when

$$P = EI\frac{\pi^2}{l^2}.$$

Section III. *Clamped-Clamped Unloaded Massive Bar. Axial Pull* = P.

§ 4. When the ends are clamped, using equation (2), the conditions at the ends and centre give

$$x = 0, \qquad y = 0, \qquad \therefore \quad C = -A,$$

$$x = 0, \qquad \frac{dy}{dx} = 0, \qquad \therefore \quad D = -\frac{\alpha}{\beta}B,$$

$$x = l, \quad y = \frac{dy}{dx} = 0; \quad \text{and} \quad x = \frac{l}{2}, \quad \frac{dy}{dx} = 0;$$

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from which

$$\left.\begin{aligned} &A(\cosh\alpha l-\cos\beta l) &&+B(\sinh\alpha l-\frac{\alpha}{\beta}\sin\beta l) &&=0\\ &A(\alpha\sinh\alpha l+\beta\sin\beta l) &&+B(\alpha\cosh\alpha l-\alpha\cos\beta l) &&=0\\ &A(\alpha\sinh\tfrac{1}{2}\alpha l+\beta\sin\tfrac{1}{2}\beta l) &&+B(\alpha\cosh\tfrac{1}{2}\alpha l-\alpha\cos\tfrac{1}{2}\beta l) &&=0 \end{aligned}\right\} \quad (3)$$

The last of these holds only for the fundamental and even harmonics.

If we put

$$\gamma=\frac{\alpha\sinh\alpha l+\beta\sin\beta l}{\cosh\alpha l-\cos\beta l},$$

$$\eta=\cosh\tfrac{1}{2}\alpha l-\frac{\gamma}{\alpha}\sinh\tfrac{1}{2}\alpha l-\cos\tfrac{1}{2}\beta l+\frac{\gamma}{\beta}\sin\tfrac{1}{2}\beta l,$$

the constants are

$$A=\frac{y_1}{\eta},\quad B=-\frac{\gamma}{\alpha}\frac{y_1}{\eta},\quad C=-\frac{y_1}{\eta},\quad D=\frac{\gamma}{\beta}\frac{y_1}{\eta},$$

and the solution becomes

$$y=\frac{y_1}{\eta}\left(\cosh\alpha x-\frac{\gamma}{\alpha}\sinh\alpha x-\cos\beta x+\frac{\gamma}{\beta}\sin\beta x\right). \quad (4)$$

The equation of the type used in the previous case is

$$-EI\frac{d^2y}{dx^2}=\rho\omega\ddot{y}_1\left\{\int_0^x\frac{y_z}{y_1}(x-z)dz-\frac{x}{2}\int_0^l\frac{y}{y_1}dx\right\}-Py+M;$$

and we find that for (4) to be the solution of this the following three conditions obtain

$$\left.\begin{aligned} \alpha^2&=\frac{P}{2EI}\left(\sqrt{1-\frac{4EI\rho\omega}{P^2}\frac{\ddot{y}_1}{y_1}}+1\right),\\ \beta^2&=\frac{P}{2EI}\left(\sqrt{1-\frac{4EI\rho\omega}{P^2}\frac{\ddot{y}_1}{y_1}}-1\right),\\ \alpha\tanh\tfrac{1}{2}\alpha l&=-\beta\tan\tfrac{1}{2}\beta l. \end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

From these equations elimination of α and β would give an expression for $\frac{\ddot{y}_1}{y_1}$.

The couples required to fix the ends are each given by

$$M=-\frac{\rho\omega\ddot{y}_1}{\eta}\left\{\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\left(\frac{\gamma l}{4}-1\right)-\frac{\nu l}{4}\right\},$$

where

$$\nu=\frac{1}{\alpha}\sinh\alpha l-\frac{\gamma}{\alpha^2}\cosh\alpha l-\frac{1}{\beta}\sin\beta l-\frac{\gamma}{\beta^2}\cos\beta l.$$

§ 5. Of equations (5), the first two could have been

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obtained directly from the general differential equation, and the last is the result of eliminating A and B from any two of equations (3).

An approximate determination of $\frac{\ddot{y}_1}{y_1}$ from equations similar to (5) has been made by Donkin and by Seebeck for very long and fine wires on the assumption that the vibration is but slightly affected by the existence of rigidity, namely

$$-\frac{\ddot{y}_1}{y_1}=\frac{i^2\pi^2}{l^2}\frac{P}{\rho\omega}\left(1+\frac{2}{l}\sqrt{\frac{EI}{P}}\right)^2;$$

as a closer approximation Seebeck found

$$-\frac{\ddot{y}_1}{y_1}=\frac{i^2\pi^2}{l^2}\frac{P}{\rho\omega}\left(1+\frac{4}{l}\sqrt{\frac{EI}{P}}+\frac{12+i^2\pi^2}{l^2}\frac{EI}{P}\right)$$

from which to estimate the period of the ith tone.

An approximation of a very different character is given in the next paragraph.

§ 6. Writing

$$\phi=\tfrac{1}{2}\alpha l,\quad \theta=\tfrac{1}{2}\beta l, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

we have in the case of no tension and dealing with the fundamental only

$$\phi=\theta=2{\cdot}365,$$

and, since for such values $\tanh\phi$ varies very slowly with ϕ, when $\frac{Pl^2}{2EI}$ is small we may put

$$\tanh\phi=1{\cdot}0178.$$

The third of equations (5) becomes

$$\phi=-\frac{1}{1{\cdot}0178}\theta\tan\theta.$$

Under these circumstances $\frac{d\phi}{d\theta}$ does not vary rapidly with θ. Hence, when the effect of P is small,

$$\phi=\int\left[\frac{d\phi}{d\theta}\right]_{\theta=2\cdot365}d\theta+K.$$

Determining the constant K by $\phi=\theta=2{\cdot}365$ and using (6) we find

$$\alpha l=21{\cdot}806-3{\cdot}61\beta l. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Combining this with equations (5)

$$\beta^2l^2=46{\cdot}08+\frac{Pl^2}{12{\cdot}03EI}-23{\cdot}7\left(1+{\cdot}02536\frac{Pl^2}{EI}\right)^{\frac{1}{2}},$$

and expanding by the Binomial as far as the second term,

$$\beta^2l^2=22{\cdot}38-{\cdot}218\frac{Pl^2}{EI};$$

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whence

$$-\frac{\ddot{y}_1}{y_1}=500{\cdot}6\frac{\mathrm{EI}}{\rho\omega l^4}+12{\cdot}6\frac{\mathrm{P}}{\rho\omega l^2}-{\cdot}171\frac{\mathrm{P}^2}{\rho\omega\mathrm{EI}}. \quad . \quad (8)$$

§ 7. Equation (8) is of course for the fundamental. When dealing with harmonics we may in all cases take

$$\tanh\phi=1,$$

and, when $\mathrm{P}=0$, we have for the ith tone

$$\alpha l=\beta l=\tfrac{1}{2}(2i+1)\pi. \quad .$$

Hence

$$\left[\frac{d\phi}{d\theta}\right]_{\theta=\frac{1}{2}\alpha l}=\tfrac{1}{2}(2i+1)\pi+(-1)^i=\chi\,(\text{say})$$

$$\therefore \qquad \phi=\tfrac{1}{2}\{\chi-(-1)^i\}(\chi+1)-\chi\theta.$$

Equations (5) then give

$$\theta=\xi\chi-\sqrt{\frac{\mathrm{P}l^2}{4\mathrm{EI}(\chi^2-1)}+\xi^2},$$

where

$$\xi=\frac{\pi}{4}\frac{2i+1}{\chi-1}.$$

Expanding as before

$$\beta^2l^2=4\xi^2(\chi-1)^2-\frac{\mathrm{P}l^2}{\mathrm{EI}(\chi+1)}.$$

Whence

$$-\frac{\ddot{y}_1}{y_1}=\left\{2\xi(\chi-1)\right\}^4\frac{\mathrm{EI}}{\rho\omega l^4}+4\xi^2\frac{(\chi-1)^3}{\chi+1}\frac{\mathrm{P}}{\rho\omega l^2}-\frac{\chi}{(\chi+1)^2}\frac{\mathrm{P}^2}{\rho\omega\mathrm{EI}}.$$

§ 8. The solution is restricted, by the use of equation (7), to small values of $\frac{\mathrm{P}l^2}{\mathrm{EI}}$.

If the axial force P is compressive, vibration will become impossible when

$$\mathrm{P}=4\mathrm{EI}\frac{\pi^2}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and the bar will be unstable.

In practice, when using equation (9) a factor of safety of at least 10 is allowed, in which case

$$\frac{\mathrm{P}l^2}{\mathrm{EI}}\not>4.$$

It can be found by trial that in this case equation (8) is a very close approximation. Generally it may be said that for the problems met with in constructional work equation (8) may be used with fair accuracy; but when dealing with long and fine rods or wires, as in acoustical problems, the

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necessary conditions are not fulfilled and this solution is inapplicable.

§ 9. A solution of the present problem free from the restrictions of the previous paragraphs may be obtained by the method of continuous approximation *.

Thus in the equation

$$-\mathrm{EI}\frac{d^2y}{dx^2}=\rho\omega\ddot{y}_1\left\{\int_0^x\frac{y_z}{y_1}(x-z)dz-\frac{x}{2}\int_0^l\frac{y}{y_1}dx\right\}-\mathrm{P}y+\mathrm{M}$$

assume

$$y=\frac{16y_1}{l^2}\left(x^2-\frac{2x^3}{l}+\frac{x^4}{l^4}\right). \quad . \quad . \quad . \quad . \quad (10)$$

Integrating twice,

$$-\mathrm{EI}y=\frac{\rho\omega\ddot{y}_1}{l^2}16\times10^{-3}\Big(1{\cdot}785715\,l^4x^2-2{\cdot}\dot{7}\,l^3x^3+2{\cdot}\dot{7}\,x^6$$
$$-2{\cdot}\dot{3}80952\frac{x^7}{l}+{\cdot}595238\frac{x^8}{l^2}\Big)$$
$$-\frac{16\mathrm{P}y_1}{l^2}\Big(-{\cdot}01\dot{6}\,l^2x^2+{\cdot}08\dot{3}\,x^4-{\cdot}1\frac{x^5}{l}+{\cdot}0\dot{3}\frac{x^6}{l^2}\Big)$$

and

$$-\frac{\ddot{y}_1}{y_1}=\frac{494{\cdot}74(\mathrm{EI}+{\cdot}025\mathrm{P}l^2)}{\rho\omega l^4}.$$

from which the approximate frequency may be determined. Proceeding to a second approximation we obtain a simple but lengthy expression for the vibration type, viz.:—

$$-\mathrm{EI}y=\frac{{\cdot}0079158}{l^6}\left(1+\frac{\mathrm{P}l^2}{40\mathrm{EI}}\right)\Big[\rho\omega\ddot{y}_1\Big(3{\cdot}5573\,l^8x^2-5{\cdot}5113\,l^7x^3$$
$$+4{\cdot}9603\,l^4x^6-3{\cdot}3069\,l^3x^7+{\cdot}5511\,x^{10}-{\cdot}3006\frac{x^{11}}{l}+{\cdot}0501\frac{x^{12}}{l^2}\Big)$$
$$+\mathrm{P}y_1\Big(42{\cdot}99\,l^6x^2-66{\cdot}1377\,l^5x^3+46{\cdot}2963\,l^2x^6-49{\cdot}6032\,x^8$$
$$+33{\cdot}0688\frac{x^9}{l}-6{\cdot}6138\frac{x^{10}}{l^2}\Big)\Big]+\frac{16\mathrm{P}}{\mathrm{EI}l^2}10^{-4}\Big[\rho\omega\ddot{y}_1\Big(-{\cdot}330685\,l^6x^2$$
$$+1{\cdot}488093\,l^4x^4-1{\cdot}3\dot{8}\,l^3x^5+{\cdot}496032\,x^8-{\cdot}330688\frac{x^9}{l}+{\cdot}066138\frac{x^{10}}{l^2}\Big)$$
$$-\mathrm{P}y_1\Big(3{\cdot}968250\,l^4x^2-13{\cdot}\dot{8}\,l^2x^4+27{\cdot}\dot{7}\,x^6-23{\cdot}809524\frac{x^7}{l}$$
$$+5{\cdot}952381\frac{x^8}{l^2}\Big)\Big].$$

and

$$-\frac{\ddot{y}_1}{y_1}=\frac{500{\cdot}4(\mathrm{EI})^2+12{\cdot}1526\mathrm{EI}\mathrm{P}l^2-{\cdot}01266\mathrm{P}^2l^4}{(\mathrm{EI}-{\cdot}0003838\mathrm{P}l^2)\rho\omega l^4}.$$

* See Phil. Mag. ser. 6, vol. x. no. 55, p. 113.

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The last terms in the numerator and denominator of this expression are very small and not sufficiently accurate to be retained except as a step in the third approximation if such were required.

Hence the frequency should be given with considerable accuracy by

$$-\frac{\ddot{y}_1}{y_1}=500{\cdot}4\frac{\mathrm{EI}}{\rho\omega l^4}+12{\cdot}153\frac{\mathrm{P}}{\rho\omega l^2}. \quad . \quad . \quad . \quad (11)$$

The terminal couples are given in this approximation by

$$\mathrm{M}={\cdot}079158\left(\mathrm{EI}+\frac{\mathrm{P}l^2}{40\mathrm{EI}}\right)({\cdot}71145\rho\omega l^2\ddot{y}_1+8{\cdot}5980\mathrm{P}y_1)$$
$$-\frac{16\mathrm{P}l^2}{\mathrm{EI}}10^{-4}\,({\cdot}66137\rho\omega l^2\ddot{y}_1+7{\cdot}9365\mathrm{P}y_1).$$

§ 10. Another approximation may be obtained by the dynamical method, in which a type of vibration is assumed*.

Thus, taking equation (10) for the type, the potential energy at any instant is

$$\mathrm{V}=\tfrac{1}{2}\,\mathrm{EI}\int_0^l\left(\frac{d^2y}{dx^2}\right)^2dx+\tfrac{1}{2}\,\mathrm{P}\int_0^l\left(\frac{dy}{dx}\right)^2dx$$
$$=\left(\frac{512\,\mathrm{EI}}{5\,l^3}+\frac{256\,\mathrm{P}}{105\,l}\right)y_1^2,$$

and the kinetic energy of the system is

$$\mathrm{T}=\frac{1}{2}\int_0^l\rho\omega\left(\frac{dy}{dt}\right)^2dx$$
$$=\frac{64}{315}\rho\omega l\left(\frac{dy_1}{dt}\right)^2.$$

The equation

$$\frac{d\mathrm{T}}{dt}+\frac{d\mathrm{V}}{dt}=0$$

gives

$$-\frac{\ddot{y}_1}{y_1}=504\frac{\mathrm{EI}}{\rho\omega l^4}+12\frac{\mathrm{P}}{\rho\omega l^2}. \quad . \quad . \quad . \quad . \quad (12)$$

A solution which is based on a very reliable method, but necessarily overestimates the value of $-\frac{\ddot{y}_1}{y_1}$, since equation (10) does not exactly represent the true type of displacement.

* See Rayleigh's 'Theory of Sound,' vol. i. § 182.

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Section IV. *Clamped-Supported Bar under Axial Pull.*

§ 11. When the bar is clamped at one end and merely supported at the other, the solution can be deduced from that of the previous section. Thus in the final equation of § 7 we may give to i, in succession, the values of the positive integers and at the same time write $2l$ for l.

For the fundamental

$$-\frac{\ddot{y}_1}{y_1}=\left(\frac{5\pi}{4}\right)^4\frac{\mathrm{EI}}{\rho\omega l^4}+12{\cdot}29\,\frac{\mathrm{P}}{\rho\omega l^2}-{\cdot}0912\,\frac{\mathrm{P}^2}{\rho\omega\mathrm{EI}}\,.$$

l is the length of the bar, and the result is still confined to small values of $\frac{\mathrm{P}l^2}{\mathrm{EI}}$.

Section V. *Bar of Negligible Mass, Supported at each end, Compressive Axial Force* $=\mathrm{P}$, *having a single load concentrated at some point in its length.*

§ 12. When dealing with bars of negligible mass it is convenient to assume the positive P to be compressive. This change of notation is therefore made in the remainder of the paper.

Let the mass m divide the bar into segments a and b.

The equations are :—

For $x<a$ $$-\mathrm{EI}\frac{d^2y}{dx^2}=\mathrm{P}y-m\ddot{y}_a\frac{b}{l}x,$$

for $x>a$ $$-\mathrm{EI}\frac{d^2y}{dx^2}=\mathrm{P}y-m\ddot{y}_a\frac{a}{l}(l-x).$$

These have respectively the solutions

$$\left.\begin{aligned} y&=\frac{c}{n^2}x+\mathrm{B}\sin nx-\mathrm{A}\cos nx,\\ y&=\frac{h}{n^2}(x-l)+\mathrm{D}\sin nx-\mathrm{C}\cos nx.\end{aligned}\right\}$$

Where $$n^2=\frac{\mathrm{P}}{\mathrm{EI}},\ c=\frac{m\ddot{y}_a}{\mathrm{EI}}\frac{b}{l},\text{ and }h=-\frac{m\ddot{y}_a}{\mathrm{EI}}\frac{a}{l}.$$

The conditions at each end are

$$\left.\begin{aligned} x=0,\quad y=0;\quad &\therefore\ \mathrm{A}=0\\ x=l,\quad y=0;\quad &\therefore\ \mathrm{D}\sin nl=\mathrm{C}\cos nl\end{aligned}\right\}\quad .\quad .\quad (13)$$

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When $x=a$ the two expressions for $\frac{dy}{dx}$ are the same, whence

$$\frac{c}{n^2}+n\mathrm{B}\cos na=\frac{h}{n^2}+n\mathrm{D}\cos na+n\mathrm{C}\sin na. \quad . \quad (14)$$

Similarly the two values of y are equal at the point $x=a$.

$$\therefore \frac{c}{n^2}a+\mathrm{B}\sin na=\frac{h}{n^2}(a-l)+\mathrm{D}\sin na-\mathrm{C}\cos na. \quad (15)$$

Equations (13), (14), and (15) give

$$\mathrm{B}=\frac{c-h}{n^3}(\sin na\,.\cot nl-\cos na)$$

$$\mathrm{C}=\frac{c-h}{n^3}\sin na,$$

$$\mathrm{D}=\frac{c-h}{n^3}\sin na\,.\cot nl.$$

The equations of the elastic central line are therefore for $x<a$,

$$y=\frac{m\ddot{y}_a}{\mathrm{P}}\left\{\frac{l-a}{l}x+\frac{1}{n}(\sin na\,.\cot nl-\cos na)\sin nx\right\}, \quad (16)$$

and the expression when $x>a$ is obtained by writing x for a and a for x in the above.

By putting $x=a$ we find

$$\frac{\ddot{y}_a}{y_a}=\frac{\mathrm{P}}{m}\left[\frac{ab}{l}+\frac{1}{n}\sin na(\sin na\cot nl-\cos na)\right]^{-1} \quad (17)$$

§ 13. As a particular case, if the mass m be at the centre of the span,

$$\frac{\ddot{y}_a}{y_a}=\frac{4\mathrm{P}}{ml}\left(1-\frac{2(1-\cos nl)}{nl\sin nl}\right)^{-1}, \quad . \quad . \quad . \quad (18)$$

and vibration is impossible when

$$\sin nl=0,$$

agreeing with the condition of instability of § 3.

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Section VI. *Loaded Massless Bar Clamped at each End.*

§ 14. Here for $x<a$,

$$-\mathrm{EI}\frac{d^2y}{dx^2}=\mathrm{P}y-m\ddot{y}_a\frac{b}{l}x+\mathrm{M}_1+(\mathrm{M}_2-\mathrm{M}_1)\frac{x}{l},$$

where M_1 and M_2 are the terminal couples.

This may be written

$$\frac{d^2y}{dx^2}+n^2y=hx-k,$$

where $$n^2=\frac{\mathrm{P}}{\mathrm{EI}},\quad h=\frac{m\ddot{y}_a}{\mathrm{EI}}\frac{b}{l}-\frac{\mathrm{M}_2-\mathrm{M}_1}{\mathrm{EI}l},\quad k=\frac{\mathrm{M}_1}{\mathrm{EI}}.$$

The solution is

$$y=\frac{1}{n^2}(hx-k)+\mathrm{D}\sin nx-\mathrm{C}\cos nx. \quad . \quad . \quad . \quad (19)$$

The conditions at the origin give

$$\mathrm{C}=-\frac{k}{n^2},\quad \mathrm{D}=-\frac{h}{n^3}.$$

For $x>a$ we must add $m\ddot{y}_a(x-a)$ to the right-hand side of (19). Then, if

$$h'=-\frac{m\ddot{y}_a}{\mathrm{EI}}\frac{a}{l}-\frac{(\mathrm{M}_2-\mathrm{M}_1)}{\mathrm{EI}l}\quad\text{and}\quad s=\frac{m\ddot{y}_a a-\mathrm{M}_1}{\mathrm{EI}},$$

the solution is

$$y=\frac{h'}{n^2}x-\left\{\frac{s+h'l}{n^2}\sin nl+\frac{h'}{n^3}\cos nl\right\}\sin nx$$
$$+\left(\frac{h'}{n^3}\sin nl-\frac{s+h'l}{n^2}\cos nl\right)\cos nx+\frac{s}{n^2}. \quad . \quad (20)$$

Equating the two values of $\left[\frac{dy}{dx}\right]_{x=a}$ and of y_a we get, after some reductions,

$$\mathrm{M}_1=m\ddot{y}_a\alpha \qquad \mathrm{M}_2-\mathrm{M}_1=m\ddot{y}_a\gamma.$$

Where

$$\alpha=\frac{a+b\cos nl-l\cos nb+\frac{1}{n}(\sin nb-\sin nl+\sin na)}{2(1-\cos nl)-nl\sin nl}$$

and

$$\gamma=\frac{(b-a)(1-\cos nl)+l(\cos nb-\cos na)}{2(1-\cos nl)-nl\sin nl}.$$

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For $x=a$ we find

$$\frac{\ddot{y}_a}{y_a}=\frac{P}{m\left\{(b-\gamma)\left(1-\frac{1}{n}\sin na\right)+\alpha(\cos na-1)\right\}}. \quad . \quad (21)$$

§ 15. If the load be at the centre, $a=b=\frac{l}{2}$ in this result, and

$$\frac{\ddot{y}_a}{y_a}=\frac{4P}{ml}\left\{1-\frac{2\left(1-\cos\frac{nl}{2}\right)}{\frac{nl}{2}\sin\frac{nl}{2}}\right\}^{-1}, \quad . \quad . \quad (22)$$

vibration ceasing when

$$\sin\frac{nl}{2}=0.$$

Section VII.

Deduction of Solutions to some Static Problems.

§ 16. The calculations contained in Sections V. and VI. are the same as those involved in the corresponding statical problems of the deflexion of a bar of negligible mass carrying a concentrated load and subjected to the axial compressive force P.

If both ends are supported, instead of $m\ddot{y}_a$ we must put the magnitude of the load in the formulæ of Section V. and so obtain an expression similar to (16) for the curve of deflexion of the central line, and to (17) and (18) for the displacement at the point at which the load is concentrated.

Making the same change in notation in Section VI. we get equations (19), (20), (21), and (22) for the case of the bar clamped at both ends.

An expression equivalent (except for a numerical error) to (22) has been given by Kirchhoff*, but I am not aware that the more general solutions for any position of the load have hitherto, in either case, been recorded.

University College, Bristol,
March 1906.

* *Vorlesungen über mathematische Physik*, Bd. i. Lecture 29, § 3, See also Todhunter and Pearson's 'History of Elasticity,' vol. ii. pt. 2. § 1289.

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XXXII. *The Ionization Ranges of the α Rays of Actinium.* *By* O. Hahn, *Ph.D.**

In previous papers † I gave an account of the methods employed to determine the ranges in air of the ionization due to the various α ray products of thorium. This paper contains a similar investigation for the ranges of the α rays emitted by the products of actinium. Apart from the physical importance of accurately determining the range in air of the α rays from each of the products, I have found these investigations of great value in throwing light on the successive changes which occur in the new product radioactinium ‡.

In this paper, therefore, an account will be given of the determination of the ranges in air of the α particles from the following products:—

(1) Active Deposit of Actinium.
(2) Actinium X.
(3) Radioactinum.
(4) Actinium emanation.

The preparations used for the experiments were:—

(1) Actinium (Emanium), activity about 300, prepared by Giesel, provided by Rutherford.

(2) Actinium, activity about 700, prepared by Debierne, kindly lent to me by Sir William Ramsay.

The apparatus employed was the same as previously used for thorium, and similar to that employed by Bragg and Kleeman in their investigations of the ranges of the α rays from the radium products. It is, therefore, not necessary to give again the details of its construction and the method of use.

As I pointed out in my last paper, the chief difficulty in the investigations on thorium arose from the strong emanating power of all the preparations containing thorium X. In order to avoid, as far as possible, the disturbances due to the emanation, the active preparation was placed inside a funnel, and a constant air-current passed through this funnel in order to carry away the emanation. This arrangement had worked satisfactorily at some distance of the active preparation from the ionization-chamber, but for small distances disturbances were not to be avoided. In some cases, a thin mica screen completely enclosed the radioactive preparation in order to prevent the escape of the emanation.

* Communicated by Prof. E. Rutherford, F.R.S.
† Phil. Mag. June & July 1906.
‡ O. Hahn, 'Nature,' April 1906.

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In the case of actinium, the disturbances due to its emanation were even greater than in the case of thorium, since the decay period of actinium emanation (half value in 3·7 seconds) is so rapid that by far the greater part of it is transformed before it can be removed even by a very strong air-current. Consequently, disturbances due to the emanation were always encountered, and the only way of avoiding this difficulty completely was to cover the emanating preparation by a mica screen and to determine the "stopping power" of the covering screen by a special experiment.

Ionization Curve of the Active Deposit of Actinium.

It was of special interest to determine the ionization-distance curve of the active deposit of actinium, as previous experiments with the active deposit of thorium had conclusively shown that the latter consisted of two well-marked α ray products instead of one as formerly supposed.

The investigation of the active deposit of actinium does not present any of the above-mentioned difficulties, as it can easily be obtained in a pure state in the form of an extremely thin film.

The active deposit was obtained on a thin copper wire, about 1 cm. long, by making it the only negatively-charged body exposed to the emanation of a fairly large amount of actinium. The measurements of the ionization curve were begun about 10 or 15 minutes after removal from the emanation. After that interval, the activity of the active deposit decays nearly according to an exponential law with the period of 36 minutes. The values of the current obtained at different times had, consequently, to be corrected for their decay of activity, and this was easily done by extrapolation from the known decay curve.

Since the activity of the wire was initially not very great and decayed rapidly during the experiment, only about one-half of the ionization curve could be accurately obtained with one active wire before the activity became very small. The curve was, consequently, completed by using another active wire. In this way the ionization curve could be obtained with accuracy.

Fig. 1 shows the curve obtained for the active deposit of actinium.

From the shape of the curve it is obvious that the active deposit of actinium contains only one α ray product. The ionization of the α rays begins at a distance of 5·50 cms., increases very rapidly, reaches a maximum at about 3·6 cms., and decreases slowly with the distance from the source. The

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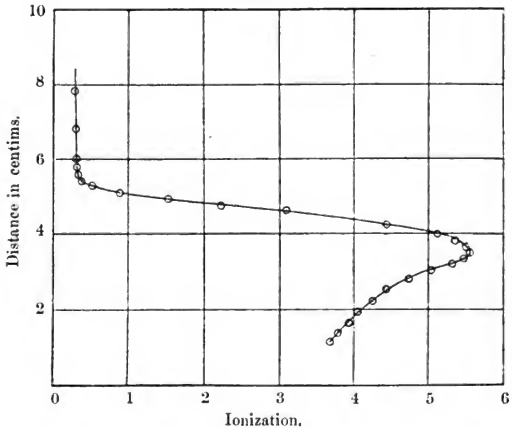
ionization was measured within 1·0 cm. from the active wire; and as it is very unlikely that a new set of α particles should enter the ionization-chamber below 1·0 cm., we have to conclude that the active deposit of actinium emits one set of

Fig. 1.



α particles which are able to traverse a distance of 5·50 cms. of air. Since the active deposit of thorium emits two different kinds of α rays, we see here one interesting point of distinction in the disintegration series of thorium and actinium.

Ionization Curve of Actinium X.

The determination of the exact range of the α particles emitted from actinium X presents very great difficulties on account of the continuous production of emanation from it. The actinium X was prepared by the well-known method used for the separation of thorium X as described in the previous communication. When precipitating the actinium solution with ammonia, the precipitate as a whole does not come down at once. If one filters therefore too early, the filtrate, at first quite clear, soon becomes cloudy, and another small precipitate can be obtained. For the purpose of the experiment a very thin film of actinium X is required, and for this purpose a

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clear filtrate must be used. Great care has to be taken to use pure chemicals, and it is always advisable to use as little liquid and reagents as possible.

The actinium X is evaporated on a flat platinum cover and heated to drive off any remaining ammonia salts. As the active deposit is regenerated very quickly, it is difficult to obtain an ionization curve of actinium X alone, for there is always some actinium B present. Since, as we shall see later, the range of the α particles of actinium X is higher than the range of those of actinium B and of the emanation, the presence of the latter products does not interfere with the accurate determination of the maximum range of actinium X.

In the earlier experiments, great difficulties were encountered in determining the real range of the α particles of actinium X, on account of disturbances due to the emanation even when a strong air-current was used. A small ionization was observed even to a distance of more than 8 cms.; and, in consequence, it was at first thought that traces of thorium X and, therefore, thorium C were present. Finally, however, these effects were found to be due only to the presence of the emanation.

In order to obtain accurate results, it was found necessary to cover the actinium X hermetically with a thin layer of mica. The mica plate was waxed down close to the bottom of the dish, and the small amount of the active matter on the sides of the dish was covered with tinfoil to absorb the α rays, but even in that case a small amount of emanation coming from the walls escaped into the gas.

A separate experiment showed that the mica screen was equivalent in stopping power to 1·20 cm. of air, and in the curves given in fig. 2 the distance is corrected for this absorption of the screen.

Fig. 2 (p. 248) shows the curves obtained for actinium X.

Curve *a* represents the ionization curve of the freshly prepared product.

Curve *b* represents the ionization curve 15 days later.

Curve *c* represents the ionization curve 42 days old.

In the figure, the ionization due to the α rays begins at about 6·55 cms. This number gives the average of several experiments, and is probably correct within 1 mm. The ionization curves, after different intervals, are given to show that the curves obtained are really due to actinium X, which Godlewski has shown decays to half value in 10 days.

In consequence of the use of the mica screen, it is not possible to obtain the shape of the ionization curve closer to

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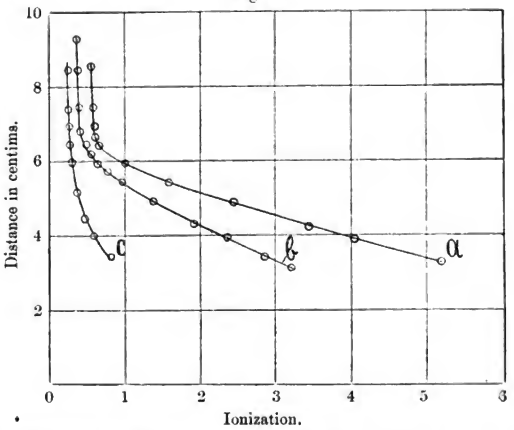
the source than is shown in fig. 2. Although the rays of the emanation and actinium B are present, they do not interfere with the determination of the maximum range of actinium X. The absence of any well-marked breaks in the

Fig. 2.



curves is to be expected since the emanation and actinium B have about the same range in air, and the cone of rays was not very narrow.

The result thus shows that the α particles of actinium X have an ionization range in air of about 6·55 cms.

Ionization Range of the α Particles of Radioactinium.

This product in the series of changes is intermediate between actinium and actinium X. It is transformed into actinium X and its successive products. Radioactinium emits only α rays, and is half transformed in about 20 days*. A full account of its properties is reserved for a later paper, but it is of interest to show how the determination of the ionization curves of this product at different times throws light on its position among the transformation products of actinium.

The radioactinium used in this investigation was separated by several distinct methods, but the simplest form of separa-

* The period of 20 days, as given above, is only approximate and probably slightly too high.

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tion was similar to that employed for the separation of radiothorium from its products. The actinium solution was precipitated successively five times with ammonia in order to completely remove the actinium X. As in the case of radiothorium, the later precipitations are not complete, that is, ammonia is not added to a sufficient extent to carry down the actinium itself. By this procedure, the radioactinium is concentrated in the first precipitate, while the actinium remains for the most part in the solution. It is not difficult to obtain a layer thin enough for the determination of the ionization curve.

Fig. 3.

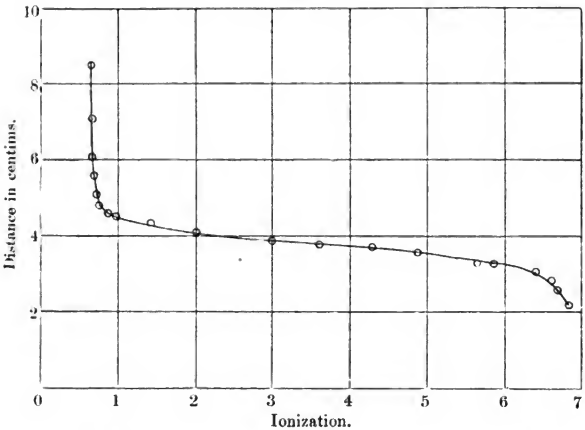

Fig. 3 shows the curve for radioactinium when completely freed from actinium X and also the active deposit. As the activity of the specimen was not very strong, the scale of abscissæ is increased in the ratio 2·5 of that employed in the other figures.

It is seen that the curve of radioactinium has the characteristic shape for a thin film of a single α ray product. The maximum range of the α rays is 4·8 cms., and this value is probably correct within half a millimetre. Since radioactinium at first increases in activity due to the production of actinium X and its successive products, we should expect to obtain evidence from the ionization curve of the presence of these products. This is seen to be the case.

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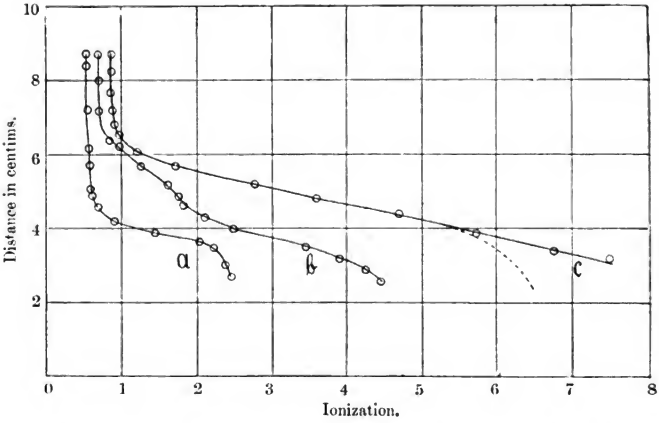
The curves *b* and *c*, fig. 4, show the curves obtained 4 and 21 days respectively after preparation. The curve *a* represents the normal curve of radioactinium itself immediately after

Fig. 4.



separation on the same scale. It is seen from curves *b* and *c* that the ionization commences to increase at a distance of about 6·6 cms. This is due to the α rays from actinium X, which we have seen have a range of 6·55 cms.

The activity of a preparation of radioactinium reaches a maximum about 20 days after preparation. The curve *c*, obtained about that time, is more regular than *b*, and no sudden changes of slope of the curve can be noticed.

The marked disturbance produced by the emanation was clearly shown by variation of the experimental conditions. The set of brass tubes 1·1 cm. high was removed and a similar set of tubes 0·4 cm. high substituted. In this case, the air-current removes a greater portion of the emanation. The lower part of the curve obtained in this way showed a distinct bend downwards. In another experiment the air-current was interrupted. The emanation consequently diffused upwards, and the curve obtained near the source bent upwards instead of downwards. Possibly better results would have been obtained if holes had been bored through the active porcelain crucible in order to allow a more direct action of the air-current.

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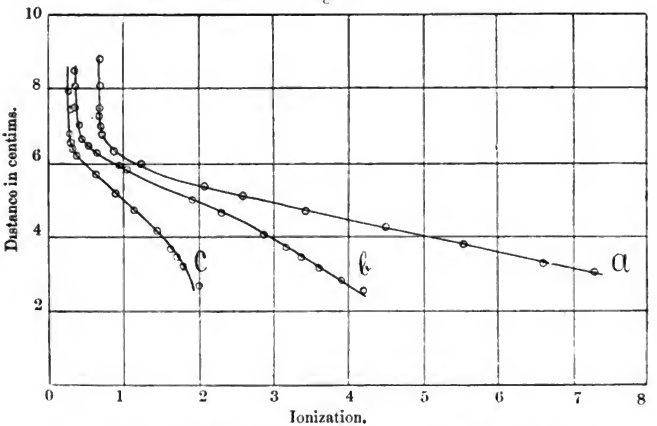
The curves in fig. 4 do not of themselves show whether the radioactinium preparation was initially freed from actinium, for the latter does not emit any rays. The activity of radioactinium and its products after reaching a maximum in about 20 days commences to decay and ultimately, according to an exponential law, with a period of about 20 days. If actinium were present the activity of the preparation would not decay to zero, but would finally reach a constant value depending on the amount of actinium present. This is obviously the case, since actinium gives rise to radioactinium and its successive products.

Fig. 5.



We see, however, from the curve shown in fig. 5 that no sensible amount of actinium could have been present. Curve *a* is the same as curve *c* in fig. 4, and is due to radioactinium at its maximum activity when in equilibrium with its successive products. The curves *b* and *c* were obtained 39 and 71 days respectively after a maximum had been reached. It is seen that the curves are all similar, showing that the activity due to each product decays at the same rate. The initial activities, due to the β rays and the natural leak of the vessel, are partly increased (curve *a*), partly decreased (curve *c*), in order to show distinctly the initial portions of the three curves separated from one another. The ionization (curve *c*) begins very nearly at the correct maximum range

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of the α particles of actinium X, viz. 6·6 cms., but the ionization due to the α rays, shown in the curves *a* and *b*, begins somewhat higher than this—a difference due to the presence of the emanation for the reasons discussed above.

The three curves are very similar to the curves obtained for actinium X, shown in fig. 2, the only difference being that the rays of radioactinium begin to add their effect at a distance of 4·8 cms. This, however, is not easy to observe in the figure.

A striking difference is shown, however, in the rates at which the ionization falls off with time. The ionization of actinium X decreases exponentially with a period of ten days, and is hardly distinguishable after about 50 days. The ionization of radioactinium, however, decreases at a much slower rate, and the ionization after two months' interval is still very marked.

The decay of the radioactinium preparation could, of course, have been measured in an ordinary electroscope, but this of itself would not have shown whether actinium X and its products were present when the activity had reached a small value. *In fact, the ionization curves serve as a means of radioactivity analysis to show what α ray products are present.*

The Range of the α Particles of the Actinium Emanation.

The method used for determining the range of the α particles of the actinium emanation was exactly the same as that used by the writer for the thorium emanation, which was described in the previous paper*. The emanation was passed with a slow air-current through a flat vessel, the top of which consisted of a thin mica plate, the stopping power of which had been determined by a special experiment. A small sensitive zinc-sulphide screen was placed vertically above the mica and could be brought to any desired distance. The point, where scintillations just began to be visible, was taken as the maximum range of the α particles when using the scintillation method.

I previously pointed out that the ranges of α particles determined by the electrical method seemed to be nearly 3 mms. greater than those determined by the scintillation method as above described. This difference may, possibly, vary somewhat with different observers.

Taking the mean of several measurements, which were in

* *Loc. cit.*

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good agreement with one another, the range at which the α particles of the actinium emanation cease to ionize the air (including the correction of 0·3 cm. for the electrical method and 1·2 cm. for stopping power of the mica screen) was found to be about 5·8 cms.

The ranges of the α ray products of actinium are shown in the following table. For completeness the ranges of the products of thorium, determined by the writer, and of radium, determined by Bragg and Kleeman, are added for comparison.

With the exception of the α rays from uranium, the ranges in air of the α particles expelled from all the products of the radioelements have now been determined accurately.

TABLE I.

Product.	Range of α Particles in cms. in air for atmospheric pressure and temperature*.	Product.	Range.	Product.	Range.
Radioactinium ...	4·8 cms.	Radiothorium ...	3·9 cms.	Radium	3·50 cms.
Actinium X	6·55 „	Thorium X	5·7 „	Emanation...	4·33 „
Emanation	5·8 „	Emanation	5·5 „	Radium A ...	4·83 „
Actinium B	5·50 „	Thorium B	5·0 „	Radium C ...	7·06 „
		Thorium C	8·6 „	Radium F †	3·86 „

* The pressure during the period of this investigation did not vary to such an extent as to produce marked differences in the ranges.

† Determined by M. Levin, Americ. Journ. Science, July 1906.

It is seen that the ranges of the α particles from thorium differ among each other to a greater extent than those of radium, and the latter to a greater extent than those of actinium. For example, the extreme difference in the ranges for the thorium products is 4·7 cms., for radium products 3·56 cms., while for actinium it is only 1·75 cm.

A comparison of the ranges of the α particles from thorium and actinium shows a very interesting relationship. In the table below the products are arranged in the order of magnitude of the ranges of their α particles.

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TABLE II.

Range of α Particles.	Product.	Range of α Particles.	Product.
4·8 cms.....	Radioactinium.	3·9 cms.....	Radiothorium.
5·50 „	Actinium B.	5·0 „	Thorium B.
5·8 „	Emanation.	5·5 „	Emanation.
6·55 „	Actinium X.	5·7 „	Thorium X.
		8·6 „	Thorium C.

It is seen that each of the products of thorium and actinium occupies a corresponding place in the table, the only difference being that there is no product in actinium corresponding to thorium C in thorium.

The similarity in the number and nature of the products of these two elements has often been pointed out, and the relationship described above is probably more than a coincidence, and indicates that there is a close connexion in the mode of transformation of the atoms of thorium and actinium.

In conclusion I again wish to express my best thanks to Professor Rutherford for his usual interest he took in this work, and for the kind advice I received from him.

The Macdonald Physics Building,
Montreal, June 10th, 1906.

XXXIII. *The Molecular Structure of Metals.* *By* J. A. EWING, *LL.D., F.R.S., M.Inst.C.E.**

I INTEND to devote this Address to considering in certain aspects the inner structure of metals and the manner in which they yield under strain. It will not be disputed that this is a primary concern of the engineer, who in all his problems of design is confronted by the limitations imposed on him by the strength and elasticity of the materials he employs. It is a leading aim with him to secure lightness and cheapness by giving to the parts such dimensions as are no larger than will secure safety; and hence it is of the first importance to know in each particular case how high a stress may be applied without risk of rupture or of permanent

* Communicated by the Author, being the Presidential Address to the Engineering Section of the British Association for the Advancement of Science, York, 1906.

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alteration in form. Again, the engineer recognizes the merit, for structural purposes, of plasticity as well as strength, and in many of his operations he makes direct use of that property, as in the drawing of wires and tubes or the flanging of plates. He is concerned, too, with the hardening effect that occurs in such processes when work is expended in permanently deforming a metal in the cold state, and also with the restoration to the normal condition of comparative softness which can be brought about by annealing. Nor can he afford to be indifferent to the phenomena of "fatigue" in metals, which manifest themselves when a piece is subjected to repeated alterations or variations of stress—fatigue of strength and fatigue of elasticity, which, like physiological fatigue, admits under some conditions of rest-cure, inasmuch as it tends to disappear with the lapse of time. No apology need be made in selecting for a Presidential Address to Section G a subject that touches so many points of direct practical interest to engineers. It is a subject which has for me the additional attraction of lying in the borderland between engineering and physics—a borderland in which I have often strayed, and still love to stray, and I enter it to-day even at the risk of wandering into regions which, to engineers, may seem a little remote from home, regions where the landscape has, perhaps, a suspicious likeness to that of the country over which the learned men of section A hold rule.

To engineers, quite as much as to physicists and chemists, we owe in recent years an immense extension of knowledge regarding the structure of metals. This has come about mainly by the intelligent use of the microscope. Take any piece of metal, in the state in which an engineer makes use of it, polish and lightly etch its surface, and examine it under the microscope, and you find that it is a congeries of a multitude of grains, every one of which may be proved to be a crystal. It is true that the boundaries of each grain have none of the characteristics of geometrical regularity which one is apt to look for in a crystal, but the grain is a true crystal for all that. Its boundaries have been determined by the accident of its growth in relation to the simultaneous growth of neighbouring grains—the grains have grown, crystal fashion, until they have met, and the surface of meeting, whatever shape it may happen to take, constitutes the boundary. But within each grain there is the true crystalline characteristic—a regular tactical formation of the little elements of which the crystal is built up. It is as if little fairy children had built the metal by piling brickbats in a nursery. Each child starts wherever it happens to be,

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placing its first brickbat at random, and then piling the others side by side with the first in geometrical regularity of orientation, until the pile, or the branches it shoots out, meets the advancing pile of a neighbour; and so the structure goes on, until the whole space is entirely filled by a solid mass containing as many grains as there have been nuclei from which the growth began.

We now know that this process of crystal growth occurs not only in the solidification of a metal from the liquid state, but in many cases during cooling through a "critical" temperature when the metal is already solid. We know also that the process may in certain conditions go on slowly at very moderate temperatures. We know also that the process of annealing is essentially the raising of the metal to a temperature at which recrystallization may take place, though the metal remains solid while this internal rearrangement of its particles goes on. Whether crystallization occurs in solidifying from the liquid or during the cooling of an already solid piece, it results in the formation of an aggregate of grains, each one of which is a true crystal. Their size may be large or small—in general, quick cooling means that crystallization starts from many nuclei, and the resulting grains are consequently small; with very slow cooling you get a gross structure made up of grains of a much larger size.

For simplicity of statement I shall ask you in what follows to confine your attention to simple metals, omitting any reference to alloys. Alloys present many complexities, into which we need not at present enter. With simple metals every crystalline grain is made of the same substance: the elementary brickbats are all exactly alike, though there may be the widest variation from grain to grain as regards the form of the grain, and also as regards the direction in which the elementary brickbats are piled. In any one grain they are piled with perfect regularity, all facing one way, like a regiment of perfectly similar soldiers formed up in rows, where each man is equidistant from his neighbours, before and behind, as well as to right and to left. Or perhaps I might compare them to the well-drilled flowers of an early Victorian wall-paper.

It was shown by Mr. Rosenhain and myself * that when a piece of metal is strained beyond its limit of elasticity, so that permanent set is produced, the yielding takes place by means of slips between one and another portion of each crystal grain. A part of each crystal slides over another part of the same

* Ewing and Rosenhain, "The Crystalline Structure of Metals," Bakerian Lecture, Phil. Trans. Roy. Soc. vol. cxciii. A, 1899.

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crystal, as you might slide the cards in a pack. It is as if all the soldiers to one side of a given line were to take a step forward, those on the other side remaining as they were, or as if all the men in the front rows took a step to the left, while those in the rows behind kept their places. In other words, the plasticity which a metal possesses is due to the possibility of shear on certain planes in the crystal that are called "cleavage" or "gliding" planes. Plastic yielding is due to the occurence of this shear; it may take place in three or more directions in a single grain, corresponding to the various possible planes of cleavage, and in each direction it may happen on few or many parallel planes, according to the extent of the strain to which the piece is subjected. Examine under the microscope the polished surface of a piece of metal which has been somewhat severely strained after polishing, and you find that the occurrence of this shear or slip is manifested on the polished surface by the appearance of little steps, which show themselves as lines or narrow bands when looked at from above. To these we gave the name of slip-bands. Just as the piece of metal is an aggregate of crystal grains, the change of shape which is imposed upon it in straining is an aggregate effect of the multitude of little slips which occur in the grains of which it is made up. Each grain, of course, alters its form in the process.

Speaking broadly, this distortion of the form of any one grain by means of slips leaves it still a crystal. If part of the group of brickbats move forward, keeping parallel to themselves and to the others, the formation remains regular, except that a step is formed on the outermost rows; the orientation of the elements continues the same throughout. Considerations which I shall mention presently lead to some qualification of this statement. I now see reason to believe that in the process of slip there is a disturbance of the elementary portions or brickbats adjoining the plane of slip, which may alter their setting, and thereby introduce to a small extent some local departure from the perfectly homogeneous orientation which is the characteristic of the true crystal. In very severe straining there may even be a wide departure from true crystalline character. We shall recur to this later; but meanwhile it will suffice to say that substantially the slip which is involved in a plastic strain of moderate amount is a bodily translation, parallel to themselves, of part of the group of elementary brickbats or molecules which build up the grain. If a crystal whose form has been altered, even largely, by such straining is cut and polished

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and etched it appears, under the microscope, to be to all intents and purposes as regular in the tactical grouping of its elements as any other crystal.

Further, in the process of straining we have, first, an elastic stage, extending through very small movements, in which there is no dissipation of energy and no permanent set. When this is exceeded, slip occurs suddenly; the work done in straining is dissipated; if the straining force is removed a strain persists, forming a permanent "set"; if the force continues to act it goes on (within certain limits) producing augmented strain. In general a large amount of strain may take place without the cohesion between the gliding surfaces being destroyed. Immediately after the strain has occurred there is marked fatigue, showing itself in a loss of perfect elasticity; but this will disappear with the lapse of time, and the piece will then be harder than at first. If, on the other hand, a process of alternate straining back and forth be many times repeated, the piece breaks.

These are now familiar facts. Can we attempt to explain them on the basis of a molecular theory which will at the same time offer a clue to the process of crystal-building as we find it in metals? I venture to make this Address the occasion of inviting attention to some more or less speculative considerations which may be held to go some little way towards furnishing the material for such an explanation

At the Leeds Meeting of this Association, in 1890, it was my privilege to bring forward certain contributions to the molecular theory of magnetism, and to show a model which demonstrated that the rather complex phenomena of magnetization were explainable on the very simple assumption that the magnetic molecules are constrained by no other forces than those which they mutually exert on one another in consequence of their polarities*. From this were found to result all the chief phenomena of permeability and magnetic hysteresis. Let us attempt to-day to apply considerations of a similar character to another group of physical facts, namely, those that are associated with the crystalline structure of metals and with the manner of their yielding under strain. Just as in dealing with magnetic phenomena, I take as starting-point the idea that the stability of the structure is due to mutual forces exerted on one another by its elementary parts or molecules, and that the clue to the phenomena is to be sought in the play of these mutual forces when displacement of the molecules occurs.

* "Contributions to the Molecular Theory of Induced Magnetism," Proc. Roy. Soc. vol. xlviii. June 19, 1890, or Phil. Mag. September 1890.

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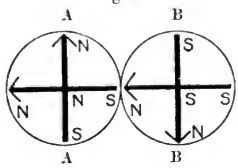
Iron and most of the useful metals crystallize in the cubic system; for simplicity we may limit what has to be said to them. Imagine a molecule possessing polarity equally in three directions, defined by rectangular axes. We need not for the present purpose inquire to what the polarity along the axes is due; it will suffice to assume that the molecule has six poles, three positive and three negative, and that these repel the like and attract the unlike poles of other molecules. We may make a model by using three magnetized rods fixed at right angles to one another at their middle points. I imagine, further, that the molecule has an envelope in the shape of a sphere, which touches the spherical envelopes of its neighbours, and assume that these spheres may turn on one another without friction *.

Think now of the process of crystal-building with a supply of such spherical molecules for brickbats. Starting with one molecule, let a second be brought up to it and allowed to take up its place under the action of the polar forces. It will have a position of stability when a positive pole in molecule A touches (or lies in juxtaposition to) a negative pole in molecule B, with the corresponding axes in line, and when the further condition is satisfied that the axes in molecule B whose poles are not touched by A are stably situated with respect to the field of force exerted by the poles of A.

In other words, we have this formation :—

Fig. 1.



For convenience of representation in the diagram the poles are distinguished by the letters N and S; but it must not be assumed that the polarities with which we are here concerned have anything to do with magnetism.

Suppose, now, that the crystal is built up by the arrival of other molecules, each of which in its turn assumes the position of maximum stability consistent with formation in cubic or normal piling. The group in that case takes an arrangement which is essentially a repetition of this quartette:—

* Or, let the envelope be a shell of any form, inside of which the axes of polarity are free to turn as a rigid system.

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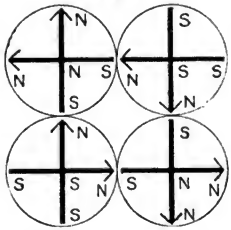
Fig. 2.



Along each row the polarity preserves the same direction, but the polarity of each row is opposite to that of each contiguous parallel row. This description applies equally to all three axes. The whole group (fig. 3) consists of the quartettes of fig. 2 piled alongside of and also on top of one another. In this way we arrive at what I take to be the simplest possible type of cubic crystal.

In this grouping each molecule has the alignment giving maximum stability, and it seems fair to assume that it will take that alignment when the crystal grain is formed under conditions of complete freedom, as in solidifying from the liquid state. As a rule the actual process of crystal-building goes on dendritically; branches shoot out, and from them other branches proceed at right angles, leaving interstices to be filled in later. We have, therefore, to conceive of the molecules as piling themselves preferably in rows rather than in blocks, though ultimately the block form is arrived at. In this position of maximum stability each molecule has its six poles touching poles of contrary name.

Now comes a point of particular importance. Imagine two neighbouring molecules in the same block to be turned round, each through one right angle, in opposite senses. They will now each have five poles touching five poles of contrary name, but the sixth pole will touch a pole of the same name as itself. They are still stably situated, but much less stably than in the original configuration, and they will revert to that configuration if set swinging through an angle sufficient to exceed the limited range within which they are stable in the new position.

Similarly we may imagine a group of three, four, or more molecules, each to be turned through a right angle, thereby constituting a small group with more or less stability, but always with less than would be found if the normal configuration had been preserved. The little group in question may be made up of molecules in a row, or it may be a quartette or

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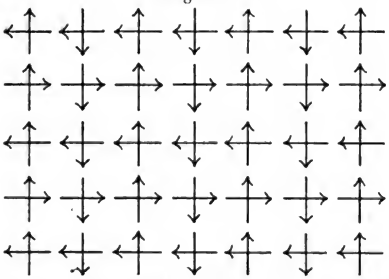
block, or take such a form as a T or L. A sufficient disturbance tends to resolve it into agreement with the normal tactics of the molecules which build up the rest of the grain.

Fig. 3.



It is conjecturally possible that small groups of this kind, possessing little stability, may be formed during the process of crystallization, so that here and there in the grain we may have a tiny patch of dissenters keeping one another in countenance, but out of complete harmony with their environment.

If this happens at all during crystallization, it would seem less likely to happen in free crystallization from a liquid state than in the more constrained process that occurs when a metal already in the solid state crystallizes at a temperature far below its melting-point. Though rare or absent in the first case, it might occur frequently in the second. There are differences in the appearance of crystal grains under the microscope in metal as cast and in metal as recrystallized in the solid state, of which this may be the explanation. It may also explain a difference pointed out by Rosenhain *, that the slip lines in cast metal are straight and regular, whereas in wrought iron and other metals which have recrystallized in the solid state they rarely take a straight course across the crystal, but proceed in jagged, irregular steps. These may be due to the presence here and there of small planes of weakness, resulting from the existence of what I have called dissenting groups. Again, these groups, possessing, as they do, less stability than their normal neighbours, may be conjectured to differ from the normal parts of the grain in respect of electrolytic quality, and to be more readily attacked by an etching reagent. Hence, perhaps, the conspicuous isolated geometrical pits that appear on etching a polished surface of wrought iron.

It will help in making clear these points, and others that

* W. Rosenhain, "The Plastic Yielding of Iron and Steel," Journal of Iron and Steel Institute, No. 1 for 1904, p. 335.

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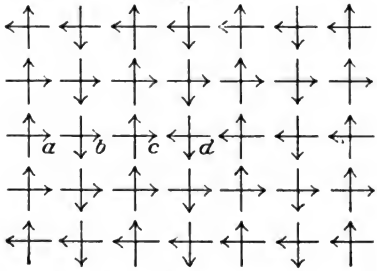
are to follow, if we study the action of a model formed by grouping a number of polarized "molecules" in one plane, supporting them on fixed centres about which they are free to turn. In the model before you the centres are uniformly spaced in rectangular rows, and the "molecules" are + shaped pieces of hardened steel, strongly magnetized along each of the crossed axes, each having, therefore, two north poles and two south poles. The third axis is omitted in the model, the movement to be studied with the help of the model being movement in one plane. On placing these "molecules" on their centres they readily take up the position already indicated in fig. 3. Each one within the group has its four poles in close proximity to four poles of contrary name, and is, therefore, highly stable. If disturbed by being turned through a small angle, and let go, it swings back, transmitting a wave of vibration through the group, which is reflected from the edges, and is finally damped out in the model by pivot friction and air friction. We may assume some damping action (say by the induction of eddy-currents) in the actual solid, of which the model may be taken as a very crude representation.

By turning two molecules carefully round together, each through one right angle in opposite senses, we set up a dissenting pair, whose equilibrium has feeble stability. A slight displacement, such as might be produced by the transmission of a vibrational wave, breaks them up, and they swing back to the normal configuration, giving out energy, which is taken up by the rest and is ultimately dissipated. By making the dissenting coterie consist of three or more we can give it additional strength.

Fig. 4.



An example is shown in fig. 4, where the three molecules marked *a*, *b*, and *c* are turned round in this way.

Notice that the normal molecule *d*, adjoining a line of such dissenters, is in a peculiar position. His neighbours present

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to him three N poles and one S pole. He has the choice of conforming to the majority, or of throwing in his lot with the dissenters; and he has a third possible position of equilibrium (very feeble equilibrium) which is reached when his two S poles are turned until the one neighbouring south pole faces just between them. I have laboured these points a little because they seem important when we come to speak of the effects of strain.

Consider now a straining action, which we may imitate in the model by sliding one part of the group past the other part. For this purpose the centres on which the "molecules" are pivotted are cemented to two glass plates which can slide parallel to one of the axes.

At first, when the displacement by sliding is exceedingly small, the strain is a purely elastic one. The molecules adjacent to the plane of sliding pull one another round a little, but without breaking bond, and if in this stage the strain is removed, by letting the plate slide back to its original position, there is no dissipation of energy. The work done in displacing the molecules is recovered in the return movement. We have here a representation of what happens between each pair of adjoining rows in the elastic straining of a metal. So far the action is within the limit of elasticity; it leaves no permanent effect: it is completely reversible.

But now let the process of straining be carried further. The opposing molecules try to preserve their rows intact, but a stage is reached when their resistance is overcome; the bonds are broken, and they swing back, unable to exert further opposition to the slip. The limit of elasticity has now been passed. Energy is dissipated; set has been produced; the action is now no longer reversible. The model shows well the general disturbance that is set up in the molecules adjoining the plane of slip, which we may take to account for the work that is expended in a metal in producing plastic strain.

Moreover, when the slip on any plane stops and the molecules settle down again, the chances are much against their all taking up the normal orientation which they had before the disturbance. What I have called dissenting groups or unstable coteries are formed as a result of the disturbance. Here and there like poles are found in juxtaposition. Viewed as a whole, the molecular constitution of the metal in the region adjacent to the plane of slip is now uncertain and patchy. It includes parts whose stability is much less than normal. Individual molecules or small groups in it are very feebly stable; a touch would make them tumble into positions of greater stability.

Observe how all this agrees with what we know about the

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nature of plastic strain through experiments on iron and other metals. Its beginning is characteristically jerky. Once the critical force is reached which is enough to start it, there is a big yield, which will not be stopped even by reducing the amount of the straining force.

Again, we know that there is a slow creeping action that continues after the straining force has done its main work. I ascribe this to the gradual breaking up of the more unstable groups which have been formed during the subsidence of disturbance in the earlier stage of the slip.

Further, we know that overstrained iron is very imperfectly elastic until it has had a long rest, or until it has been raised for a short time to a temperature such as that of boiling water *. This is to be expected when we recognize the presence of unstable individuals or groups resulting from the overstrain. When the elasticity of the overstrained piece is tested by removing and reapplying the load, some of these tumble into new positions, making irreversible movements, which dissipate energy and produce hysteresis in the relation of the strain to the stress although the strain is quasi-elastic. At ordinary temperatures these unstable groups are gradually becoming resolved, no doubt through the action of the molecular movements that are associated with heat, and hence the slow progressive recovery of perfect, or nearly perfect, elasticity shown by the experiments of Muir. Let the temperature be raised and they disappear much more quickly; in warm surroundings the rest-cure for elastic fatigue does not need to be nearly so long.

Rosenhain † has recently shown that after the slip-bands on the surface of an overstrained specimen have been obliterated by polishing, traces of them will reappear on etching if only a short interval of time is allowed to lapse since the overstraining; but if time is given for complete recovery no traces are found. This is in remarkable agreement with the view now put forward, that the layers contiguous to the surface of slip contain for a time comparatively unstable groups. They are consequently different from the normal metal until the unstable groups are resolved, and the temporary difference manifests itself on etching, providing that is done while the difference still exists.

From the engineer's point of view a much more important matter than this fatigue of elasticity is the fatigue of strength that causes fracture when a straining action is very frequently repeated. Experiments which I made with Mr. Humfrey ‡

* J. Muir, "On the Recovery of Iron from Overstrain," Phil. Trans. vol. cxciii. A, 1900.

† Journ. Iron and Steel Institute, 1906.

‡ Ewing and Humfrey, "The Fracture of Metals under Repeated Alternations of Stress," Phil. Trans. vol. cc. A, 1902.

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showed that this action begins with nothing more or less than slight slip on surfaces where the strain is locally sufficient to exceed the limit of elasticity. An alternating stress, which makes the surfaces slip backwards and forwards alternately many thousands, or it may be millions of times, produces an effect which is seen on the polished surface as a development of the slip-lines into actual cracks, and this soon leads to rupture.

We have, therefore, to look for an effect equivalent to an interruption of continuity across part or the whole of a surface of slip, an effect progressive in its character, becoming important after a few rubbings to and fro if the movement is violent, but only after very many rubbings if the movement is slight.

That there is a progressive action which spreads more or less into the substance of the grain on each side of the original surface of slip was clearly seen in the experiments referred to. It was found that a slip-band visible on the polished surface of the piece broadened out from a sharply defined line into a comparatively wide band with hazy edges, and this was traced to an actual heaping up of material on each side of the step which constituted the original line.

I think this suggests that under alternating stresses which cause repeated backward and forward slips, these do not occur strictly on the same surface in the successive repetitions, and hence the disturbance spreads to some extent laterally. It may be conjectured that slip on any surface leaves a more or less defective alignment of the molecular centres; that is to say, the rows on one side of the plane of slip cease to lie strictly in line with those on the other side. If this occurs over neighbouring surfaces, as a result of slips on a number of parallel planes very close together, the metal throughout the affected region loses its strictly crystalline character, and with it loses the cohesion which is due to strict alignment.

Mr. G. T. Beilby, in a very suggestive paper *, has advanced grounds for believing that portions of a metal may pass from a crystalline to an amorphous formation under the mechanical influence of severe strain, as in the hammering of gold-leaf or the drawing of wire, and that this occurs in the polishing of a metallic surface, and also in the internal rubbing which takes place at a surface of slip within the grain. In both cases he suggests the formation of an altered layer. When a polished metal surface is etched, the altered layer is dissolved away, and the normal structure below it is revealed.

Without accepting all Mr. Beilby's conclusions, I think the idea of an altered and more or less amorphous layer is supported by the considerations I am now putting forward. We have assumed that in normal crystallization the intermolecular forces lead to a normal piling, in which each molecule touches

* Beilby, "The Hard and Soft States in Metals," Phil. Mag. August 1904.

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six neighbours. But it may be conjectured that some of them may take up pyramidal piling (touching twelve others) under the compulsion of strong forces—such forces, for example, as act on the superficial molecules of a surface that is being polished.

If this also occurs at a surface of slip, it gives us a clue to several known facts. It at least assists in explaining the familiar result that metal is hardened by straining in the sense of being made less plastic. Again, it accounts for the general increase of density which is found to take place in such an operation as wire-drawing. Further, if a local in-increase of density occurs in the interior of a grain through piling of some molecules in the closer manner where repeated slips are going on, the concentration of material at one place requires it to be taken from another; in other words, the closer piling tends to produce a gap or crack in the neighbourhood where it occurs. This is consistent with what we know of the development of cracks through repeated alternations of strain.

Recourse to the model shows that with pyramidal piling the polar axes point in so random a manner that the aggregate may fairly be called amorphous. To illustrate this a group is shown with centres fixed at the corners of equilateral triangles.

It is obvious that any pyramidal piling at a surface of slip tends to bar further slip at that particular surface. Hence not only the augmented hardness due to strain, but the tendency in repeated alternations to lateral spreading of the region on which slip occurs. The hardness due to straining is, of course, removed when we raise the metal to such a temperature that complete recrystallization occurs, normal piling being then restored in the new grains.

Taking a previously unstrained piece, it is clear that the facility with which slip will occur at any particular surface of slip in any particular grain depends not only on the nature of the metal and on the orientation of the surface in question to the direction of the stress, but also on the amount of support the grain receives from its neighbours in resisting slip there. In other words, for a given orientation of surface the resistance to slip may be said to consist of two parts; one is inherent in the surface itself, and the other is derived from the position of the grain with reference to other grains.

To make this point clear, think of a grain (under stress) in which there is a gliding surface oriented in the most favourable direction for slipping. Slip on this surface can take place only when its yielding compels the neighbours (which are also under stress) to yield with it, and the surfaces in these on which slip is compelled to occur are, on the whole, less favourably situated. Hence the original grain cannot yield

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until the stress is considerably in excess of that which would suffice to make it yield if it stood alone, or had neighbours equally favourably inclined.

Apply this consideration to the case of steel, where there are two classes of grains; the ferrite, which is simply iron, and the pearlite, which is a harder structure. Slip on any ferrite grain is resisted partly by the strength of the surface itself, and partly by the impossibility of its yielding without forcing slip to take place on neighbouring (harder) grains. Now suppose the structure is a very gross one, such as Mr. Stead has shown may be found in steel that is seriously overheated. On the large grains of ferrite in overheated steel the resistance to slip will be but little greater than it would be in iron, and, consequently, under an alternating stress fatigue of strength, leading to rupture, may be produced by a very moderate amount of load. Mr. Stead * has shown how the effects of overheating can be removed by the simple expedient of raising the steel to a temperature sufficient to cause recrystallization—a homœopathic remedy that transforms the gross structure of the overheated metal into an ordinarily fine structure, where no ferrite grain can yield without compelling the yielding of many pearlite grains. Hence we find, as Rogers † has demonstrated by experiment, that steel cured by reheating from the grossness of structure previously produced by overheating, has an immensely increased power to resist the deteriorating effects of often repeated stress.

I trust you will not feel I have abused the license of the Chair in presenting contributions to molecular theory that are for the most part in the nature of speculative suggestions, thrown out in the hope that they may some time lead to fuller and more definite knowledge. Remote as they may seem to be from the concerns of the workaday engineer, they relate to the matter which it is his business to handle, and to the *rationale* of properties without which that matter would be useless to serve him. We have attempted to penetrate into its very heart and substance in order the better to comprehend the qualities and functions on which the practical work of engineering relies. The man whose daily business leads him through familiar tracks in a forest does well to stray from time to time into the shady depths that lie on either hand. The eyes of his imagination will be opened. He will at least learn his own limitations, and, if he is fortunate, he may gain some clearing on a hilltop which commands a wider view than he has ever had before.

* See especially a paper by J. E. Stead and A. W. Richards on "The Restoration of Dangerously Crystalline Steel by Heat Treatment," Journ. of the Iron and Steel Inst. No. 2, 1903.

† F. Rogers, "Heat Treatment and Fatigue of Steel," Journ. of the Iron and Steel Inst., No. 1, 1905.

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XXXIV. *The Dielectric Strain along the Lines of Force.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,

IT has naturally pleased me to have the support of Messrs. Wüllner and Wien in regard to my views on electrostriction, especially as there is still considerable opposition to my conclusion, that æther stresses are not communicated to nonconductors when electrified. But their letter in the April number of the Philosophical Magazine seems to me to require an answer, and I should be much obliged if you would publish this letter.

As regards my criticisms of the methods employed by Messrs. Wüllner and Wien, I came to the opinion expressed in my paper only after much hesitation and thought, and I do not feel justified in changing it. Where I misunderstood them, I am glad to be corrected, and also to express my regret that I overlooked their second paper. Criticism of methods of experimentation is a minor matter; and the important thing to me is, that our results are in agreement.

Their manner of stating this agreement I must refer to. They say: "We are glad to see that in the principal points Mr. More agrees totally with our views, viz., that there exist no other real forces in electrostriction than the pressure of the armatures. But we think we have done a little more than he." Now, I published my first paper on this subject in the Philosophical Magazine in 1900, and stated then, in spite of the conclusions of many investigators to the contrary, that there was no expansion in glass when electrified. Since then I have published papers frequently, all supporting this view, varying methods and working with glass, paraffin, ebonite, and a shellac mixture, both for transverse and longitudinal effects. When, also, it is remembered that they published their first article two years later, acknowledging then that they undertook the work because of my disagreement with Quincke and Cantone, it seems to me it would be proper to give me what credit the discovery may be worth and to say, that they are glad to agree totally with my view. I should naturally have supposed that their form of statement was due to the letter being written in a foreign language, if I had not previously received a letter from them, in German, expressing the same idea,—that I had now been convinced of my errors and had adopted their views. Whether they or I have done the major part of the work will have to be decided by others, and it is rather futile for us to discuss it.

I remain, Gentlemen,

Very faithfully yours,

Cincinnati, 26 April, 1906. LOUIS T. MORE.

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XXXV. *The Theory of "Moving Coil" and other kinds of Ballistic Galvanometers.* By Prof. HAROLD A. WILSON, *M.A., D.Sc., M.Sc., F.R.S.**

A FORMULA which is usually given for ballistic galvanometers is

$$Q=\frac{HT}{G\pi}\sin\frac{\theta}{2},$$

where Q = quantity of electricity passed through galvanometer.
H = magnetic field controlling the galvanometer-needle.
G = magnetic field at needle, supposed perpendicular to H, due to unit current in the coil.
T = time of a complete oscillation of the needle.
θ = angle of swing of the needle from rest corrected for damping.

If ϕ is the steady angular deflexion due to a current i, then $i=\frac{H}{G}\tan\phi$; so that

$$Q=\frac{Ti\sin\theta/2}{\pi\tan\phi}.$$

The above formulæ are of course, strictly speaking, only applicable to ballistic galvanometers consisting of a coil of wire having a single small magnet freely suspended at the centre of the coil, and arranged so that at its equilibrium position the axis of the magnet is in the plane of the coil.

In the following paper the proper formulæ for ballistic galvanometers of several types in general use are obtained, and it is found that in several cases they differ appreciably from the above.

In Prof. Fleming's 'Handbook for the Electrical Laboratory and Testing Room,' vol. ii., the formula for a moving-coil galvanometer is worked out; but owing to an approximation used in the calculation it is not obtained exactly, and it is afterwards given as applying to both moving-coil and moving-needle galvanometers.

The first type of galvanometer that will be considered is the moving-coil type with a rectangular coil, cylindrical iron core and cylindrical pole-pieces symmetrically arranged. In galvanometers of this type the magnetic field is approximately radial, so that the couple on the coil due to a steady current is proportional to the current and independent of the deflexion. The controlling couple is due to the torsion of the wire by which the coil is suspended, so that the work required to turn

* Communicated by the Physical Society: read May 25, 1906.

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the coil through an angle θ from its zero position is $\frac{\alpha\theta^2}{2}$, where α is the couple exerted by the wire when the coil is turned through the unit angle. Let Ci denote the couple due to a current i, then we have $Ci=\alpha\phi$. When a current i is passed through the galvanometer for a short time dt, we have

$$Ci\,dt=CdQ=Kd\omega,$$

where K is the moment of inertia of the coil and ω its angular velocity. Hence

$$Q=\int i\,dt=\int\frac{Kd\omega}{C}=\frac{K\omega}{C}.$$

We have also

$$T=2\pi\sqrt{\frac{K}{\alpha}} \text{ and } \frac{\alpha\theta^2}{2}=\tfrac{1}{2}K\omega^2;$$

hence

$$T=2\pi\sqrt{\frac{\theta^2}{\omega^2}}=\frac{2\pi\theta}{\omega}.$$

Thus

$$Q=\frac{K\omega}{C}=\frac{\alpha\theta^2}{C\omega}=\frac{T\alpha\theta}{2\pi C}.$$

But $\frac{\alpha}{C}=\frac{i}{\phi}$; so that finally

$$Q=\frac{Ti\theta}{2\pi\phi} \text{ exactly.}$$

Since the couple on the coil due to the current is independent of θ, it is not necessary with this type of instrument that the time during which the transient current passes should be very small compared with T.

Another type of moving-coil instrument in common use has a narrow coil suspended between the poles of a magnet and no iron core. In this case, the couple on the coil due to a current i will be nearly $Ci\cos\phi$, so that $Ci\cos\phi=\alpha\phi$ or $\frac{\alpha}{C}=\frac{i\cos\phi}{\phi}$; hence the exact formulæ for this type of instrument are

$$Q=\frac{T\alpha\theta}{2\pi C}=\frac{Ti\theta\cos\phi}{2\pi\phi}.$$

In these formulæ it is assumed that the plane of the coil at its zero position is parallel to the magnetic field.

It appears, therefore, that the first type of moving-coil ballistic galvanometer considered is superior to the second in respect of the simplicity of the exact formulæ to be used with it, and the absence of error when the transient current lasts an appreciable time.

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Moving-needle instruments will now be considered. The formulæ

$$Q = \frac{HT \sin \theta/2}{G\pi} \text{ and } Q = \frac{Ti \sin \theta/2}{\pi\phi}$$

apply to what may be called a tangent ballistic galvanometer. The time of the transient current must of course be small compared with T, and its greatest value must not be sufficient to appreciably change the magnetism of the needle.

In many ballistic galvanometers an astatic system of needles is used. First, suppose the system is exactly astatic and that the torsion of a wire or quartz fibre supplies the controlling couple. In this case, $Ci \cos \phi = \alpha\phi$ and $K\omega^2 = \alpha\theta^2$, so that the formulæ are

$$Q = \frac{T\alpha\theta}{2\pi C} = \frac{Ti\,\theta \cos \phi}{2\pi\phi},$$

as with the second type of moving-coil instrument. Also $C = M(G_1 + G_2)$, where M is the moment of each needle and G_1 and G_2 the fields due to unit current at the two needles respectively.

One other case will be considered, viz. when the needle system is more or less astatic and is controlled by magnetic fields at each needle. Let the horizontal component of the controlling field at one needle be H_1 making an angle α_1 with the plane of this needle, and let M_1 be the moment of this needle. Let M_2, H_2, and α_2 be the corresponding quantities at the other needle. Then in the equilibrium position,

$$0 = M_1H_1 \sin \alpha_1 + M_2H_2 \sin \alpha_2.$$

The controlling couple when the needle is deflected through an angle ϕ is

$$M_1H_1 \sin (\phi + \alpha_1) + M_2H_2 \sin (\phi + \alpha_2)$$
$$= \{M_1H_1 \cos \alpha_1 + M_2H_2 \cos \alpha_2\} \times \sin \phi;$$

and if ϕ is the deflexion due to a current i, then

$$iM_1G_1 \cos \phi + iM_2G_2 \cos \phi = \sin \phi \{M_1H_1 \cos \alpha_1 + M_2H_2 \cos \alpha_2\}.$$

Hence

$$i = \frac{M_1H_1 \cos \alpha_1 + M_2H_2 \cos \alpha_2}{M_1G_1 + M_2G_2} \tan \phi.$$

It can now be easily seen that the formulæ for a ballistic galvanometer of this type are

$$Q = \frac{T(M_1H_1 \cos \alpha_1 + M_2H_2 \cos \alpha_2) \sin \theta/2}{\pi(M_1G_1 + M_2G_2)} \text{ and } Q = \frac{Ti \sin \theta/2}{\pi \tan \phi}.$$

So far the correction of the observed deflexion θ for damping has been neglected. The method usually described depends on the "logarithmic decrement," and is cumbrous, and moreover inexact unless the damping is very small. The

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following method has the great advantages of simplicity and exactness; it is not new: the writer learned it about ten years ago from Prof. W. Stroud.

Let θ_1, θ_2, θ_3, &c. be the successive swings of the galvanometer-needle. Then

$$\frac{\theta_1}{\theta_2}=\frac{\theta_2}{\theta_3}=\frac{\theta_3}{\theta_4}=\&c.=f \text{ say.}$$

To find f observe, say, θ_1 and θ_3 or θ_1 and θ_5.

Then
$$f^2=\frac{\theta_1}{\theta_3} \text{ or } f^4=\frac{\theta_1}{\theta_5}.$$

Also the corrected value of the first swing is

$$\theta_1\sqrt{f}=\theta_1\left(\frac{\theta_1}{\theta_3}\right)^{\frac{1}{2}}=\theta_1\left(\frac{\theta_1}{\theta_5}\right)^{\frac{1}{4}}.$$

In most cases the angular deflexions may be taken as proportional to the deflexions on the scale. Let these be δ_1, δ_2, δ_3, &c. Then the corrected value of the first swing is

$$\delta_1\left(\frac{\delta_1}{\delta_3}\right)^{\frac{1}{2}} \text{ or } \delta_1\left(\frac{\delta_1}{\delta_5}\right)^{\frac{1}{4}}.$$

If the damping is small the corrected value is very nearly $\delta_1+\frac{1}{4}(\delta_1-\delta_3)$, and this formula is sufficiently exact for most purposes in practice.

The following table contains a summary of the results obtained:—

Type of Galvanometer.	*Formulæ.*
(1) Tangent Galvanometer with small needle at centre of coil.	$Q=\frac{HT}{G\pi}\sin\frac{\theta}{2}$, $Q=\frac{Ti\sin\theta/2}{\pi\tan\phi}$.
(2) Astatic Galvanometer with purely torsional control.	$Q=\frac{T\alpha\theta}{2\pi M(G_1+G_2)}$, $Q=\frac{Ti\,\theta\cos\phi}{2\pi\phi}$.
(3) Astatic Galvanometer with purely magnetic control.	$Q=\frac{T(M_1H_1\cos\alpha_1+M_2H_2\cos\alpha_2)}{\pi(M_1G_1+M_2G_2)}\sin\frac{\theta}{2}$, $Q=\frac{Ti\sin\theta/2}{\pi\tan\phi}$.
(4) Moving-coil Galvanometer with iron core and radial magnetic field.	$Q=\frac{T\alpha\theta}{2\pi C}$, $Q=\frac{Ti\,\theta}{2\pi\phi}$,
(5) Moving-coil Galvanometer with narrow coil and no core.	$Q=\frac{T\alpha\theta}{2\pi C}$, $Q=\frac{Ti\,\theta\cos\phi}{2\pi\phi}$.

It will be seen that type (4) is superior to all the others as regards the simplicity of the exact formula to be used with it. It is also superior in most other respects. Galvanometers are often met with in practice which strictly speaking do not belong to any of the simple types considered in this paper. In such cases the exact formulæ are more complicated, and the best plan to adopt is to determine experimentally the relation between θ and Q.

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THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

OCTOBER 1906.

XXXVI. *On the Recombination of Ions made by α, β, γ, and X Rays.* By R. D. KLEEMAN, *B.Sc.*, 1851 *Exhibition Scholar of the Adelaide University* *.

IN a paper read before the Royal Society of South Australia† it was shown by Professor Bragg and the writer that the current through a gas ionized by α rays was still unsaturated when calculation showed that the number of ions lost by general recombination was small.

This effect was ascribed to some of the ions being only partially separated from their parent molecules by the action of the α particles. In the absence of an external electric field these ions fall back on to their parent molecules. A strong electric field is supposed to be able to complete the separation of the ions, and thus under a strong electric field the number of ions is materially increased.

The electric field has no influence on the rate of general recombination of the ions after the separation of ion from parent molecule, for Langevin has shown that the rate of recombination of ions uniformly distributed is not affected by an external electric field.

The recombination of an electron with its parent molecule was, in the above-mentioned paper, termed initial as distinguished from general recombination.

The writer undertook some experiments at the Cavendish Laboratory to see whether initial recombination depended in

* Communicated by Professor J. J. Thomson, F.R.S.

† Trans. Roy. Soc. of S. A., October 1905; Phil. Mag. April 1906.

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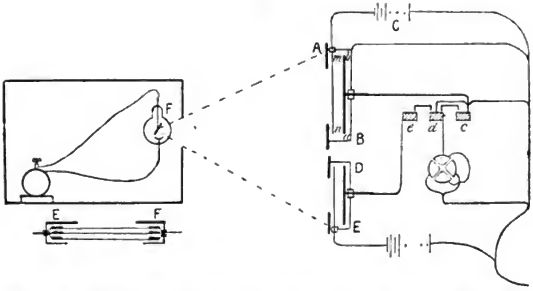
some way on the nature of the ionizing agent. The experiments were begun with X rays.

§ I.

The X-ray bulb does not, as is well known, give a constant source of radiation, and the problem that presented itself at the beginning was how to overcome this difficulty. A null-method was first used. Two ionization chambers were connected to an electrometer, so that their leaks were in opposite directions. The potential of one of them, A, was varied, while that of the other, B, was kept constant, but the leak in B could be adjusted and made equal to that in A by opening or closing a sliding door which admitted the X rays. But this method, since the displacement of the sliding door could not be made greater than 5 cms. without introducing different proportions between its parts, was not considered so sensitive as the following:—In this method the leak of the constant potential chamber was stored up, while that of the other chamber was measured directly by means of an electrometer, the stored up charge was then discharged into the electrometer, and two swings of the needle taken.

The experimental arrangement is shown in fig. 1. The

Fig. 1.



chamber DE is the constant potential or standard chamber. AB is the chamber of which the potential was varied, and it will therefore be called the adjustable chamber. The adjustable chamber used for investigating secondary X rays consisted of a circular metal box containing two metal electrodes at a distance of ·5 cm. from one another, one of them, *ab*, was a thick lead plate connected with the mercury cup *c*, the other, *mn*, was a thin aluminium plate connected with the battery C. The standard chamber DE was of

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identical construction, though somewhat smaller, and its lead plate was connected with the mercury cup *e*.

The X rays were generated by an X-ray bulb F, which was enclosed, together with the induction-coil, in a lead box having a window for the exit of the rays. When X rays were allowed to fall on the two chambers, some of the rays penetrated the aluminium plates, and meeting the lead plates gave rise to an intense secondary radiation from the surface of the lead, so that the air between the plates was principally ionized by secondary X rays. The intensity of the primary rays was cut down to the required amount by placing sheets of tinfoil over the window of the lead chamber in which the X-ray bulb and induction-coil were enclosed.

The primary X rays were investigated by means of two chambers of the form EF. The electrodes of these chambers consisted of brass rings over which tissue-paper had been tightly stretched, and this covered with thin aluminium-leaf. This arrangement introduced little secondary radiation. The electrodes were at a distance of ·5 cm. from one another. The central electrode of the standard chamber was connected with the mercury cup *e*, and that of the adjustable chamber was connected with the mercury cup *c*. Each pair of outside electrodes was connected with a battery.

If *na* denote the leak per second in the adjustable chamber, the leak in the standard chamber may be denoted by *nb*, and the ratio of the leaks of the two chambers, when taken for a time *t*, is $\left(\frac{tna}{tnb}\right)$, that is $\left(\frac{a}{b}\right)$, a quantity independent of the time of leak and the variations in intensity of the rays. If the leak in the adjustable chamber changes when a different E.M.F. is applied, the change is proportional to *a*, that is to $\frac{a}{b}$, since *b* is constant.

It was found that if the electrodes of the two chambers were not of exactly the same material, the compensation was not perfect. This appeared to be due to the X rays varying not only in intensity but also in nature, and their variations in nature not giving rise to equal relative changes in the intensity of the secondary radiations, on account of the materials being different.

The observations were carried out in the following manner:—

At the beginning of an observation the mercury cup *d* was kept earthed, and each of the cups *e* and *c* kept connected with *d*. The current was then started in the primary circuit

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of the induction-coil. After a few seconds, the connexions of d with e and the earth were broken simultaneously: this allowed the leak in the adjustable chamber to run into the electrometer and that in the standard chamber to get stored up. At the end of about twenty seconds, the current through the primary circuit of the induction-coil was broken, and the first two swings of the electrometer-needle were read.

After having discharged the electrometer, the connexion between c and d was broken, and d connected with e, thus allowing the charge in the standard chamber to be shared by the electrometer. The third and fourth swing of the needle were taken in this case, since the first and second were difficult to read accurately on account of the rapid motion of the needle.

When a quantity of electricity is suddenly discharged into an electrometer, the swings of the needle may be large; and if the mean of two large swings is taken as the ultimate position of rest of needle, a considerable error is introduced on account of the damping of the needle.

It became necessary, therefore, to devise a formula for obtaining the correct position of rest of needle from two of its swings.

Let V denote the potential-difference of the quadrants of the electrometer, and x_0 the corresponding reading on the scale. Then the forces of restitution acting on the needle oscillating about x_0, are $VK_2 - xK_1$, $xK_1 - VK_2$, where K_1, K_2 are constants. Since $x_0 K_1 = VK_2$, each of these forces is equal to $K_1(x_0 - x)$, and therefore the needle executes an harmonic vibration about x_0, whose period is independent of the value of x_0.

But since there is a damping effect on the vibrations of the needle, the amplitudes of the vibrations will decrease, but any two successive amplitudes should bear a constant ratio to one another. This was found to be the case: for example, the ratios of a number of pairs of consecutive amplitudes were found to be ·54, ·54, ·55, ·55.

Let $\frac{1}{K}$ denote the ratio of two successive amplitudes of the swinging needle, and let x_1, x_2 denote two successive extreme deflexions, of which x_1 is the further away from zero. Then

$$x_0 = x_1 - \{(x_1 - x_2) - K(x_1 - x_2) + K^2(x_1 - x_2) - \ldots (-1)^n K^n (x_1 - x_2)\},$$

where $n = \infty$; therefore $x_0 = x_1 - (x_1 - x_2)\frac{1}{1+K}$.

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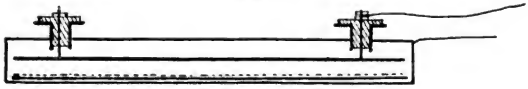
The swinging needle was allowed to settle down in a few cases, and its deflexion compared with that calculated by means of the above formula from two swings: the agreement was very good. The value of $\frac{1}{1+K}$, calculated from observations of K, was found to be ·65 when either the lead or aluminium chamber was in connexion with the electrometer.

The deflexion proper (x_0), when the adjustable chamber was connected with the electrometer, was also calculated by means of this formula.

The deflexions of the electrometer-needle were obtained from the motion of the image of a scale across the cross-wire of a telescope, a scale being placed in such a position that an image of it was reflected into the telescope by the mirror of the electrometer-needle.

Besides the experiments with X rays, a set of experiments was carried out with the α rays of uranium. The contrast which appears in the behaviour of these two different ionizing agents will, I think, be brought out better if these experiments are described before any of the actual observations made with X rays are given. The apparatus used in these experiments consisted of a square flat metal box (see fig. 2),

Fig. 2.



19 cms. by 19 cms., and 1·5 cm. high. The electrode connected with the electrometer was a thick lead plate, 18 cms. by 18 cms., suspended from insulating supports. The case was connected with the high potential. A plate, covered with a very thin layer of uranium oxide, could be slid in from one side. The distance between the layer of uranium oxide and the lead plate was ·5 cm.

The following table gives some of the readings taken. The leak was allowed to run into the electrometer for 13 seconds, the connexion between chamber and electrometer was then broken and the first and second swing of needle read. The deflexion proper (x_0) was calculated by the formula given, the value of $\frac{1}{1+K}$ having been found to be ·61.

The natural leak of chamber and electrometer (when the

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uranium oxide was out) was found to be 10 divisions for 13 seconds, and practically independent of the potential applied; and 10 divisions have therefore been subtracted from each leak.

TABLE I.

Volts per cm.	Deflexion (each reading repeated).		Volts per cm.	Deflexion (each reading repeated).	
8	1647 1641	Lack of saturation 22 per cent.	1200	2121 2135	Lack of saturation 15 per cent.
1200	2128 2158		20	1844 1829	
8	1676 1663		1200	2095 2117	
1200	2079 2145		20	1847 1855	

It will now be shown that the lack of saturation obtained when 8 or 20 volts per cm. were applied, was not due to the existence of general recombination of the ions.

Let us consider the case when 8 volts per cm. were applied. The capacity of the uranium chamber could be calculated, and by means of it, making use of the method of mixtures, the capacity of electrometer and chamber was found to be about 230 E.S. Us. The deflexion of needle for one volt was 4800 scale-divisions. The leak per second was about 130 scale-divisions. The area of the electrode upon which the current was received was 324 cm.^{2} The current per cm.^{2} per second received on the plate was therefore equal to $\frac{130 \times 230}{4800 \times 300 \times 324}$ E.S. Us., that is $6{\cdot}5 \times 10^{-5}$ E.S. Us. The number of ions received per cm.^{2} is therefore 2×10^5 approximately. The longest life of an ion was about $\frac{{\cdot}5}{8 \times 1{\cdot}5}$ of a second, if the velocity of an ion under the action of one volt is taken to be 1·5 cm. per second. Let us assume that the air between the plates was uniformly ionized, and let n denote the number of ions per c.c. at any instant. Then $\frac{n \times 24}{2} = 2 \times 10^5$, which gives $n = 1{\cdot}7 \times 10^4$.

The number of ions lost by general recombination is therefore $(1{\cdot}7 \times 10^4)^2 \frac{10^{-6}}{2}$, that is $1{\cdot}4 \times 10^2$. The ratio of the number of ions lost by recombination to the total number received is therefore $\frac{1}{1400}$. Thus only a small fraction of the ions made are lost by recombination.

Since the ionization is not uniform between the plates, let us assume that it is equivalent to all the ionization taking

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place in the mm. next to the uranium oxide. The number of ions lost by recombination will then be about five times greater; but it still remains a small fraction of the total number received.

In order to test still further whether the lack of saturation was due to non-uniformity of the ionization, a thinner layer of uranium oxide was used, having only about half the ionizing power of the previous layer.

The readings obtained are given in the following table, the leak being taken for 22 seconds.

TABLE II.

Volts per cm.	Deflexion (each reading repeated).	
1200	2565 2553	Lack of saturation 22 per cent.
8	2002 2007	
1200	2593 2562	
8	2001 2042	

It will be seen that the lack of saturation when 8 volts per cm. were applied is the same as obtained with the thicker uranium layer. The lack of saturation in both cases is therefore not due to non-uniformity of ionization.

This establishes thus another similarity between the α rays of uranium and radium.

I now append some of the observations made with X rays.

The following table contains some of the observations made with the aluminium chambers, in which the air was principally ionized by primary rays.

TABLE III.

Volts per cm.	Deflexion (adjustable chamber).	Deflexion (standard chamber).	Ratio.	
1200	1696	594	286	Lack of saturation 5 per cent.
8	1703	624	273	
1200	1641	567	289	
8	2117	769	275	
20	1914	671	285	Lack of saturation 2 per cent.
1200	1302	446	292	
20	1298	458	283	
1200	1696	589	289	

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The leaks were taken for 20 seconds.

The X-ray bulb was worked by a coil having an ordinary interrupter.

When the two chambers were disconnected from the electrometer and the X-ray bulb sending out rays, there was a leak of about 30 scale-divisions in 20 seconds, due to the natural leak of electrometer and ionization in the connexion tubing by stray X rays. This small leak appeared in the leak of the adjustable chamber, but it was not subtracted since its exact value was not known on account of the intensity of the rays not being constant. Its value must be, however, almost proportional to the total leak, and it reduces, therefore, in each ratio of leaks to an approximately equal quantity. The natural leak of the standard chamber, found by subtracting the leak of the electrometer from that of electrometer and chamber, was very small. The disturbing effect of these small leaks upon the readings must be very small, as is shown by the consistent readings obtained, though the induction-coil worked very irregularly, and the leak had sometimes to be taken for a longer or shorter time than 20 seconds.

The following table gives some of the readings obtained with the lead chambers.

TABLE IV.

Time.	Volts per cm.	Deflexion (adjustable chamber).	Deflexion (standard chamber).	Ratio.	
20 sec.	1200	2339	1327	176	Lack of saturation 8 per cent.
,,	8	2405	1458	165	
,,	1200	2400	1339	179	
,,	8	2138	1305	163	
20 sec.	20	1251	623	201	Lack of saturation 2 per cent.
12 ,,	1200	1208	587	206	
18 ,,	20	1169	586	200	
24 ,,	1200	594	291	204	

The readings for the voltages 20 and 1200 were taken with an X-ray bulb which had been run so hard that the coil (giving a six-inch spark) was scarcely able to work it. It was found at the end of these experiments that the platinum of the spark-gap was almost worn away, which accounts, to some extent, for the irregularity of the rays.

General recombination could scarcely have affected the above readings. The capacity of lead chamber together with the electrometer was found to be about 180 E.S. Us., while

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that of the aluminium chamber together with the electrometer was found to be about 150 E.S. Us. In the lead chamber the rays fell on an area of 140 cm.^{2}, and in the aluminium chamber on an area of 50 cm.^{2} If we take 100 scale-divisions per second as the average leak in each chamber, we get a current of 9×10^{-5} E.S. Us. per cm.^{2} for the lead chamber, and a current of 10×10^{-5} E.S. Us. per cm.^{2} for the aluminium chamber. The number of ions lost by recombination is therefore, in both cases, somewhat greater than occurred in the experiment with uranium oxide (about three times greater in the case of the aluminium chamber), but it is still a small fraction, in each case, of the total number received.

It appears from these observations that there is very little lack of saturation (when the number of ions lost by general recombination is small) with primary and with secondary X rays, and initial recombination is therefore practically absent in air ionized by X rays.

The greater lack of saturation in the case of the lead plate chamber is probably due to non-uniformity of its ionization, which is certainly very much more dense near the lead plate than elsewhere.

It may, perhaps, be argued that when the air was ionized by the α rays from a layer of uranium oxide, the ionization was very dense near the layer, and lack of saturation was due to loss by diffusion to the plate. It becomes of importance, therefore, to compare the density of ionization at various distances from the uranium oxide with that in the lead chamber due to X rays, for the experiments show that in the latter case the loss of ions is comparatively small.

Let us take an element of surface δs on the surface of the uranium oxide, and draw a thin cylinder through it in any direction, and produce it both ways. The number of uranium molecules per c.c. which eject their α particles along the cylinder towards the surface of the uranium, is everywhere the same. Therefore, since the range of an α particle is diminished in proportion to the matter it has to traverse, the ranges of the α particles along the cylinder from the surface of the uranium consist of a set of ranges diminishing in arithmetical progression, beginning with the maximum range of an α particle in air and ending with zero. In the last case the α particle from some depth in the uranium oxide just manages to reach the surface. The total ionization in the cylinder may therefore be approximately represented by the area of a triangle, and written $\frac{r^2 K}{2}$, where K denotes a

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constant and r the maximum range of an uranium α particle in air. The total ionization in the cylinder for a length r_1 measured from the uranium becoming consequently

$$\left(r_1 r - \frac{r_1^2}{2}\right)\mathrm{K}.$$

The ionization I in terms of the saturation current between the plate on which the uranium oxide is spread and a plane at a distance x, is then given by

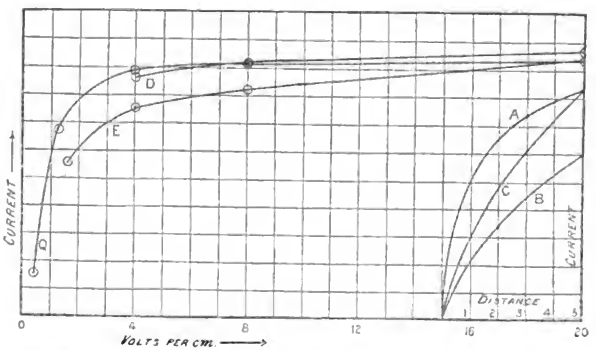
$$\mathrm{I} = \frac{r^2 \mathrm{K} n}{2} \int_{\theta_1}^{\frac{\pi}{2}} r d\theta \,.\, 2\pi r \sin\theta + \mathrm{K} n \int_0^{\theta_1} r d\theta \,.\, 2\pi r \sin\theta \left(r_1 r + \frac{r_1^2}{2}\right).$$

where $\frac{x}{r} = \cos\theta_1$, $\frac{x}{r_1} = \cos\theta$, and n denotes a constant, and θ the angle the projected particle makes with x. Integrating this equation we obtain

$$\mathrm{I} = \mathrm{K} n \pi r^2 \left\{ x^2 - 2rx \log \frac{x}{r} \right\}.$$

Curve A in fig. 3, taken from one of Langevin's papers,

Fig. 3.



shows the relation between current and distance of a plane from a lead plate when the intervening space is ionized by the secondary X rays produced by primary rays falling on the lead plate, the abscissæ denoting distances from the lead plate and the ordinates the corresponding currents.

If the ordinate 8·3, corresponding to the distance ·5 cm.

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on Langevin's curve, represents the leak in the lead chamber, the corresponding ordinate representing the uranium chamber leak is $\frac{8{\cdot}3 \times 6{\cdot}5}{9}$, that is 6, where $\frac{6{\cdot}5}{9}$ is the ratio of the current in the uranium chamber to that in the secondary X-ray chamber.

If we put $I=6$, $x={\cdot}5$, and $r=3$ in the foregoing equation, we get $Kn\pi r^2=1{\cdot}07$. We are now able to plot the relation between x and I. This was done, and is shown by the curve B. Curve C was obtained by taking the current in the uranium chamber equal to that in the secondary X-ray chamber.

It will be seen that the ionization was far more dense near the lead plate of the X-ray chamber than near the uranium oxide, and lack of saturation in the latter case was therefore not due to diffusion, or the ionization not being uniform.

The effect of diffusion must always be small when the field applied gives to an ion a velocity which is great in comparison with the velocity of diffusion. Consider a slab of ionized gas of thickness one or two mean free paths of a molecule and lying close to a metal plate: it will rapidly lose its ions on account of their rapid to and fro motion bringing them in contact with the plate, whereby they lose their charges. But the ions adjacent to this slab will cross, as a whole, the boundary with a velocity which is the sum of the velocity of diffusion and the velocity given to them by the attraction of their induced charges, for the ions that cross the boundary through their rapid to and fro motion do not at once give up their charges, and therefore we have also ions which cross the boundary in the opposite direction.

The force on an ion due to the induced charge in a metal plate at a distance of 5×10^{-4} cm. from the ion is about $\frac{1}{10}$ of a volt.

Thus the drift of ions towards a metal plate is equivalent to the application of a fraction of a volt to the ions of both signs, and the loss of ions therefore confined to a slab of ionized gas of thickness several mean free paths of a molecule, when the external field is strong. The loss is therefore in general very small.

It can be shown that the lack of saturation observed with weak ionization by α rays is independent of general recombination, whatever the value of the recombination coefficient, and independent of the velocity of an ion under unit electromotive force.

Consider an ionized gas between two metal electrodes at a difference of potential.

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As we pass from one plate towards the other the number of ions moving in the same direction will in general increase with the distance from the plate. Suppose that the lack of saturation observed is due to general recombination. Then, if we increase the ionization, the density of the moving ions of each sign becomes greater everywhere, and we increase general recombination, whatever the value of the recombination coefficient, and therefore the lack of saturation. But it has been shown in this paper, and in the paper by Prof. Bragg * and the writer, that the lack of saturation is independent of the density of the ions, when on the whole it is small.

If the ions are of a very slow moving kind, then the increase in density of the ions of one sign with the distance from the plate from which they are receding, is greater than before, and therefore general recombination as a whole greater. If the observed lack of saturation is due to the existence of slow moving ions, it is caused by greater general recombination taking place, and, therefore, if we increase the ionization we ought to increase the lack of saturation. But since the lack of saturation has been shown to be independent of the density of the ions, it follows that it is also independent of their velocity. That the lack of saturation is independent of the velocity and recombination coefficient of the ions, follows also from the fact that lack of saturation is independent of the depth of chamber.

In the paper on the recombination of ions by Prof. Bragg and the writer, an experiment of Rutherford's was quoted in support of the initial recombination theory. Rutherford found, in the experiment mentioned, that the ionized gas drawn away from the uranium which ionized it, and treated in a separate vessel, was easier to saturate than when it was in contact with the uranium and ionization going on continuously.

In a later paper † Prof. Bragg describes an experiment by Mr. Madsen, who found that the current through a mixture of air and ether drawn away from the uranium which ionized it, was still hard to saturate.

But such an effect may appear quite apart from initial recombination. At the instant a portion of the ionized gas comes under the action of the field, it must contain some ions in the act of undergoing general recombination, and a greater or smaller number of these ions will be separated again, according to the strength of the field.

It should be observed that the electric field has no further

* *Loc. cit.* † Phil. Mag. May 1906.

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effect on the ions after they are separated, for, as has been mentioned before, an electric field has no influence on the rate of recombination of ions after they have once been uniformly distributed in the field. The number of ions undergoing recombination in the stream of gas at any instant depends, of course, on the density of the ions. Thus we may have a lack of saturation of any magnitude, and in this way both Rutherford's and Madsen's results can be explained.

On the other hand, experiments of this kind are liable to a source of error, if no special precautions are taken. The stream of ionized air passed between two plates at a difference of potential, must necessarily pass first through a field whose intensity is less than that which exists exactly between the plates, and a number of ions, depending on the velocity of flow of the gas, may have reached the plates before they reach the maximum field. An increase of potential might then give rise to an increase of current on account of the velocity of the ions becoming greater in this weaker part of the field, and general recombination therefore becoming less.

In fig. 3 I have plotted α and X ray current-voltage curves. The curve Q was plotted from aluminium-chamber readings. The part of the curve corresponding to low potentials has been plotted because each electrode was of the same material on the outside, and the disturbing effect of the Volta difference of potential therefore small.

Curves D and E were plotted from lead and uranium chamber readings respectively, but were not plotted for low potentials because here the Volta couples consisted of lead-aluminium and lead-brass respectively, and the curves therefore were not comparable with the curve Q or with one another at low voltages.

The lack of saturation of current corresponding to any point on the α ray curve, indicated by the curve's slope, is very marked. The curve D (from lead chamber) has a slightly greater slope than the curve E, which is probably due to the ionization in the chamber not being uniform.

The initial recombination was next studied in some gases and mixtures of a gas and air.

Since no difference appeared in the behaviour of primary and secondary X rays with respect to the initial recombination of the ions made, secondary X rays have been used throughout these experiments.

The uranium chamber used in the previous experiments (fig. 2), being provided with two tubes communicating with the interior, was used as the adjustable chamber by fixing an aluminium plate with a suitable lead guard to the bottom.

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The depth of the chamber was ·5 cm. and the area ionized 256 cm.^{2}

Readings with the α rays from uranium could be taken by sliding a plate covered with a thin layer of uranium oxide into the chamber through a side door.

The chamber was made gas-tight by waxing the joints.

The observations were carried out in the same manner as before.

Since air containing a small proportion of a gas heavier than air greatly increases the lack of saturation when the mixture is ionized by α rays, air bubbled through a volatile liquid has been used in several experiments.

In Tables V. and VI. are given some of the observations made. Observations made with the α rays from uranium have been placed alongside for comparison.

TABLE V.

	X Rays.			α Rays.		
	Volts per cm.	Ratio.	Lack of saturation and time of leak.	Volts per cm.	Deflexion.	Lack of saturation and time of leak.
AIR.	1200 8	111 105	13 sec. 5 p. cent.			
Air bubbled through chloroform for 1 minute.	8 1200 8 1200 20	114 123 113 122 117	13 sec. 7 p. cent. 4 ,,	1200 400 80 20 8	2182 2058 1853 1624 1451	10 sec. 6 p. cent 15 ,, 26 ,, 38 ,,
AIR.	1200 8	122 114	16 sec. 7 p. cent.			
Air bubbled through carbon-tetrachloride for 1¼ minutes.	8 1200 8 1200	129 142 129 141	14 sec. 9 p. cent	700 80 20 8	2794 2369 2067 1845	15 sec. 15 p. cent. 26 ,, 34 ,,
AIR.	20 1200 20 1200	126 132 126 129	3 p. cent.			
Air bubbled through ether for 1 minute.	1200 20 8 1200	152 142 138 152	7 p. cent. 9 ,,	1200 400 80 8	2796 2599 2069 1306	7 p. cent. 26 ,, 53 ,,

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TABLE VI.

	X RAYS.			α RAYS.		
	Volts per cm.	Ratio.	Lack of saturation and time of leak.	Volts per cm.	Deflexion.	Lack of saturation and time of leak.
AIR.	...	...	...	700 80 8	2642 2539 2023	15 sec. 4 p. cent. 23 „
Air bubbled through methyl-alcohol for 2 min.	20 710 8	159 158 154	20 sec.	700 89 8	2640 2437 1855	15 sec. 8 p. cent. 30 „
A stream of CO_2 passed through chamber for 2 min.	1200 8 1200 8	134 124 136 129	10 sec. 8 p. cent. 4 „	660 79 20 7·9	3008 2459 1963 1544	13 sec. 18 p. cent. 35 „ 49 „
Ethyl chloride passed into chamber.	1200 20 1200 20 1200	346 296 313 298 332	20 sec. 10 p. cent.	1050 660 390 79 20 7·9	2869 2691 2306 1470 997 817	10 sec. 6 p. cent. 20 „ 40 „ 65 „ 72 „
AIR.	800	162	20 sec.			

The ratios only of the deflexions of the two chambers are given in the tables.

Calculation showed that general recombination could not have appreciably affected the above readings.

The X-ray bulb used in these experiments gave fairly constant rays. When the X-ray bulb was sending out rays and the two chambers disconnected from the electrometer, an approximately constant leak (largely due to stray X rays) was obtained. This leak was subtracted from the adjustable chamber reading.

The natural leak of each of the two chambers was so small as to be negligible.

The X-ray readings for the different gases and mixtures of air and a gas are not strictly comparable with one another, except in the cases stated or indicated in the tables, because any slight displacement of the X-ray bulb or chambers between a set of experiments would alter the ratio.

The uranium readings in Table V. are also not comparable with one another because the layer of uranium oxide had to be replaced by another once or twice on account of some liquid getting accidentally into the chamber, but the uranium

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readings in Table VI. are comparable with one another. Each uranium leak is the mean of two readings, which seldom differed by more than two per cent.

In the case of a mixture of air and a vapour, the air was drawn by means of a water-pump through a wash-bottle containing the volatile liquid.

It will be seen that the uranium saturation leak was not appreciably affected when the air was bubbled through methyl alcohol, but the lack of saturation for low voltages was approximately doubled.

In the case of ethyl chloride, an amount of ethyl chloride gas, about double the cubical contents of the chamber, was passed into the chamber so that the ethyl chloride entered at the bottom while the air was expelled at the top; the chamber ought to have contained a large percentage of the gas at the end of the process. It will be seen that the uranium saturation leak is almost double the air leak, and the lack of saturation for low voltages is increased enormously, while the X-ray saturation leak is also about doubled, but the lack of saturation for low voltages only slightly increased.

On the whole, it appears from these observations that the effect of initial recombination is also small in other gases besides air.

§ II.

The fact that initial recombination is practically absent in a gas ionized by X rays is due, probably, to the electron being ejected with a greater speed from its parent molecule than is the case when a molecule is ionized by an α particle. We know from Dorn's experiments that the electrons ejected from a metal plate on which X rays fall, have a speed varying from $1{\cdot}8 \times 10^9$ to $8{\cdot}5 \times 10^9$ cm./sec., and it is very probable that an electron is ejected from its parent molecule with a velocity of the same order.

Curie and Sagnac have shown that the penetrating power of these ejected electrons is of the same order as that of the Lenard rays, a piece of aluminium $\cdot 46 \times 10^{-4}$ cm. thick reducing the stream of electrons about 40 per cent. If we take 3·42 for their coefficient of absorption in air, as found by Lenard for the cathode rays, and apply it to the electrons ejected from air molecules, we find that less than one per cent. is absorbed in a distance of ·003 cm., and about 50 per cent. in a distance of ·2 cm.

The electric field at a distance of ·003 cm. from a charged molecule is $\frac{e\,300}{(\cdot 003)^2} = 10^{-2}$ of a volt per cm. approximately.

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The electrons thus get well out of the spheres of influence of their parent molecules before getting attached to other molecules.

But this is also true for a dense gas. If we take ethyl chloride, for example, we find that one per cent. of the electrons are absorbed in a distance of ·0006 cm. and five per cent. in a distance of ·0034 cm., the electric field at these distances from a charged molecule being respectively 3×10^{-1} and 10^{-2} of a volt.

Since X rays and α rays differ from one another in their power of effecting a complete separation of electron from its parent molecule, it will be of interest to compare them more closely.

The α particle seems to effect the ionization of a molecule by means of its external field, for Professor Thomson has shown that the charge of an α particle will get neutralized when its velocity falls below a certain limit, which is approximately that at which, as Rutherford showed, it ceases to ionize.

And it seems that the α particle ionizes a large number of the molecules that lie within a limiting distance from its path. This distance, as well as the number of molecules ionized in it, depends probably on the velocity of the α particle and the nature of the gas. Rutherford, by assuming that the α particle in air ionizes all the molecules in a cylinder of diameter twice the diameter of a molecule, obtained for the number of α particles projected per sec. from a gram of radium a value that agreed well with those obtained by other methods.

The mechanism of the ionization of the X ray seems to be essentially different from that of the α ray. When a cathode particle strikes the anode in a vacuum-tube a spherical pulse of force travels away from it, and probably encounters all the molecules in the volume of gas under investigation, but ionizes only a very small fraction of them, possibly none at all sometimes.

It appears from some experiments by Barkla, that the electrons of a molecule emitting secondary X rays move in the same direction as the direction of the force in the primary pulse, and it seems reasonable to suppose, therefore, that the electron from an ionized molecule is ejected in the same direction as the force in the ionizing pulse. And it seems that a molecule is only ionized when one of certain directions in its interior is parallel or approximately parallel to the ionizing pulse.

Now, it appears from the electron-theory of the atom, so ably worked out by Professor Thomson, that some of its

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electrons can be displaced only in certain directions, and when an electron is displaced beyond a certain point, the direction of the forces acting on it, due to the other electrons and positive charge of molecule, will be such as to eject it from the molecule.

The electron would then derive, in part at least, its energy from the potential energy of the molecule.

This would account for the great penetrating power of the electrons ejected from molecules ionized by X rays.

§ III.

For investigating the initial recombination of ions made by β rays, the β rays of uranium oxide were used.

About 30 gms. of uranium oxide were spread uniformly in a shallow metal dish, and covered with a sheet of aluminium of ·27 mm. thickness. This thickness of aluminium was sufficient to stop all the α particles.

This dish fitted into the chamber used in the previous experiments.

The electrode was raised by means of its supporting screws till the distance from the aluminium plate was ·5 cm.

Table VII. gives some of the readings obtained.

It appears from these readings that there is little initial recombination of ions in a gas ionized by β rays.

This result might have been expected, since the secondary β rays from a solid have considerable penetrating power, and there is no reason why molecules should behave differently in the gaseous state.

An experiment was also carried out to test whether the ions made by β rays that had been allowed to pass through a considerable thickness of aluminium, showed greater initial recombination than before. The observations (see Table VII.) showed that there is still little initial recombination.

This probably means that, since initial recombination in the case of the α particle depends on its velocity, as will be shown further on, the rays which penetrated the aluminium plate suffered scarcely any decrease in velocity, and therefore the absorption of β rays by a slab of matter consists more in the deflexion of the rays than absorption of their energy.

§ IV.

The initial recombination of ions made by γ rays was next investigated, the same ionization chamber as before being used.

The electrode was raised till it occupied the middle of the

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chamber, its distance from either the top or bottom being then 7 mm.

A quantity of radium bromide, surrounded by 3 cms. of lead, was placed at a distance of 6 cms. from the chamber.

Table VII. gives some of the observations made. Each deflexion in the table is the mean of two readings, the leak of the electrometer having been, of course, subtracted from each reading.

TABLE VII.

	β RAYS.			γ RAYS.		
	Volts per cm.	Deflexion.	Lack of saturation and time of leak.	Volts per cm.	Deflexion.	Lack of saturation and time of leak.
AIR.	8 80 400	1002 1025 1029	3 p. cent. 20 sec.	6 60 286	1832 1868 1854	1 p. cent. 12 sec.
Air bubbled through ether for 1 min.	8 400	1427 1516	5 p. cent. 20 sec.	6 286	1768 1832	3 p. cent. 9 sec.
CO_2 passed into chamber.	8 400	1502 1505	 20 sec.			
Ethyl chloride passed into chamber.	8 80 400	1734 1834 1918	9 p. cent. 4 ,, 16 sec.	6 60 286 536	1639 1760 1841 1865	12 p. cent. 6 ,, 1 ,, 7 sec.
AIR. Uranium covered with an additional thickness of ·7 mm. of aluminium.	8 80	1546 1528	 3 min.			

It will be seen that the initial recombination of the ions made by γ rays is also small.

With α rays the lack of saturation is greater the denser the gas, and although the lack of saturation is small in gases ionized by β, γ, or X rays, it is also more marked the denser the gas.

Eve has shown that the secondary rays from a metal plate on which γ rays fall consist in part of projected electrons of great penetrating power; and it is very probable, therefore, that the ejected electrons in a gas ionized by γ rays possess a velocity of the same order, which enables them to get well away from their parent molecules.

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I also tried whether there there is any abnormal initial recombination effect when air is ionized by α and γ rays at the same time.

The result is given in Table VIII.

TABLE VIII.

AIR.	Volts per cm.	Deflexion.	Time of leak.	γ leak.
α rays.	6 286	1135 1478	20 sec.	$1215-\frac{1135}{2}=648$
α and γ rays.	6 286	1215 1387	10 sec.	$1387-\frac{1478}{2}=648$

A plate covered with a thin layer of uranium oxide was put into the chamber and a set of readings taken. The radium bromide was then placed near the chamber, and another set of readings taken, the air being now ionized by α and γ rays.

If we take the value of an α ray leak for 10 seconds, and subtract it from the α and γ leak corresponding to the same time and voltage, we ought to get the same remainder in each case if the ionizing effects of α and γ rays are simply additive.

This, as shown in the table, is the case.

The electrons ejected from the molecules of a gas ionized by β, γ, or X rays have very likely themselves ionizing power, and these electrons may cause the ejection of others which also have ionizing power. But the number of electrons ejected ultimately with a velocity so small that only a partial separation of electron from molecule takes place, would be a fraction of the whole number of ions made, and the effect of initial recombination therefore small.

§ V.

Some experiments were also carried out to test whether there is any connexion between the velocity of an α particle and the initial recombination of the ions made.

The apparatus used is shown diagrammatically in fig. 4.

A is a flat ebonite ring to which is attached a piece of wire gauze a, which is connected with a battery. The ebonite ring supports a metal case B, in which an electrode b is suspended by a central rod which is connected with an electrometer but insulated from the case, the case being connected

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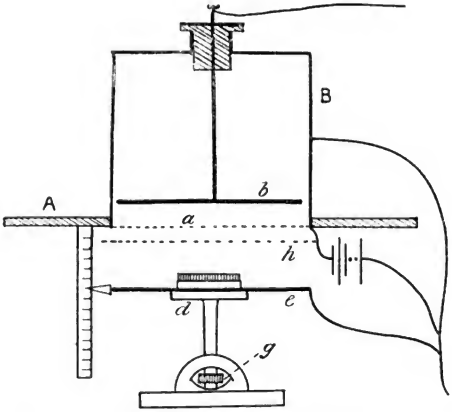
to earth. The distance of the metal electrode from the gauze was 2·5 mms.

Fig. 4.



A film of polonium was used as the source of ionization. A bundle of tubes, each ·5 cm. long, was placed on the disk with the axes of the tubes vertical in order to obtain approximately parallel and vertical rays.

The disk was placed on a large metal plate *e*, supported by the table *d*, which could be raised or lowered by means of the screw *g*, the distance of the polonium film from gauze being indicated by a pointer on a scale.

In one of the experiments another gauze *h*, insulated from *a* and connected to earth, was placed at a distance of 3 mms. from *a*, the object being to prevent a drift of ions into the chamber. This arrangement was used to ascertain if the variable field between *e* and *a*, in the absence of the gauze, was strong enough to prevent a drift of ions into the chamber, and did not otherwise affect the leak.

This apparatus served also to determine the range of the α particle from polonium in air at atmospheric pressure, and the ionization along its course.

Curves have been plotted from the observations showing the relation between ionization and distance of polonium film from gauze.

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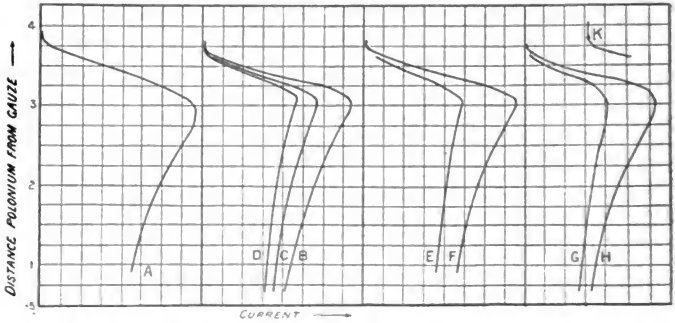
Curve A in fig. 5 was obtained with a film of polonium about eighteen months after it had been prepared.

Fig. 5.



The tubes used were ·5 cm. high and ·04 cm. in diameter. The gauze was kept at a potential of 160 volts, and the leak allowed to run into the electrometer for one minute.

The readings were taken without the protecting gauze *h*, since it appeared from some experiments, which will be described further on, that no drift of ions into the chamber took place, even with low voltages.

The curve shows that the α particle from polonium, like the α particle from radium, is a more efficient ionizer towards the end of its course.

It was also found that the greater the velocity of the α particle, the better the ions are able to get away from their parent molecules.

For example, when the distance of the polonium film from gauze was about ·7 cm., the leaks for 8 and 160 volts per cm. were respectively 475 and 628 scale-divisions per minute, each leak being the mean of two readings. The lack of saturation when 8 volts are applied is therefore 24 per cent., the current being practically saturated with 160 volts. When the distance of polonium from gauze was about 2·7 cms., the leaks for 8 and 160 volts were respectively 629 and 967 scale-divisions, each leak being, as before, the mean of two readings. This gives a lack of saturation of 35 per cent. for 8 volts, or a lack of saturation about 1·5 times that obtained when the distance was ·7 cm.

This is brought out very nicely by the curves B, C, D, which correspond to a difference of potential of 320, 24, and

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8 volts respectively between gauze and plate. A polonium film of greater strength was used in these experiments, the tubes now used being ·5 cm. high and ·15 cm. in diameter. Both the polonium films were purchased from Dr. Sthamer, of Hamburg.

In order to test whether the leaks were affected by a drift of ions into the chamber, the two curves E and F, corresponding to 8 and 320 volts respectively, were plotted from readings taken with the protecting gauze *h*. It will be seen that the curves are of the same shape as D and B.

Curves G and H were plotted from readings taken with a different apparatus, but constructed on the same lines. The distance of plate from gauze was 4 mms., and the potential-differences used were 10 and 500 volts respectively.

I append the readings from which these curves have been plotted, and also the lack of saturation corresponding to different distances of polonium from gauze.

Distance from gauze.	Leak for 500 volts per cm.	Leak for 10 volts per cm.	Lack of saturation.
·7 cm.	489	395	19·2 p. cent.
1·0	518	408	21·2
1·3	558	425	23·8
1·6	592	442	25·3
1·9	635	469	26·1
2·2	692	494	28·6
2·4	737	520	29·4
2·6	787	552	29·9
2·8	863	571	33·8
3·0	903	592	34·4
3·1	887	585	
3·2	800	532	
3·3	637	436	
3·5	245	183	
3·6	110	79	
3·7	28	24	
3·8	5	5	
4·1	5		

It will be seen that the lack of saturation increases with distance from gauze.

General recombination could scarcely have affected any of the observations made. The greatest leak observed was about 120 scale-divisions per second. This leak was received on an area of 20 cm.^{2} The capacity of the system was about 130 E.S.Us. From this we obtain that about $\frac{1}{600}$ of the number of ions received on the plate were lost by general recombination.

The increase of lack of saturation with decrease of velocity of the ionizing α particles is probably due to the velocity of

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the ejected electrons being smaller the smaller the velocity of the α particles. The electrons are very likely ejected at right angles to the motion of the α particles. And we know that the electric force at right angles to a moving electric charge is greater the greater the velocity of the charge.

*The range of the α particle of polonium determined by this apparatus was found to be 3·80 cms. at a pressure of 77·34 cms. of mercury and a temperature of 12° C.

The range was obtained from the curve K, in which the leaks were plotted against distances of polonium film to the side of gauze nearest the electrode.

§ VI.

Summarizing the results recorded in this paper we have:—

1. The lack of saturation with weak ionization by α rays is not due to diffusion of ions, nor does it depend on the recombination coefficient or velocity of the ions.
2. Initial recombination is very small in gases ionized by β, γ, or X rays; in other words, these ionizing agents effect a complete separation of the negative ions from their parent molecules.

   This appears to be related to the fact that the secondary cathode rays from a metal plate on which β, γ, or X rays fall, have considerable penetrating power, while no such penetrating cathode rays have been shown to exist in the case of the α particle. And it seems reasonable to conclude, from this and other facts, that the velocities of the electrons ejected from molecules in the gaseous state ionized by β, γ, or X rays are of the same order, while the velocity of an electron ejected by an α particle is much smaller.
3. This property of the β rays is not destroyed by passing them through a thick aluminium plate.
4. The initial recombination observed when a gas is simultaneously ionized by α and γ rays is equal to their separate effects.
5. Initial recombination is greater the slower the velocity of the ionizing α particle, which probably means that the slower the velocity of the α particle the slower the velocity of the ejected electrons.

* Before the publication of this work a paper appeared in the *Phys. Zeit.*, May 1906, by Kučera and Mašek, who describe some experiments having the object of finding the range of the α particle of polonium and the ionization along its course. These experimenters make the range 4·1 cms. (measured from gauze) at a pressure of 73·3 cms. of mercury. Allowing for the greater atmospheric pressure in my experiment, my range is slightly lower than theirs.

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In conclusion I wish to express my thanks to Professor J. J. Thomson for his kindly interest and advice during the research.

Cavendish Laboratory, June 30, 1906.

Note.—The fact that lack of saturation in a gas depends on the velocity of the ionizing α particle explains the deviation in certain ratios occurring in the paper on the recombination of ions by Prof. Bragg and the writer. (See Phil. Mag. April 1906, p. 475.) Prof. Bragg was the first who observed that lack of saturation was smaller when the ionization-chamber was brought nearer to the radium; but he seems not to have been quite sure that it was a true effect, since he did not mention it in the above or in any of the subsequent papers.

But in a letter to the writer just come to hand he mentions that he has established that there is a distinct change in the lack of saturation with velocity of the α particle from radium.

XXXVII. *The Use of the Secohmmeter for the Measurement of Combined Resistances and Capacities.* By S. R. Milner, *D.Sc., Lecturer in Physics, University of Sheffield* *.

PROFESSORS Ayrton and Perry's secohmmeter is an instrument which consists essentially of two rotary commutators mounted on the same axle, by the rotation of which rapid reversals of the battery and galvanometer terminals of a Wheatstone's-bridge system may be produced. It was originally designed for comparison of inductances, but it may be used equally well for the measurement of any single electrical quantity (*e. g.* capacity or electrolytic resistance) which is best measured by an alternating-current balance on a Wheatstone's bridge. So long as it is a question of measuring single resistances or capacities only, the secohmmeter and galvanometer are indeed much superior to the induction-coil and telephone of the older method. The chief advantage is of course the much greater sensitiveness than the telephone which the galvanometer has as a balance indicator; but in addition to this, an important point in practice is the convenience arising from the fact that the galvanometer deflexions themselves indicate the direction in which the adjustable arm requires to be varied.

Several classes of electrochemical measurements have hitherto had to be carried out by the induction-coil and telephone method, since in them the use of the secohmmeter

* Communicated by the Author.

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fails to give an easily interpretable result. Cases in point are the determinations of (1) the resistance of an electrolyte between small electrodes which become appreciably polarized in a smaller time than that elapsing between successive reversals of the battery; (2) the surface capacity of electrodes in a solution the resistance of which is not negligible; (3) the capacity of a condenser which has an appreciable leakage, such as is used in determining the S.I.C. of poorly insulating materials. Here we are dealing with what is equivalent to a resistance and a capacity combined in such a way that they cannot be taken apart and measured separately. (1) and (2) may be looked on as a combination in series, (3) in parallel, of a resistance and a capacity with each other. For the accurate determination of the members of such a combination by the Wheatstone's bridge, each must be individually balanced by that of a similar combination in one of the other arms. The balancing thus requires the adjustment of two variables instead of one, and this is a very difficult operation in practice unless the two adjustments are independent of each other. Nernst has shown that with the telephone the two adjustments can be made independently by reason of the fact that the loudness of the sound becomes a minimum when one member of the combination is balanced independently of the value of the other. With the secohmmeter, however, the two adjustments are not independent of each other in general, although they become so under certain conditions which are elucidated in the present paper. In some recent measurements of the kind included in classes (1) and (2), I found a telephone was wholly unavailable by reason of its want of sensitiveness. An investigation in some detail of the theory and practice of the use of the secohmmeter for the purpose, showed that methods for adjusting to the double balance, as simple and effective as those of Nernst with the telephone, although based on different principles, were available with the secohmmeter. As the instrument will probably be used more and more in the future in view of its superiority in sensitiveness to the telephone, it is hoped that a description of these may be of value.

In fig. 1, S and K are an unknown resistance and capacity supposed inseparably connected in series and forming one arm of a Wheatstone's-bridge system A B C D. P, Q, R, F, and f are the remaining resistances of the bridge, and H is a known capacity in series with R. X and Y are the commutators of the secohmmeter, shown separately, but really mounted on the same axle. Each reverses the current passing through it four times during a complete revolution of the

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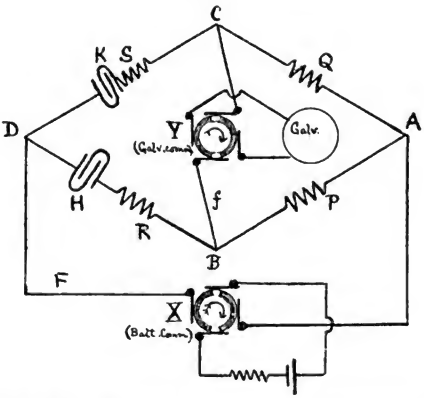
axle. In general the reversals of the battery commutator produce an alternating current in the arms of the bridge, which gives rise to a direct current through the galvanometer in consequence of the second set of reversals. The general

Fig. 1.



theory of the bridge shows that the alternating current in the arm B C, and with it the direct current through the galvanometer, will vanish when P, Q, R, and H are adjusted so that the double equality is satisfied,

$$\frac{P}{Q} = \frac{R}{S} = \frac{K}{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus if any two of these four variables, say Q and H, be fixed arbitrarily, an adjustment of both P and R is required to effect a complete balance and determine S and K. P and R are taken throughout in this paper as the adjustable variables.

Incomplete Balances.—The use of the secohmmeter presents at the outset an effect which does not exist in the telephone methods in the fact that, besides the complete balance, an infinite number of balances may be obtained which are characterized by zero galvanometer deflexion. In fact, generally, R being given *any* value, P may always be adjusted so as to bring the galvanometer-needle to zero. The following pairs of values of R and P taken with the battery and galvanometer reversed simultaneously are examples of this effect.

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TABLE I.

S, 7650 ω. K, ·4314 mf. Q, 4000 ω. H, $\frac{1}{3}$ mf. $\frac{K}{H} = 1{\cdot}291$.

R arbitrary ..... ω	5000		15000	
R/S	·6356		1·961	
Reversals per sec.	69·2	130	78	131·5
P required for balance ... ω	4260	3585	6490	7170
P/Q	1·065	·896	1·622	1·792
P calc. (see p. 308)..... ω	4317	3621	6503	7120
Diff. $P_{exp.} - P_{calc.}$ ..... ω	−57	−36	−13	+50

These results become understandable if we represent by time-curves the effect of the commutators on the E.M.F. of the battery and the current through the galvanometer.

In the usual form of the instrument the two commutators do not reverse instantaneously; one, X (in fig. 1 the battery commutator), breaks the circuit for a certain fraction of the whole period between two successive reversals ("reversal period"); the other, Y, short-circuits for the same fraction of the reversal period. (In Nalder's instrument the fraction is almost exactly $\frac{1}{6}$ in each case.) The four brushes of X which make contact in pairs on the two insulated brass pieces attached to the rotating axle are fixed on a plate which can be turned by hand around the axle. This is very convenient, as it enables the position of the brushes to be adjusted (even while the axle is in rotation), so that the reversals of the battery occur either simultaneously with, or at any assigned fraction of the reversal period before, those of the galvanometer. We shall call this fraction the lead, δ, of the battery over the galvanometer commutator.

In fig. 2, Curve I. represents the E.M.F. of the battery after commutation plotted with the time. *ac* is the reversal period, *ab* the "contact," and *bc* ($= \frac{1}{6}ac$) the "break" period. During *ab* the E.M.F. has a finite value E, while during *bc* we may represent it as zero. With non-inductive resistances only in the arms, the curve for the current in the galvanometer branch before commutation by Y will be similar in shape to that for the E.M.F., and may also be represented by I. The introduction of the capacities H and K into the arms will in general produce distortion and give exponential curves for the galvanometer current in both *ab* and *bc* (see II., fig. 2). To determine the effect on the current through the galvanometer of the reversals of the commutator Y at any lead, mark on the diagram the centre points of the short-circuiting periods of Y at a distance, proportional to the lead, to the

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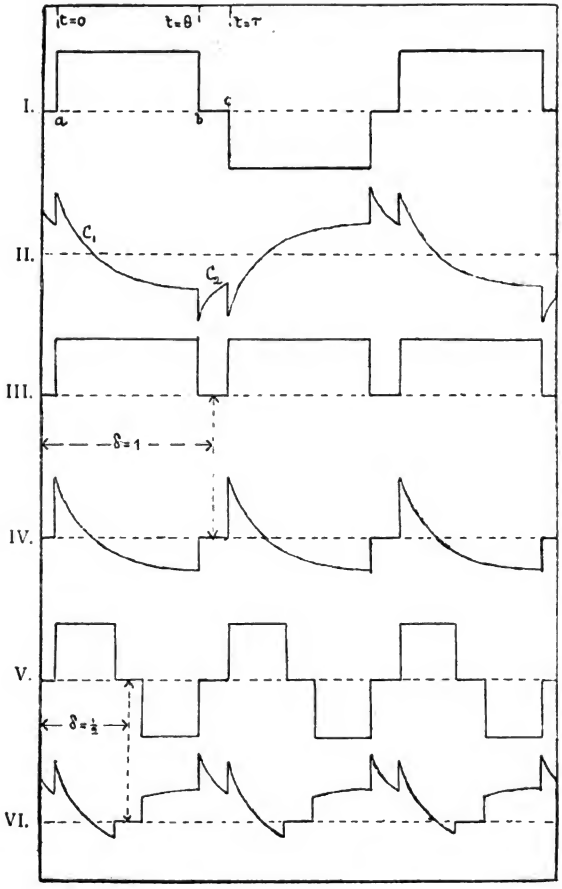
Fig. 2.



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right of the centres of the break periods of X. In every alternate section thus produced the ordinates of curves I. and II. become reversed, while during the short-circuiting periods themselves the galvanometer current is cut out altogether and the ordinates of the curves become zero. Curves III. and IV. show in the resistance and in the capacity case the resulting curve for the galvanometer current at lead $\delta=1$ (X and Y reversals simultaneous); V. and VI. show the same thing at a lead of $\frac{1}{2}$ (Y reversals midway between those of X). The mechanical effect on the galvanometer is proportional to the mean ordinate of the curve in each case. As the lead is continuously increased the mechanical effect will pass through a maximum value (approximately when the whole area of the current curve, thus reversed and cut out, is above the axis), diminish to zero (equal areas above and below), and increase again to a maximum in the opposite direction (whole area below). We see from curves III. and V. that when resistances only are in the bridge-arms, the mechanical effect has its maximum values at leads $\delta=0$ and $\delta=1$, and is zero at $\delta=\frac{1}{2}$*. When capacities are present, however, the leads at which the mean galvanometer current is a maximum and zero respectively will no longer have these fixed values. Nevertheless in every instance they will have definite values for given values of the resistances, the capacities, and the speed, or more generally for a definite relation between these variables. Conversely, if the lead be given any value and the existing galvanometer deflexion be reduced to zero by a suitable adjustment of the variables, the result of the adjustment is to bring about the corresponding relation.

These considerations give a qualitative explanation of the results detailed on p. 300, and, generally, of incomplete balances obtained at any lead. The methods of adjusting to the complete balance, which are described below, are based on the fact that, by making suitable arrangements in the bridge, the relation resulting from an incomplete balance, which is very complicated in the general case, degenerates for suitable

* This gives a simple practical means of marking out accurately a scale of δ. With non-inductive resistances only in the arms (the effect of the inductance of the galvanometer should be made negligible by inserting a high resistance in series with it or with the battery) a suitable deflexion of the spot is obtained. Then, if while the secohmmeter axle is kept in rotation the plate to which the brushes of the commutator X are attached is turned slowly round by hand, the positions of the plate which correspond to the zero and maximum deflexions are easily ascertained. These positions $\delta=0$, $\frac{1}{2}$, and 1 may be marked once for all by scratching a line on the plate and three corresponding ones on the brasswork of the instrument. I have used a uniform paper-scale pasted on to the brasswork to read the intermediate values of δ.

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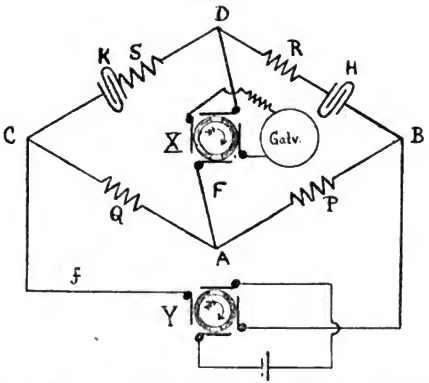
values of the lead into each one of the three equalities included in (1), and that there exist in each case simple and accurate means of adjusting the commutator to the corresponding positions. This is shown by the following analytical investigation of the current through the galvanometer, and the values of the lead which correspond to each one of the three equalities (1).

In a Wheatstone's-bridge system the combinations may be arranged either as in fig. 1, or as in fig. 3, which is obtainable

Fig. 3.



from fig. 1 by simple interchanges of the battery and galvanometer without altering the lettering. In both arrangements when the system comprises only the resistances P, Q, R, S, F and f, and the capacities H and K, situated as in the figures, the differential equation of the current C through the galvanometer branch due to an E.M.F. E in the battery branch is given by the expression

$$C=\frac{(PS'-R'Q)E}{Ff(P+Q+R'+S')+F(P+Q)(R'+S')+f(P+R')(Q+S')+PQ(R'+S')+R'S'(P+Q)}, \quad . \quad (2)$$

where R′ and S′ denote differential operators such that

$$R'=R+\frac{1}{HD}, \quad S'=S+\frac{1}{KD}, \quad D=\frac{d}{dt}. \quad . \quad . \quad (3)$$

The relations corresponding to incomplete balances which

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result from (2) in its general form become complicated by the presence in them of F and f, the resistances of the battery and galvanometer arms; and it is only by assuming certain simplifications in (2) that results which are practically useful are obtained. The most useful simplification, and that on which the methods described below are based, is that in which F is supposed to be arbitrarily made very large and f very small, compared with the combinations, and in the following these conditions are assumed to be complied with*. Equation (2) then becomes

$$C=\frac{PS'-QR'}{(P+Q)(R'+S')}\frac{E}{F},$$

or if we write

$$p=\frac{P}{P+Q},\qquad r'=\frac{R'}{R'+S'},$$

we get the very simple form

$$C=(p-r')\frac{E}{F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If we put

$$r=\frac{R}{R+S},\quad k=\frac{K}{K+H},\quad z=\frac{1}{R+S}\left(\frac{1}{H}+\frac{1}{K}\right),$$

we get by (3)

$$r'=\frac{rD+kz}{D+z},$$

and (4) becomes on multiplying up to eliminate reciprocals of D,

$$(D+z)C=\left\{(p-r)D+(p-k)z\right\}\frac{E}{F}. \quad . \quad . \quad (5)$$

When the variations of E/F are those given by the battery commutator of the secohmmeter, and represented in curve I (fig. 2), this equation gives the "unreversed" current C through the galvanometer branch (*i.e.* previous to the second set of reversals). In any period of time during which E/F

* Practical considerations on the necessary magnitudes of F and f are given later, p. 313. In order to avoid clashing with these conditions it is theoretically necessary to situate the "breaking" commutator X in the high-resistance arm F, and the short-circuiting commutator Y in the low-resistance arm f (*i.e.* in the arrangement of fig. 1, X, in that of fig. 3, Y, should be the battery commutator, as shown in the figures). In practice the rather more convenient use of X as the battery commutator in all cases makes, however, no appreciable difference to the results.

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is constant, the integration of (5) gives

$$C=(p-k)\frac{E}{F}+A\,e^{-zt},$$

where A is an arbitrary constant. This shows that the expression for the current is different during the contact period *ab* (fig. 2) of the battery commutator, where E/F has a finite constant value, and during *bc* where E/F is zero. Let t be the time reckoned from the instant a at which the battery commutator just begins to make contact, τ the reversal period *ac*, θ the contact period *ab*. We may then put

$$\left.\begin{aligned} C=C_1&=(p-k)\frac{E}{F}+A\,e^{-zt} \text{ between } t=0 \text{ and } t=\theta \\ C=C_2&=B\,e^{-z(t-\theta)} \qquad \text{,,} \qquad t=\theta \text{ and } t=\tau \end{aligned}\right\}, \quad (6)$$

where B is another arbitrary constant.

The constants A and B may be determined on the assumption that the state is steady. Integrate (5) between two times infinitely close together which include any sudden finite variation in E/F, and we see at once, since the only finite terms are those which contain D as a factor, that

$$\left\{\begin{matrix}\text{Sudden increase}\\ \text{in current}\end{matrix}\right\} \quad (p-r)\times\left\{\begin{matrix}\text{Sudden increase}\\ \text{in E/F}\end{matrix}\right\}.$$

Applying this formula to the sudden variations of E/F which occur at the instants $t=\theta$ and $t=0$, we get

$$[C_2-C_1]_{t=\theta}=-(p-r)\frac{E}{F}$$

and

$$[C_1-C_2']_{t=0}=\ \ (p-r)\frac{E}{F},$$

where C_2' is the final value of the current during the break period just preceding the instant $t=0$. But from the consideration that the state is steady we must have

$$[C_2']_{t=0}=-[C_2]_{t=\tau}.$$

These equations determine A and B. Substituting the resulting values in (6), and putting $\theta=\frac{5}{6}\tau$ (p. 300) we get

$$\left.\begin{aligned} C_1&=\frac{E}{F}\left\{p-k-(r-k)\frac{1+e^{-\frac{1}{6}z\tau}}{1+e^{-z\tau}}\,e^{-zt}\right\} \\ C_2&=\frac{E}{F}(r-k)\frac{e^{\frac{5}{6}z\tau}-1}{1+e^{-z\tau}}\,e^{-zt} \end{aligned}\right\}, \quad (7)$$

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for the unreversed currents through the galvanometer branch during the contact and break periods of the battery respectively. When $p=r=k$, which is in effect equation (1), C_1 and C_2 are zero. When $r=k$ only, the exponential terms vanish, and the current becomes

$$C_1=(p-r)\frac{E}{F}, \quad C_2=0.$$

This is the same as it would be if the capacities were absent and there were the resistances only in circuit; a result which might have been inferred without calculation, from the fact that when $r=k$ the operator D cancels out from both sides of the differential equation (5) for the current.

Curve I. (fig. 4) shows the calculated values of C_1 and C_2 with the following values of the variables:—

E, 2·05 volt; F, ·52 Ω; P, 5000; Q, 4000; R, 5000; S, 7650 ω; H, $\frac{1}{3}$ mf.: K, ·4314 mf.; τ, $\frac{1}{82}$ sec.

The ×'s are experimental points taken as follows. By releasing two opposite brushes of the short-circuiting commutator Y from contact with the brass pieces, an instantaneous contact becomes made between the two remaining brushes twice in each revolution of the axle, that is once in each complete alternation of the battery current when the battery is being commutated by X. The phase of the alternation at which this contact is made may be given any value by suitably adjusting the lead of X. The instantaneous P.D. between any two points of the circuit due to the battery current may therefore be measured direct by connecting through this contact a potentiometer on to the points. The method is rather interesting as it forms a very simple way of illustrating the distortion produced in the current by any arrangement of condensers, &c. for the simple type of alternating E.M.F. given by the secohmmeter. The contact is, however, only approximately instantaneous, it really extends over $\frac{1}{12}$ of the complete period; the experimental results consequently do not show the sudden changes in the value of the current, but they agree well with the calculated mean values during the interval of contact.

Effect of Galvanometer Reversals.—We see from the curves of fig. 2 that, if δ is the lead of the battery commutator X, the effect of commutator Y is to reverse the ordinates of the curves for C after the instant $t=\delta\tau$, and by short-circuiting to make them zero from the instant $t=(\delta-\frac{1}{6})\tau$ up to $t=\delta\tau$.

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The mean continuous current through the galvanometer is consequently

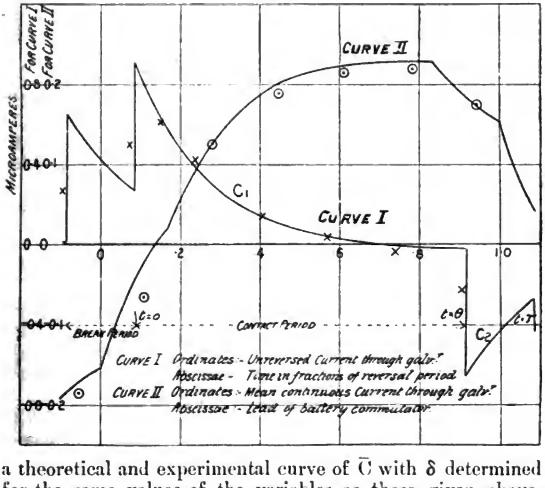
$$\bar{C} = \frac{1}{\tau}\left[\int_0^{(\delta-\frac{1}{2})\tau} C\,dt - \int_{\delta\tau}^{\tau} C\,dt\right]. \quad . \quad . \quad . \quad (8)$$

If δ lies between $\frac{1}{6}$ and $\frac{5}{6}$, we get, on carrying out the integrations by means of (7), and simplifying a little by writing hyperbolic in place of exponential functions,

$$\frac{F\bar{C}}{E} = 2\left\{(p-k)(\delta-\tfrac{1}{2}) - \frac{r-k}{z\tau}\left(1 - \frac{\cosh^2 \frac{1}{2}z\tau}{\cosh \frac{1}{2}z\tau}\, e^{-z\tau(\delta-\frac{1}{2})}\right)\right\} \quad (9)$$

$F\bar{C}/E$ is thus a function of the four variables $p-k$, $r-k$, $z\tau$, and δ. The particular expression (9) for the function will be modified somewhat when $\delta < \frac{1}{6}$ or $> \frac{5}{6}$, in consequence of the different expressions for C during the contact and the break periods of the battery commutator. The corresponding expressions for $\bar{C}$ are easily obtained, but since the general results deduced from them are the same, it is not necessary to write them down. Curve II. (fig. 4) represents

Fig. 4.



a theoretical and experimental curve of $\bar{C}$ with δ determined for the same values of the variables as those given above. The relation between the variables which corresponds to an

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incomplete balance at any lead δ is obtained by putting $\bar{C}$ zero in equation (8).

Incomplete Balance at zero lead.—This is the commutator position at which the instrument is ordinarily used. The relation between the variables corresponding to the balance is

$$\frac{p-k}{r-k} = \frac{\cosh \frac{1}{2} z\tau}{\cosh \frac{1}{2} z\tau} \cdot \frac{\sinh \frac{5}{2} z\tau}{\frac{5}{2} z\tau} = \phi \text{ say} \quad . \quad . \quad (10)$$

ϕ is a function of $z\tau$ which varies from 1 to 0 as $z\tau$ increases from 0 to ∞. Its course is shown in the curves I. and I *a*. (fig. 5), which represent ϕ plotted with $z\tau$ and with $1/z\tau$ for the large and small values of $z\tau$ respectively. The $\odot$'s give a number of experimental values of $\frac{p-k}{r-k}$ at corresponding values of $z\tau$, which were obtained by balancings similar to those the results of which are given in Table I., p. 300.

(10) may be written

$$p = \phi r + (1-\phi)k,$$

and since ϕ is always less than 1, we see that a balance at zero lead always results in a value of p which is a weighted mean between those of r and k; but, since the weighting factor is unknown, no practical conclusion can be drawn from it. The result is, however, important, as it shows that, in the ordinary way in which it is often used to measure electrolytic resistances without attempting to balance the capacities of the electrodes, the secohmmeter is very markedly inferior to the telephone in the accuracy of its results. The telephone gives the equality $p=r$ independently of the value of k.

Balance corresponding to $r=k$.—When $r=k$ we have from (12) as the condition that $\bar{C}=0$, the relation

$$\delta = \tfrac{1}{2}.$$

This result is a simple consequence of the fact referred to above that, when $r=k$, the curve for the unreversed galvanometer current C is the same as if the bridge comprised non-inductive resistances only. After reversal, the mean galvanometer current is thus reduced to zero at the same lead, $\delta=\frac{1}{2}$, as in the case of pure resistances.

The converse proposition is of course also true. If δ be made $\frac{1}{2}$, and the resistance R adjusted until the galvanometer deflexion is reduced to zero, the result of the adjustment will be to make

$$r=k, \quad \text{or its equivalent } \frac{R}{S} = \frac{K}{H} \cdot \quad . \quad . \quad (11)$$

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This proposition gives a simple means of determining the equality in practice. The commutator position $\delta=\frac{1}{2}$ may be easily and accurately obtained (see footnote, p. 302) once for all, or we may obtain it and adjust to the equality (11) by proceeding as follows. Shortcircuit R and H, then $r=k=0$, and the galvanometer deflexion will be zero only when $\delta=\frac{1}{2}$. Turn the commutator while the axle is still in rotation until the deflexion becomes zero, fix it in this position, give H its original value, and balance the resulting galvanometer deflexion by adjustment of R.

Balances corresponding to $p=k$, and $p=r$.—When $p=k$, and $C=0$ in (9) we get for the corresponding value δ_2 of δ the equation

$$e^{z\tau(\delta_2-\frac{1}{2})}=\frac{\cosh^2\frac{1}{2}z\tau}{\cosh\frac{1}{2}z\tau}, \quad . \quad . \quad . \quad . \quad (12)$$

and when $p=r$, $C=0$, and $\delta=\delta_3$,

$$e^{z\tau(\delta_3-\frac{1}{2})}\{1-z\tau(\delta_3-\tfrac{1}{2})\}=\frac{\cosh^2\frac{1}{2}z\tau}{\cosh\frac{1}{2}z\tau}, \quad . \quad . \quad (13)$$

with similar equations in the region where $\delta<\frac{1}{6}$ or $>\frac{5}{6}$. The leads δ_2 and δ_3 are each functions of $z\tau$ only. δ_2 varies from $\frac{1}{2}$ to 0, and δ_3 from 1 to $\frac{1}{2}$, as $z\tau$ increases from 0 to ∞. The course of these functions is shown in the curves of fig. 5 (II. & III. give δ_2 and δ_3 with $z\tau$, from $z\tau=0$ up to $z\tau=4$, II *a*. & III *a*. give δ_2 and δ_3 with $1/z\tau$, from $1/z\tau=0$ up to $1/z\tau=0{\cdot}4$, to include the region where $z\tau$ is too great to admit of direct plotting). The $\times$'s are experimental values obtained by simply arbitrarily making the corresponding equality and turning the commutator until the galvanometer deflexion becomes zero.

Two practical methods for adjusting to the equalities may be deduced from these results. They depend on the fact that although

$$z=\frac{1}{R+S}\left(\frac{1}{H}+\frac{1}{K}\right), \quad r=\frac{R}{R+S}, \quad \text{and } k=\frac{K}{K+H},$$

are all functions of the same variables, the ratios r and k may be altered as much as we please without affecting z, so long as $R+S$, and $\left(\frac{1}{H}+\frac{1}{K}\right)$, *i. e.* the total resistance and capacity of the branch BDC, remain unaltered. *E. g.*, if H be transferred from the arm BD to a position in series with SK in the arm CD (in fig. 1 this may be easily done by transferring the battery-lead making contact at D to the junction between R and H) we make $k=0$ without

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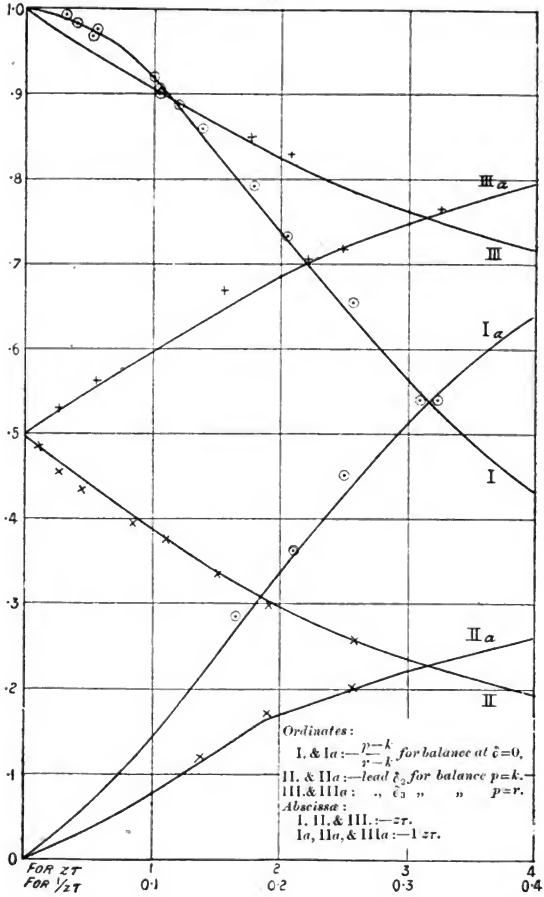
Fig. 5.



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affecting z, or, if the speed of the secohmmeter remain constant, without affecting $z\tau$. An operation of this kind is subsequently referred to for brevity as one "at constant $z\tau$," and it has the property that it in no way affects the values of the commutator leads at which zero galvanometer deflexion requires the satisfaction of the partial equalities. The $p=r$ balance only is referred to below, but the description applies equally to the $p=k$ balance by simple interchange of symbols.

(1) To obtain the equality $p=r$, make $r=0$ at constant $z\tau$ by transferring the whole resistance R from the arm BD to CD (*e. g.* in fig. 3 transfer the galvanometer contact D to the point of junction between R and H). Make also resistance P zero, and consequently $p=\frac{P}{P+Q}=0$. Then $p=r=0$, and the galvanometer deflexion will be zero only when the commutator lead has the particular value δ_3 which corresponds to the $z\tau$ at which the operation is carried out. While the secohmmeter axle is still in rotation turn the commutator X until the galvanometer deflexion is zero, and fix it in this position. Then when R is transferred back to BD, a deflexion of the galvanometer results which should now be balanced by adjustment of P. The resulting value of P will satisfy the equality $p=r$, or its equivalent $P/Q=R/S$.

(2) At the lead δ_3 which corresponds to the equality $p=r$, the galvanometer deflexions must be entirely independent of k, so long as the variations of k are made at constant $z\tau$. In every other position of the commutator a motion of the needle will result from varying k, and this will indeed be in opposite directions according as the lead is greater or less than δ_3. If we arrange that the capacity H may be transferred by a tapping key from the arm BD to CD (or in fig. 1 the battery contact altered to correspond), so that by tapping the key k will be made zero at constant $z\tau$, the commutator position at which tapping this key gives no motion to the galvanometer-needle is easily ascertained. The balance $p=r$ is obtained as before by the adjustment of P in this position.

I find that the results of these incomplete balances in practice may be obtained correct to under $\frac{1}{2}$ per cent. Theoretically, the speed should be kept constant during the adjustment of P, since δ_3 varies with the speed, but the actual effect of variations of speed on δ_3 and on the balance is very small, and no difficulty arises in practice when the secohmmeter is kept in rotation by a motor. Method (2) has the slight advantage in this respect, that the correctness of the balance can be tested after P has been adjusted by

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tapping the key which makes k zero, but method (1) is somewhat simpler to carry out.

Practical Considerations.—When any one of the three partial equalities of (1) has been obtained by a balance at the corresponding lead, the attainment of the complete equality becomes of course only a matter of the adjustment of the remaining variable to balance at any other lead. In practice, much the best method in general is to adjust to $r=k$ at $\delta=\frac{1}{2}$, as described on p. 308, and then, turning the commutator until $\delta=0$ (where the galvanometer is most sensitive to variation in p) balance the resulting deflexion by the adjustment of P or Q. The advantage of the $r=k$ balance is that the corresponding commutator position is both a fixed one independent of the speed, and easy to ascertain without the necessity for introducing any extra tapping key into the bridge system. It has the disadvantage, however, that it contains both the unknowns S and K, and consequently no information can be drawn from it until the complete balance has been ascertained. The $p=k$ and $p=r$ balances are not subject to this disadvantage, and for this reason they may occasionally be usefully employed to make a direct measurement of one member of a combination when a knowledge of the other is not required. I have found them useful in saving time in making a rapid preliminary survey of electrochemical combinations which were varying in value in the course of an experiment.

The only objectionable feature of these methods is the restriction theoretically necessary on the magnitudes of the resistances F and f of the battery and galvanometer branches (*i.e.*, F must be large and f small). Both conditions tend to reduce the sensitiveness of the arrangement, but the disadvantage in this respect is more apparent than real, and the lost sensitiveness may be easily regained. Thus in the arrangement of fig. 1, where the condition (F large) requires in practice the insertion of a high resistance in series with the battery, the E.M.F. of the battery may be increased in proportion to F without any possibility of overstepping the limit of permissible current through any part of the system. This is not the case in the arrangement of fig. 3; on the other hand, in this case the use of an electrometer in place of a galvanometer will completely satisfy the theoretical requirements.

As regards the practical obtaining of a complete balance it is not, however, necessary to attempt to satisfy accurately the theoretical conditions. The neglect of the second condition (f negligible) makes so very little difference to the

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results that in practice its satisfaction may be safely omitted. Nonsatisfaction of the first (F large) is more serious, and results in the incomplete balance failing to correspond with the partial equality. The complete balance may, however, always be arrived at by successive approximations Thus a series of successive adjustments, first of R at $\delta=\frac{1}{2}$, then of P at $\delta=0$, then of R at $\delta=\frac{1}{2}$ again, will ultimately result in the complete balance, however widely the theoretical conditions are departed from. The rapidity of the measurement may, however, be very much increased by paying attention to the theory. When F is very small, the rate of approximation of the adjustments is initially rather slow, but it becomes much more rapid as the complete balance is more nearly reached. Now in carrying out such a series it is in any case desirable to diminish the sensitiveness of the galvanometer in the earlier stages. If we do this in such a way as to at the same time partially satisfy the theoretical conditions (*i. e.* by inserting a high resistance in series with the battery in the arrangement of fig. 1, or with the galvanometer in that of fig. 3) the approximation becomes extremely rapid throughout. This is shown by the results of experiments on successive balances given below.

TABLE II.

S, 7650 ω. K, ·4314 mf. Q, 4000 ω. H, $\frac{1}{3}$ mf.
Arrangement of fig. 3. E.M.F. 2 volts.

Arbitrary initial value of resistance P.	Resistance, in series with galvanometer	Commutator lead δ.	Variable adjusted.	Balance value.	
(1) 3000.....	·5 Ω	$\frac{1}{2}$	R	9900	
	,,	0	P	5200	
	,,	$\frac{1}{2}$	R	9900	
	none	0	P	5176	
	,,	$\frac{1}{2}$	R	9908	*
	,,	0	P	5176	
(2) 3000.....	·1 Ω	$\frac{1}{2}$	R	9900	
	,,	0	P	5090	
	,,	$\frac{1}{2}$	R	9900	
	none	0	P	5176	
	,,	$\frac{1}{2}$	R	9908	*
	,,	0	P	5177	
(3) 8000.....	·1 Ω	$\frac{1}{2}$	R	10100	
	,,	0	P	5260	
	,,	$\frac{1}{2}$	R	9900	
	none	0	P	5176	
	,,	$\frac{1}{2}$	R	9908	*
	,,	0	P	5177	

* = complete balance.

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With $F = \cdot 5\ \Omega$, single adjustments of R and P practically gave the complete balance, for two figures of the resistances were as much as the galvanometer would indicate with the arrangement so unsensitive. With $\cdot 1\ \Omega$ two sets of adjustments were required, and with still smaller resistances F the approximation became laborious. But it is very simple to proceed as in the first experiment, and adjust to the first two figures of R and P with a high resistance in circuit, when the balance may be completed to the fourth figure by a single further set of adjustments with the resistance shortcircuited and the arrangement thoroughly sensitive.

General Conditions for Balance at $\delta = \frac{1}{2}$.—This commutator position is unique in the ease with which it may be adjusted to in practice, and the fact that the incomplete balance here may be obtained independent of the speed. It thus forms much the most convenient position in practice for an incomplete balance. The relation corresponding to the balance is moreover easy to investigate in the general case. Zero galvanometer deflexion at $\delta = \frac{1}{2}$ independent of the speed must imply that the curve for the unreversed current through the galvanometer branch is an undistorted copy of that for the E.M.F. The relation is consequently simply the condition that the differential coefficient D shall vanish by cancelling from the general relation (2) in which the operators R′ and S′ of (3) have been substituted. This may be determined by simple algebra. The condition is twofold: (*a*) F must be large compared with the rest of the bridge variables, and (*b*) the relation

$$\left(\frac{R}{S} - \frac{K}{H}\right) + \frac{Q}{S} \cdot \frac{f}{f + P + Q}\left(\frac{P}{Q} - \frac{K}{H}\right) = 0 \quad . \quad . \quad (14)$$

must be satisfied. This result shows the necessity for the restriction in the methods above on the magnitude of F, and also the relation which results when f is not negligible. When either f *or* P and Q are negligible (14) degenerates into $R/S = K/H$, hence the condition that f should be small in the methods may be replaced by this alternative.

Parallel Combinations:—An investigation exactly similar to the above reveals the fact that similar methods of balancing are equally applicable when R & H, and S & K, are in parallel instead of in series with each other. The operators R′ and S′ to be introduced into (2) are now such that

$$\frac{1}{R'} = \frac{1}{R} + HD, \quad \frac{1}{S'} = \frac{1}{S} + KD. \quad . \quad . \quad (15)$$

The conditions that D shall vanish from equation (2)

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become in this case as follows :—(*a*) either f, or P and Q, must be negligible compared with the combinations, and (*b*)

$$\left(\frac{R}{S}-\frac{K}{H}\right)+\frac{QR}{F(P+Q)+PQ\left(1+\frac{f}{f+P+Q}\right)}\left(\frac{P}{Q}-\frac{K}{H}\right)=0. \quad (16)$$

These are the relations corresponding to balance at $\delta=\frac{1}{2}$ independent of the speed, and we see as before that they can only have practical utility if we arbitrarily give some of the bridge variables negligible magnitudes with respect to the others. Two practical cases exist :—

(1) Make F large, and either f, or P and Q (or both), small, compared with the combinations; and the result of the balance is

$$\frac{R}{S}=\frac{K}{H}.$$

(2) Make P and Q, and also F, small compared with the combinations, and the result of the balance is

$$\frac{P}{Q}=\frac{K}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In (1) the necessary conditions and the resulting balance are exactly the same as in the series case, and the method of practical balancing described above is thus applicable to both classes of combinations. Moreover, under these conditions the form of the differential equation for the current is exactly the same as that of equation (5), and the methods of pp. 309 & 311 for determining the other partial equalities are also equally available.

In electrochemical work parallel combinations usually occur in the determination of the S.I.C. of leaking dielectrics; they then invariably are of very high impedance, and the conditions (1) would be difficult to satisfy in practice (unless an electrometer is used in place of the galvanometer in fig. 3). In such cases a better simplification of the bridge system is got by satisfying (2). The balance at $\delta=\frac{1}{2}$ by adjustment of P will then give the equality (17), and the complete balance is attained by subsequent adjustment of R at any other commutator position. This makes a very simple method in such cases, since the conditions (2) will usually be quite sufficiently satisfied in practice when P, Q, and F have any ordinary values, in consequence of the high impedances of the combinations concerned.

Method independent of Resistances of Battery and Galvanometer Arms.—It is possible to devise methods by which

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the incomplete balance $R/S = K/H$ may be adjusted to, even when no restrictions are placed on the resistances F and f. Unless F is moderately large, however (say $10000\,\omega$ in the case experimented on above), the corresponding lead is liable to vary a good deal with the speed, and this may in some cases be a disadvantage sufficient to counterbalance the advantage of the generality of the method. The following method, which I have tested experimentally and found to work well, is applicable to both series and parallel combinations. Arrange (*e. g.* by means of a Morse tapping-key T not shown in the figures) that the end B (figs. 1 & 3) of the combination R H may be transferred to the point C, thus making the arm BD infinite, and CD to consist of SK and RH in parallel with each other. Give the adjustable resistances P and Q any values small compared with that of R, and, holding down the key T, turn the commutator X until the galvanometer deflexion becomes zero. Then, releasing the key T, thus reproducing the original Wheatstone's bridge arrangement, balance the resulting galvanometer deflexion by the adjustment of R. The balance thus obtained should be tested by again tapping the key T, and the process if necessary repeated (this will not be necessary unless F is very small). When no deflexion exists whether the key is tapped or not, the balance will correspond accurately to the equality

$$\frac{R}{S} = \frac{K}{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This may be proved as follows:—When the key T is tapped, and the combinations RH and SK put in parallel with each other, the unreversed current through the galvanometer branch is easily seen, when P and Q are of negligible magnitude, to be given by the expression

$$C' = \frac{Q}{f} \cdot \frac{E}{F + \frac{R'S'}{R'+S'}},$$

where R' and S' denote the operators (3) or (15) corresponding to the combinations. Also when P and Q are small in the Wheatstone-bridge equation (2), it reduces to

$$C = \frac{PS' - QR'}{f(R'+S')} \cdot \frac{E}{F + \frac{R'S'}{R'+S'}},$$

which is the expression for the unreversed current through the galvanometer branch when the key T is not down.

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When (18) holds this simplifies still further to the form

$$C=\frac{PS-QR}{f(R+S)}\cdot\frac{E}{F+\frac{R'S'}{R'+S'}},$$

in the first factor of which pure resistances only occur. Hence

$$C=\frac{PS-QR}{Q(R+S)}C',$$

which shows that the curves for the two currents are exactly of the same shape. The commutator positions at which the galvanometer deflexions are reduced to zero in the two cases will be consequently identical.

XXXVIII. *The Disruptive Voltage of Thin Liquid Films between Iridio-Platinum Electrodes.* Part I. *Voltages* 25–400. *By* P. E. SHAW, *B.A., D.Sc.**

ELECTRIC-TOUCH measurements can readily be made under oil. If a telephone be included in the circuit, the sounds made in contact are sharp and clear for all the liquids mentioned below. They even excel air in this respect. This property of oils to render contact distinct has been used in potentiometers and other instruments. The whole instrument, or at least the slide-wire, is immersed in paraffin oil (see a paper by R. A. Lehfeldt and discussion following, in Phys. Soc. Proc. p. 479, 1901–1903). This previous use of oils suggested the present research.

The relation of spark-length to small P.D. of the electrodes has been experimentally tested for gases by Earhart (Phil. Mag. [6] i. p. 147, 1901), Shaw (Proc. Roy. Soc. vol. lxxiii. p. 337, 1903), Kinsley (Phil. Mag. May 1905, p. 692), and Hobbs (Phil. Mag. [6] x. Dec. 1905).

Earhart discovered the knee occurring in the curve at 350 volts. Shaw pushed the investigation further by using small P.D. down to ½ volt. Kinsley threw doubt on the constancy of potential for a given gas and given electrodes. Hobbs, working more exhaustively than his predecessors, has added much definite and useful information.

The discharge in gases must bear a close relationship to the discharge in liquids, but little precise work has been done on the latter, though Hughes (Proc. Inst. Elec. Eng. 1892), Swinton (Discussion on the above paper), Steinmetz (Trans. Amer. Inst. Elec. Eng. 1893), before 1903, and recently, C. E. Skinner (Nat. Elec. Light Assoc., Boston, Mass., 1904),

* Communicated by the Physical Society: read June 22, 1906.

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Przibram (*Akad. Wiss. Wien, Sitz.-Ber.* Nov. 1904), and Voege (*Elektrotechn. Zeitschr.* Dec. 1904) have tested the relationship for spark-lengths greater than $\frac{1}{20}$ mm.

In the following paper the P.D. varies from 25 volts to 400 volts, and the corresponding spark-lengths vary from $\frac{1}{10}$ micron (micron = $\frac{1}{1000}$ mm.) to about 10 microns.

Recently a book on the subject has been published giving useful information ('The Insulation of Electric Machines,' by H. W. Turner and H. M. Hobart, Whitaker & Co., Paternoster Sq., E.C.). It is made clear in this book that water, acids, and alkalies are very destructive of the insulating powers of oils; thus Skinner has shown that $\frac{1}{30}$ per cent. of water reduces the dielectric strength of transformer oil to half its value when quite dry.

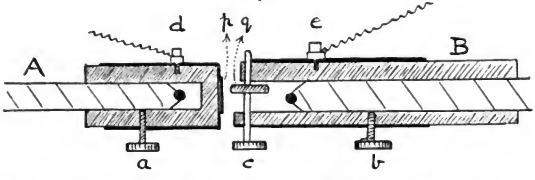
In the present work all the commercial liquids used were shaken up and allowed to remain over potassium carbonate, filtered, and then heated to 110° for several hours, then kept in a glass-stoppered bottle till used.

Method.

In a research by the present writer on the sparking-distance in air (*loc. cit.*), the electric micrometer with levers was used to measure spark-length; in the present case the electric micrometer, without levers, is used. It has been already described (Proc. Roy. Soc. April 1906) for measuring gauges; it has proved very convenient for the present experiments.

The only parts of the apparatus which need be here mentioned can be understood from fig. 1. Two spindles, A and B, are in one line, and can be worked to or from one another

Fig. 1.



by micrometer-screws not shown, and their distance apart can be read and controlled to $\frac{1}{100}\,\mu$ (*i.e.*, $\frac{1}{100,000}$ mm.). The divisions on the micrometer-head show movements of $1\,\mu$, and these, when viewed by a microscope, can be each subdivided into 100 parts.

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On A and B are fitted two caps of ebonite clamped on by screws *a*, *b*. Mounted on each ebonite is a sheath of steel. On the left cap is a plate of iridio-platinum *p*, and on the right is a disk of iridio-platinum *q*, set on a spindle turning on an axis in the plane of the paper and perpendicular to B. This disk projects, as shown, so that when B advances to A, the surfaces *q* and *p* are brought into contact. Since *q* is a diametral section of a sphere, its edge touches the plate *p* at a point. Discharge is made to occur between *p* and *q*. By causing *p* to rotate on a horizontal axis along A and *q* to rotate on a horizontal axis perpendicular to B, fresh surfaces of *p* and *q* can be presented to one another continually, since *q* touches *p* not at the centre.

The P.D. on surfaces *p*, *q* is maintained by connecting wires to terminals *d e*. The full circuit is shown in fig. 2. Here a 400-volt lighting circuit is joined through key K_1 to a water resistance. Between the fixed zinc electrodes Z_1, Z_3 is the full P.D., while between the moveable Z_2 and the fixed Z_1 is the P.D., say V, used in discharging the apparatus. The leads to the electrodes Z_2 and Z_3 are insulated from the liquids by being enclosed in glass tubing.

If switches K_2 and K_3 are put over to the right, the above voltage V will act through the voltmeter V_0 or on the discharge surfaces *p*, *q* according as the keys K_5 and K_6 are put over to the left or the right.

It is easy to vary the P.D. used by moving Z_2.

Another circuit is on the left and is brought into action by putting switches K_2 and K_3 over to the left, whilst K_5 and K_6 are kept to the right. The single cell C has resistance r_1, say 100 ohms, and r_2, say 4000 ohms, in circuit. The telephone T is in the circuit and sounds when *p* and *q* touch. This circuit is used to determine the position of contact of *p* and *q*, the P.D. between them being very small, in this case $\frac{1}{40}$ volt. The actual discharge for high potential experiments, say 100 volts, from the right circuit, takes place when *p* and *q* are separated by a small gap; but the discharge-distance for the small voltage $\frac{1}{40}$ volt of the left circuit is so minute that it can be ignored. The above apparatus can be used for discharge through gases, liquids, or solids.

The *modus operandi* for liquids is as follows:—The caps on spindles A B are removed from the spindles, the iridio-platinum surfaces ground with finest emery to remove the old surface, polished with rouge or leather, and cleaned. The caps are placed on A and B and *p*, *q* made to approach. One drop of the liquid to be tested is placed on *q*, and as *p* approaches *q* the liquid settles by capillarity between *p* and *q*.

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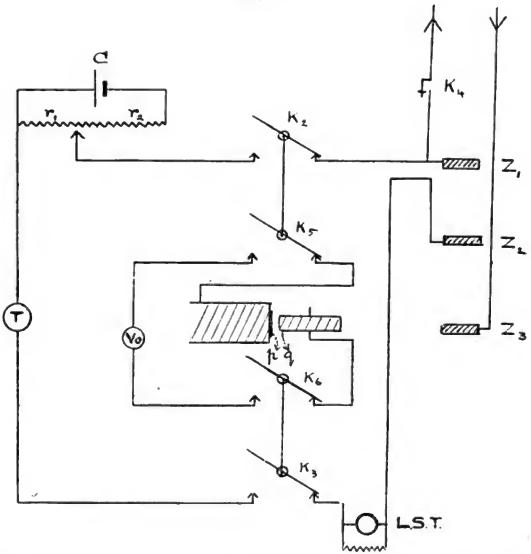
Then the spindle B is moved slightly away, the liquid being thus drawn out into a thread connecting the surfaces, and there is a guarantee that liquid without air-bubble connects the surfaces. The switches K_2 and K_3 are put to the left,

Fig. 2.



and K_5 and K_6 put over to the right. If now p is advanced and touches q, the telephone T sounds; the reading of the divided head of the right micrometer-wheel is taken. This is the first reading. The surface q is drawn away from p about twice as far as the spark-length about to occur. Switches K_2 and K_3 are put to the right, and K_5 and K_6 put to the left. The position of Z_2 is arranged so that the voltmeter shows the desired P.D. Now K_5 and K_6 are put over to the right, and q is made *very slowly* to approach p, the micrometer-scale being watched so that at the moment when discharge occurs and is indicated by the sounding of telephone L.S.T., the scale is read a second time. The discharge-distance is the difference between the first and second readings.

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These distances are entered in the tables following in units which are 0·01 μ (μ = micron = $\frac{1}{1000}$ mm).

The surfaces of p, q presented to one another are renewed, as shown above, by rotations and another discharge is made.

Most of the liquids used are of commercial quality, and are purified as shown above; others were obtained specially pure. The following liquids were tested:—

Vegetable Oils.—Olive oil, castor oil, linseed oil, rape oil. Also turpentine, fusel oil, and oil of resin.

Animal Oils.—Cod-liver oil, neats-foot oil.

Mineral Oils.—Paraffin, transformer oil.

A Homologous Series.—Pentane (C_5H_{12}); hexane (C_6H_{14}); heptane (C_7H_{16}); octane (C_8H_{18}). These were obtained from C. A. F. Kahlbaum, Berlin. Hexane and octane were specially pure, being derived from propyl iodide and octyl iodide respectively; while heptane and pentane were obtained from petroleum.

Varnishes.—Of these armacell and ohmaline are obtained from Griffiths Bros., Bermondsey, London; while Sterling varnish is made by The Sterling Varnish Co., Manchester. Though these are made to insulate when dry, they all have great dielectric strength when liquid, as shown in the curves (fig. 5).

TABLE I.

l = spark-length stated in units of 0·01μ.

P.D. in volts.	l.						
	Olive Oil.	Linseed Oil.	Castor Oil.	Rape Oil	Turpentine.	Fusel Oil.	Air.
375	560	550	570	...	...	550	1300
350	510	475	520	610	520	...	1100
300	445	350	400	430	420	...	160.250 480.320
275	...	...	...	...	...	...	120
250	400	300	350	400	340	300	118
200	...	220	...	300	220	255	100
186	340	...	318	...	165		
150	290	190	275	220	120	205	80
125	235	150	210	...	100	...	48
100	200	95	170	115	90	150	46
75	145	...	150	115	80	110	36
70	140						
68	115						
66	93						
62	80	...	110				
50	60	56	58	45	55	80	14
25	27	27	35	...	27	35	8
10	11						

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TABLE II.

P.D. in volts.	*l.*						
	Paraffin Oil.	Transformer Oil.	Cod-liver Oil.	Pentane.	Hexane.	Heptane.	Octane.
390	...	...	450				
345	breaks	breaks	395	400	260	320	350
300	down	down.	350	250	...	260	310
250	readily.	. .	290	220	220	275	
200	...	350	...	170	220	235	240
175	350						
150	300	210	175	145	170	140	180
125	220	170	150				
100	180	110	120	90	150	100	105
75	155	80					
62	125						
56	95						
50	75	70	50	50	80	50	
25	38						

As all experimenters on gaseous discharge have found, the discharge-distance is an uncertain measurement. The same order of error occurs for liquids. In the case of gases, dust or want of polish increases the distance of discharge for a given P.D. In liquids, solid or flocculent substances or minute air-bubbles may be floating, or the presence of water, acid, alkali, or salts will set up electrolysis and change the discharge-distance.

In the curves below (figs. 3, 4, 5) some liquids may be represented roughly by straight lines; other liquids certainly require irregular curves for proper representation. They may be classed as follows:—

Straight Curves.	*Irregular Curves.*
Linseed Oil.	Paraffin Oil.
Rape ,,	Transformer ,,
Fusel ,,	Castor ,,
Cod-liver ,,	Olive ,,
Pentane.	Turpentine.
Octane.	Hexane.
Armacell Varnish.	Heptane.
Ohmaline ,,	
Sterling ,,	

Though this classification is convenient, there is no sharp demarcation between the two classes above. Only those liquids are placed in the second class for which it would be absurd to suppose the departure from a straight line could be due to experimental errors.

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In the cases of castor oil, olive oil, and paraffin there appears to be a distinct and similar variation in potential gradient for discharge-distances between 1 μ and 2 μ (see fig. 4).

The air curve is given for reference.

Hobbs (*loc. cit.*) has shown that potential gradient near the origin depends on the metal of which the electrodes are made, every metal having a different gradient from every other.

The electrodes used throughout these researches are iridio-platinum. The gradient for this alloy is steeper (220 volts per micron) than any found by Hobbs, though he found platinum to have a high gradient (160 volts per micron).

Fig. 3.

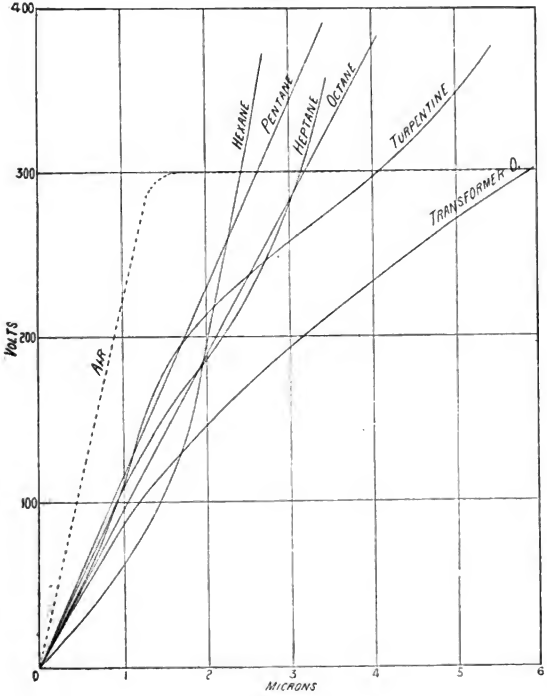

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Fig. 4.

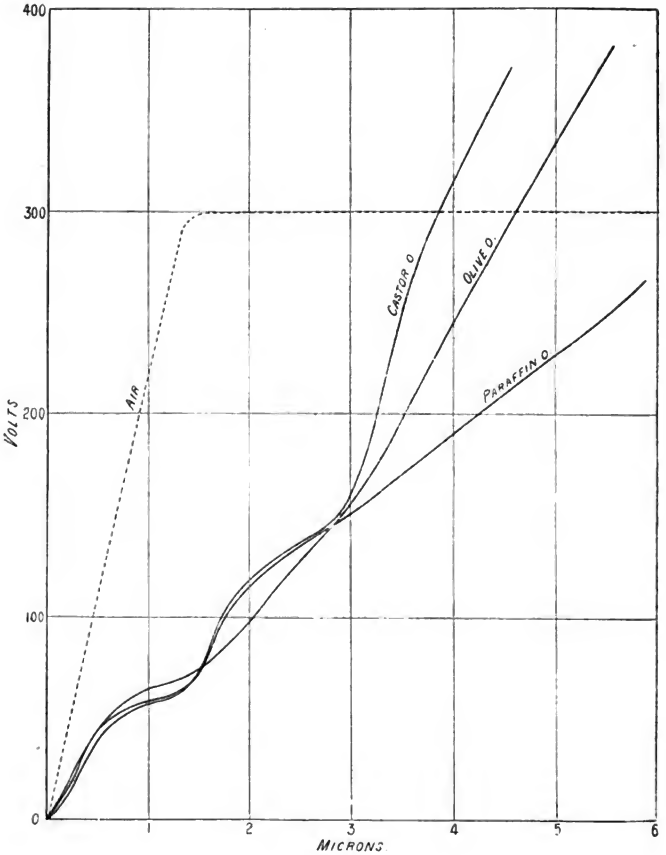

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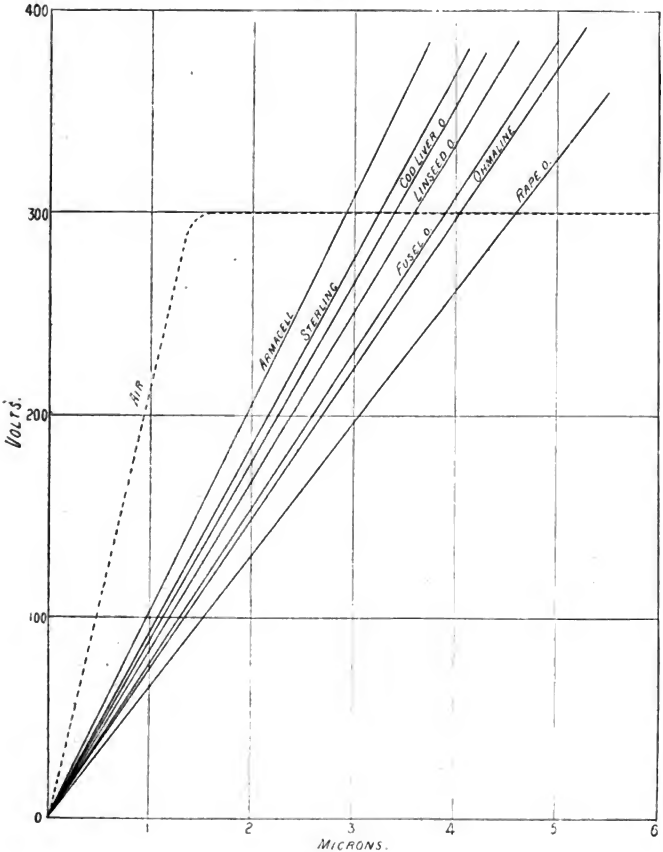
Fig. 5.



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The "knee" occurs at 300 volts, the horizontal part extending to 8μ, *i. e.* well to the right of the plotting shown. After 8μ the gradient is very small. Air has greater strength than any liquid for small distances; but the liquid curve in all cases crosses the air curve for discharge lengths varying between $2\frac{1}{2}\mu$ and 6μ. That is, for any greater distances the liquids have much greater strength than air.

The order of strength for the higher voltages used is roughly:—

Hexane, heptane, armacell, pentane, octane, Sterling varnish, cod-liver oil, linseed oil, fusel oil, ohmaline, castor oil, turpentine, olive oil, rape oil. Paraffin and transformer oil are both weak. Resin oil gave very irregular results, not showing, as might be expected, great strength.

Notes and Precautions.

1. The voltmeter was carefully standardized.

2. The loud-speaking telephone in the discharge circuit was shunted with non-inductive resistance of a few ohms. Some experimenters in previous researches have kept the discharge surfaces shunted with a voltmeter, so that the discharge is shown by the fall of the voltmeter-pointer. This seems faulty; for when discharge commences, it will be at once violently retarded by an inductive rush from the voltmeter. In the present work the shunted telephone of low resistance indicates discharge, the voltmeter being then cut out and no inductive resistance left in circuit.

3. The members of the homologous series are very volatile; the method employed is to cause drops of them to fall slowly on the discharge surfaces, a fresh drop arriving before the last has evaporated.

4. The discharge is of the nature of a volcanic eruption, the cathode loses material and a distinct pit or crater forms there, while fine dust is scattered on both electrodes. The surfaces approach, but do not always meet on discharge. To avoid dust the surfaces are wiped after each discharge.

5. The discharge surfaces should be treated with care whilst the zero is being obtained. "Make" and "break" should be made as little as possible, and the zero should not be much overrun. The highly polished surfaces, even of the very hard iridio-platinum, are easily damaged for discharge work.

6. If a contact is "wheezy," it is rejected at once; this indicates bad polish, and always produces electrolysis or premature discharge.

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7. The contact sound is invariably clearer and sharper for liquids than for air, and discharge is more definite. Air discharge often occurs partially, *i. e.* there is a rush of material from the cathode and the contacts approach one another but do not touch. Discharge does not continue; in fact, thi partial discharge seems to retain the tension and for a time stop the complete and final breakdown.

Theory.

Very short spark-lengths in gases have now been studied by Earhart, Shaw, Kinsley, and Hobbs. The accumulated results of these and the present experiments on short spark-lengths in liquids should lead inductively to a theory of the conditions of matter at and between the discharge surfaces. Recently T. Schwedoff (*Ann. d. Physik*, pp. 918–934, April 1906) has propounded a ballistic theory of the spark-discharge. The fundamental principle he uses is that the discharge between the surfaces is due to the shattering of the molecules of the gas into free ions; this occurs when the electric strength between the electrodes jointly with the kinetic energy of the ions already existing free, reaches sufficient magnitude. He uses the ballistic law of resistance, in which the frictional force opposing the passage of an ion depends on its speed and on the density of the gas. An expression is finally derived

$$v = \frac{300 h_0 . 10^{-4} . l}{1 + \frac{c}{\lambda}\left(1 - e^{\frac{l\delta}{c}}\right)},$$

where v = potential difference on the surfaces;
l = spark-length;
δ = density of the gas;
h_0, c, λ are constants.

The values of these constants are derived from consideration of the experimental results of Earhart and Shaw. Putting in the constants and the density of the gas, the P.D. required for any given spark-length is derived.

Schwedoff thus obtains a curve which fairly closely follows that of Earhart, having a knee in the curve at voltage about 350.

At first sight the similarity between Earhart's experimental curve and Schwedoff's theoretical curve appears to be a striking confirmation of the theory; but careful comparison shows that the two curves are far from coincident near the

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"knee," one set at this important point being only 50 per cent. of the other set. Again, since Earhart's values are used to give the constants in the formula, there is naturally a certain degree of coincidence. But this is not all: the most recent results by Hobbs (*loc. cit.*) are opposed to Schwedoff's theory in each of the following particulars:—

(1) The critical potential gradients for different gases are identical below the knee.

(2) The critical potential gradients vary very greatly according to the metal used in the electrodes.

(3) Although Earhart did not take note of the fact, his results indicate a short horizontal portion to the curve, the potential gradient there being nil. Hobbs has thoroughly established the horizontal portion for every gas and for every kind of electrode.

As Hobbs points out, his results in conjunction with the previous work of Peace (Proc. Roy. Soc. p. 99, 1892), Strutt (Phil. Trans. p. 377, 1900), Carr (Proc. Roy. Soc. p. 374, 1903), which showed in every case minimum spark-potential in a gas (unless the voltage is very small), seem to indicate that at atmospheric pressure the carriers of the discharge are the ions of the gas for distances greater than 6μ; while they are the ions of the metals of the electrodes for distances less than $2\frac{1}{2}\mu$; for distances between $2\frac{1}{2}\mu$ and 6μ the P.D. for discharge remains constant, and discharge occurs not at the extreme points of the opposing surfaces, but between points at the distance corresponding to the minimum sparking-potential, *i. e.* about 350 volts in air.

J. J. Thomson had previously suggested ('Conduction of Electricity through Gases,' p. 386, Camb. Univ. Press) that the metal corpuscles might play a part in these effects. A corpuscle about to leave the electrode feels an electrostatic attraction towards the electrode; if the electric intensity between the electrodes exceeds this, the corpuscle escapes. This critical field works out, from the principle of electric images, to be 8×10^3, which is the order of field in Earhart's experiments. Thus Thomson accounts for the fact that the first part of the curve is straight. This theory, however, may need some modification or amplification in view of the fact, mentioned above, discovered by Hobbs that each metal has its own gradient. Hobbs' curves indicate that great gradient goes with great atomic weight, *i. e.* the heavy metals lose their corpuscles with greatest reluctance.

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Results.

(1) The insulating liquids do not differ very much from one another in power of resisting discharge, the strength lying between the limits 110 volts per micron and 70 volts per micron, though some liquids seem to have slightly less strength. The range of voltage here spoken of is between 25 and 400.

(2) Some liquids have a peculiar variation in potential gradient for distances between 1μ and 2μ. Castor oil, olive oil, and paraffin are cases in point.

(3) All insulating liquids have greater strength than air for P.D. greater than 300 volts; for less P.D. liquids have much less strength than air.

(4) The results for a homologous series of hydrocarbons do not indicate any simple connexion between composition and power of resisting discharge; the differences found are as likely to be due to accidental impurities as to specific qualities of the liquids.

But these simple substances have specially great dielectric strength. It may be that simplicity of composition makes for strength.

(5) Whereas in the case of gases there is always a horizontal portion in the curve, in liquids there is no such distinct break in the curve between 25 and 400 volts, unless the region at about 60 volts in castor oil, olive oil, and paraffin is of this nature.

(6) No simple connexion can be observed between power of resisting discharge and specific inductive capacity, in liquids.

(7) The temperature throughout was from 15° C. to 18° C.

XXXIX. *The Fluorescence, Magnetic Rotation, and Temperature Emission Spectra of Iodine Vapour. By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University**.

[Plate IV.]

THE fluorescence of iodine vapour has been observed by Wiedemann and Schmidt, and also by Konen, but the luminosity appeared to be too feeble to allow a study, either visual or photographic, to be made of its spectrum. By studying the conditions under which the fluorescence manifests itself to the best advantage, it has been found possible to photograph its spectrum with a large three-prism spectrograph, with its slit narrowed to such a degree that the

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distance between the D lines of sodium was fully three times their width.

The magnetic-rotation spectrum which was discovered last year, which manifests itself when polarized white light is passed in succession through magnetized iodine vapour and a Nicol prism, set originally for extinction, has been photographed with a concave grating of twelve feet radius, so that a very perfect knowledge of its nature is now at our command. As is the case with the analogous magnetic-rotation spectrum of sodium vapour, many of its bright lines coincide in position with the dark lines of the absorption spectrum, though the one is by no means the complement of the other.

The remarkable emission of light which occurs when iodine is raised to a temperature of 700° or over has been carefully studied, and the condition found under which its spectrum is very distinctly discontinuous. Its spectrum is remarkably like the fluorescence spectrum, though the heads of the bands do not appear to coincide, due possibly to a difference in the densities in the two cases.

The three types of spectra enumerated above will now be considered in detail.

The Fluorescence Spectrum.

The fluorescence of iodine vapour manifests itself to the best advantage when the vapour is formed in a moderately high vacuum. A glass bulb from 5 to 10 cms. in diameter, and containing a few flakes of iodine, is exhausted on a mercurial pump and sealed. The fluorescence at once appears when a beam of sun- or arc-light is focussed at the centre of the bulb by means of a large condensing-lens, no heating of the bulb being necessary. The cone of fluorescent light is yellowish green and of considerable intensity, and can be shown to a large audience in a darkened room. If the bulb contains air at atmospheric pressure, there is absolutely no trace whatever of fluorescence. Not until the pressure has been reduced to about 15 cms. does the luminous cone appear, and it is then so feeble that only close inspection reveals its presence. The intensity gradually increases as the pressure is further reduced, the most rapid change occurring between 1 cm. and the high vacuum. The presence of air also destroys the magneto-rotatory power of the vapour, though to a less degree at fairly low pressures; in other words, the fluorescence seems to be more sensitive than the magnetic rotation. I was at first inclined to agree with Professor Larmor that the effects might be due to collisions, the phenomena requiring

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time for their development. The heating of a gas by absorption of radiant energy must in some way result from a transformation of the energy stored in the molecule during absorption (which we may regard as potential) into kinetic energy at the moment of collision. That is, the velocity of the rebound is increased by actions or motions going on within the molecule. If we consider that the molecule is capable of storing up energy to a certain point after which further absorption results in a re-emission of radiant energy (fluorescence), and if we further assume that at the moment of collision the stored energy is liberated, or spent in increasing the molecular velocity, it is clear, if the time-interval between collisions is sufficiently small, that the internal energy of the molecule cannot be increased up to the point at which fluorescence manifests itself. At atmospheric pressure, however, the average time-interval between two collisions is sufficient for the passage of about 130,000 light-waves. It may appear at first sight improbable that fluorescence would be excited at all if the passage of such an enormous number of waves failed to awaken any response. The capacity of a molecule for storing energy may be, and probably is, very great; and it is by no means unlikely that a molecule can absorb the energy of many thousand light-waves before being brought to the unstable condition at which a re-emission of radiant energy commences. If molecular impacts are responsible for the suppression of the fluorescence, it would appear probable that a light gas, such as hydrogen, would be less detrimental. Bulbs were accordingly filled with hydrogen, and the fluorescence of the iodine vapour examined as the pressure was gradually reduced. Faint traces of fluorescence appeared at a pressure of 30 cms., much higher than was the case with air; but at lower pressures there did not seem to be very much difference, the intensity of the fluorescence in air and hydrogen at 2 cms. pressure being about the same. It was found, moreover, that if a bulb containing just enough air to prevent the fluorescence were warmed, the fluorescence at once appeared. This circumstance may perhaps be taken as evidence against the theory of molecular collisions, and we may adopt as an alternative what may be termed a solution theory. I have been of the opinion for many years that we may speak with perfect correctness of a solution of a gas in a gas. Experiments made in 1896 on the absorption spectrum of iodine and bromine mixed with the vapours of bisulphide of carbon, showed that a portion of the iodine was in a state of solution, giving an absorption spectrum devoid of lines or bands, while a portion

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existed in the state of a gas, giving the fine-line absorption spectrum. With a given density of the vapour, a ceratin amount of iodine could be vaporized without showing its gas absorption spectrum: if a little more were added, however, the fine lines at once appeared. This experiment appears to me to be analogous to the one in which the fluorescence of iodine in air at moderately low pressure can be brought about by the vaporization of an excess of iodine. In other words, we may say that at room temperature, air at 15 cms. pressure will dissolve all of the iodine which will vaporize at this temperature, but if the temperature is raised the excess of iodine remains undissolved, and it is this portion only which fluoresces. Further evidence confirming this theory has been recently found by one of my students, Mr. T. S. Elston, who has been working on the fluorescence of anthracene. The vapour of this substance emits a brilliant blue-violet fluorescent light when it is formed in a vacuum, or in hydrogen, nitrogen, or carbonic acid at atmospheric pressure, but when formed in oxygen, sulpur dioxide, or cyanogen at atmospheric pressure, there is absolutely no trace of fluorescence. Heated to a higher temperature in any one of these gases, it reacts chemically with them, and we may if we choose use the term incipient chemical action instead of solution. That the union is only of a temporary nature was shown by vaporizing the anthracene in the oxygen, cooling the bulb until the anthracene had crystallized on the walls, and then opening the bulb. The oxygen was found to be at atmospheric pressure, indicating that none of it had been consumed, and on filling the bulb with hydrogen and vaporizing the anthracene, the fluorescence at once appeared again with undiminished intensity. Probably interesting results will be obtained by measuring the vapour-pressure of mixtures of anthracene and oxygen. If equal volumes at the same pressure and temperature are mixed, it seems probable that a diminution in the pressure will result.

The fluorescence was bright enough to record its spectrum on an isochromatic plate with an exposure of seven hours. The spectrum is reproduced on Pl. IV. (upper figure). The dispersion of the three-prism spectrograph was insufficient to show the structure of the bands, which extend from the orange-red well down into the greenish-blue. I feel certain that I shall be able to photograph the spectrum with the 12-foot concave grating, although it will require a very long exposure.

The magnetic-rotation spectrum and the temperature-

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emission spectrum are shown in register with the fluorescence spectrum. The dark bands of the latter appear to be pushed further towards the red end than in the magnetic-rotation spectrum, while in the temperature spectrum they are still further displaced. The magnetic spectrum shows much more structure than either of the other two.

Some visual observations were made of the fluorescence of the vapour when stimulated with fairly homogeneous light furnished by the monochromatic illuminator. The colour of the fluorescent light changed in a marked manner when the wave-length of the exciting radiation was altered. It was bluish-green when stimulated with blue light, yellow-green with green stimulation, and orange with yellow stimulation. It was only with great difficulty that the spectrum could be observed under these conditions, and about all that can be said at present is that the fluorescence spectrum covers the exciting region and extends to a short distance on the red side of it.

The Magnetic-Rotation Spectrum.

A glass bulb, 3 cms. in diameter, containing a small flake of iodine, was exhausted, sealed, and mounted in a small section of brass tubing placed between the poles of a powerful magnet. Two small holes in the brass tube permitted the passage of light, along the lines of force, through the bulb. Parallel light from an arc was passed in succession through a Nicol prism, the bulb, and a second nicol. When the latter was set at extinction, the crater of the arc could be barely seen, and appeared of a faint grey colour. On exciting the magnet, an intense blaze of emerald-green light at once appeared, wholly different in colour from that of the fluorescent light.

The spectrum of this light was photographed without difficulty with the 12-foot concave grating, with an exposure of eight hours. The absorption spectrum and the iron arc are shown on the same Plate, together with a scale of wave-lengths. These spectra are reproduced on Plate IV. (lower figure), the upper absorption spectrum taken with dense, the lower with rare vapour. At certain points the magnetic spectrum appears to be the complement of the absorption spectrum, the bright lines of the former coinciding with the dark lines of the latter. This is especially noticeable at wave-length 5125. Below this point the magnetic spectrum shows more structure than the absorption, though both were taken with the same width of slit. The absorption-lines in this

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region were too fine and close together to be completely resolved, which makes it difficult to draw conclusions. In the region around wave-length 5200, the bright magnetic lines usually coincide with dark absorption-lines, though there appears to be no relation between their relative intensities. Above wave-length 5300 the spectra at first sight appear to be the same. This is only true, however, if we confine ourselves to the general appearance of the fluted bands, careful inspection showing that the bright lines coincide with the dark.

Theoretical considerations make it seem probable that all of the magnetic lines are close doubles, since the rotation of the plane of polarization has its maximum value on *each* side of every absorption-line. The double lines have only been observed, however, in the case of the D lines of sodium. Photographs of the magnetic effects seen with sodium vapour of varying density were reproduced in connexion with an earlier paper ("Magneto-Optics of Sodium Vapour"*). It is clear that two absorption-lines very close together may under certain conditions give rise to a magnetic triplet, the rotation midway between the absorption-lines having a value of 90°, with a similar condition just above one absorption-line, and just below the other. I have been unable to find any evidence of such an effect in the iodine spectrum, though it can be observed without difficulty at the D lines, the components of the triplet being very broad in this case. With very dense sodium vapour, as I have already shown, we may have a multiplex magnetic line, the rotations at the components being 90°, 270°, 360°, 450°, &c. In the case of iodine, and also in the case of sodium in the region of the channelled absorption spectrum, the magnetic lines appear single with all resolving powers used up to the present time. Possibly the employment of an echelon or some such instrument would reveal their duplicity. The line 5001 of the magnetic spectrum of sodium would be the best one to begin with, as it is isolated and fairly bright. If the nicols were set parallel, instead of crossed, it is possible that the appearance of the lines in the absorption spectrum would be altered. We should expect the lines to appear doubled, with two fainter and very close components between them. Probably, however, this system would not be resolved with a 12-foot grating, and we should notice only a slight broadening of the line. Bromine gives a fairly bright magnetic-rotation spectrum, which has been photographed with the three-prism spectrography, but not with the grating. It does not, however, exhibit the slightest trace of fluorescence under any conditions.

* Phil. Mag. [6] x. p. 408.

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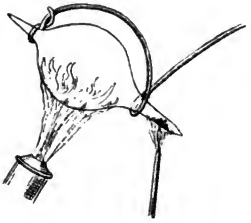
On the theory proposed, this indicates that the bromine molecule has a greater capacity for storing energy than the molecule of iodine: in other words, it takes care of the absorbed energy, without re-emission, during the interval between collisions.

The magnetic rotatory power appears to be less easily affected by molecular impacts.

The Temperature Emission Spectrum.

The temperature emission of iodine vapour can be most beautifully shown by means of a quartz bulb provided with a short tubular stem. A small flake of iodine is introduced into the bulb, which is then highly exhausted and sealed with an oxy-hydrogen flame. The bulb is supported with a stiff iron wire, and heated with a blast-lamp as shown in fig. 1. In

Fig. 1.



a dark room it will be observed that, even at the high temperature of the blast-lamp, the quartz bulb radiates little or no light, on account of its small absorption coefficient. If now a small flame is applied to the stem containing the iodine, the vapour enters the white-hot bulb, which instantly appears filled with a highly luminous red cloud, which flashes into view like a red flame. The spectrum of this light appeared almost continuous in the three-prism spectrograph. There was, however, a faint suggestion of bands in the orange region, which made it appear probable that the continuity of the spectrum might be due to too great depth of the radiating layer. As is well known, the theory of radiation shows that an absorbing vapour, when radiating a discontinuous spectrum, will emit a continuous spectrum, if the thickness of the radiating layer be sufficient. On heating the stem as well as the bulb and projecting its image on the slit of the spectro-

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graph, the bands appeared much more distinct. The layer of vapour in this case was only a millimetre or two in thickness. The bulb was now opened, and the stem drawn down to a very fine capillary, with a bore of perhaps 0·1 mm. It was then re-exhausted and sealed. The capillary was placed in the hottest part of the flame of the blast-lamp and the remainder of the bulb heated with a small Bunsen flame. The vapour in the capillary immediately became highly luminous and almost white in colour, owing doubtless to the higher temperature possible with this arrangement.

The spectrum of the capillary light was found to exhibit strong bands, and resembled closely the fluorescence spectrum. It was photographed with an exposure of 8 hours, the blast-lamp being operated with an improvised water-blower. The bands were not resolved into lines, and it is of great importance to study it under higher dispersion. It can doubtless be photographed with the large concave grating, since an exposure of several days can be given if necessary, the apparatus requiring no attention. The fluorescence will give more trouble, as a powerful beam of sun- or arc-light is necessary, either of which requires more or less constant attention.

The temperature-emission spectrum is reproduced on Pl. IV. between the fluorescence and absorption spectrum.

The bands will be seen to be shifted towards the red, with respect to the corresponding bands in the fluorescence spectrum.

XL. *Leakage Currents in the Moisture condensed on Glass Surfaces.* *By* Prof. Fred. T. Trouton, *F.R.S.*, *and* C. Searle *.

THE leakage currents over the surfaces of non-conductors exposed to ordinary atmospheric conditions behave often in a most perplexing way. The current is not only very different on different occasions, but in addition may be very different on the same occasion according to the direction it is applied. This latter effect was exhibited by one of us to the Physical Society during last Session, when it was shown that the current may be many times greater when sent in one direction over the surface of glass, than when sent

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in the other. It was found indeed possible to produce a resistance which had the value of about 2×10^5 ohms in one direction, but of 10^6 ohms for currents in the other direction.

The exact conditions necessary for bringing a conducting surface into this sensitive state of "unidirectional conductivity" are complicated. The effect was originally noticed where the current was passed over a glass surface between two platinum electrodes melted on to the glass. The effects in this case appear to be very similar to those observed where tinfoil electrodes cemented on to glass are used which are described below.

That the change in resistance observed on reversal cannot be explained by the usual back-E.M.F. of chemical decomposition, is proved by the effect appearing with differences of potential ranging up to as much as 3000 volts.

The effect was observed to be very sensitive to changes in the hygrometric state of the atmosphere, as well as to be dependent on the length of time for which the current was allowed to run before reversal.

The general character of the phenomenon is as follows. On a difference of potential being continuously applied across a surface, say of glass between two metallic electrodes, the current is observed to gradually diminish with time. In addition, if the current is reversed at any moment during the interval, it is found to be different from the direct current—sometimes being as much as six or seven times greater than the direct current. The exact ratio of the currents for any given hygrometric state depends on the length of time the current has been allowed to run before reversal.

With the view of elucidating the requirements necessary for the appearance of these sensitive conditions of the surface, it was thought advisable to investigate the following three points:—(1) The rate at which the direct current diminishes with time; (2) the law connecting the value of the *initial* current with the hygrometric state of the surrounding air; and (3) if in any given condition of sensitiveness Ohm's law holds for currents sent in either direction over the surface.

The glass surfaces used in these experiments were prepared in the following manner. A sheet of tinfoil is fixed over a piece of ordinary window-glass about 30 cms. square by means of shellac. Parallel cuts are now made on the surface at a distance of about 2 mms. and alternate strips are removed. The final condition of this plate is shown in the sketch

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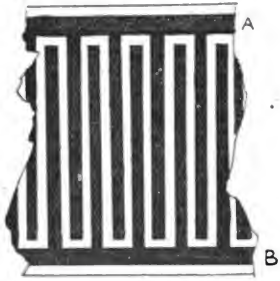
(fig. 1), which represents a portion of such a plate; the tinfoil surface is shown in black. Such a plate will be called a "grid."

Fig. 1.



Let the grid be connected up in series with a galvanometer and suitable E.M.F., say 100 volts. The current will assume a definite value and will then fall, more or less rapidly at first but at a gradually decreasing rate, until a fairly steady but relatively small value is attained.

This apparently steady value is in reality not constant. The current undoubtedly slowly falls, though at this stage the time-current curve is almost parallel to the time axis.

If, however, during the period of transition the current through the grid be reversed, it will assume a new value greater than before, and will begin more or less slowly to further increase in amount, the duration of the first stage and the rapidity of rising depending on the E.M.F., the previous history of the grid, and on the hygrometric state of the surrounding air. The initial value on reversal also depends on these conditions.

In attempting to explain this phenomenon two suggestions presented themselves, and are as follows:—

I. The fact that glass is hygroscopic leads us to suppose that what current does flow is conducted across by the film of moisture on the surface, possibly containing salts dissolved out of the glass. Taking a section of the grid perpendicular to the strips, the initial state before a current flows will be when the moisture lies evenly dispersed over the surface. If now the current be switched on, it is possible that a species of electric endosmosis takes place whereby water is heaped up at, say, the

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positive terminal while it shrinks away from the other. The heaping up of the water would cause an increase in resistance owing to the departure from uniform cross-section of the conducting film.

If this current be now reversed, water falls on one side and rises at the other. This would cause a diminution in resistance in the first instance, but subsequently an increase. Thus on reversal the current would, as observed, commence by rising, while finally falling in amount.

II. An electrolytic action may occur in which sufficient oxygen is released to produce a sensible covering of oxide, at least in the case of tin.

In this way a very considerable increase in the resistance may be brought about. On reversal of the current the hydrogen evolved would reduce the oxide, and thus diminish the resistance, and consequently the current would increase in amount. A difficulty in this second hypothesis is the rapidity under some circumstances with which the reduction must be effected; for with even a dead-beat galvanometer the current is often several times the direct value on reversal.

Rate of Change of Current with Time.

The time-rate of change of the current was experimentally determined and compared with that deduced from the second hypothesis.

Assuming the change of resistance as due to the formation of electrolytic products, we may reasonably take

$$dR/dt = a \,.\, dX/dt,$$

where X = amount of material formed deleterious to the passage of the current; but

$$dX/dt = b \times C, \qquad C = E/R;$$

so

$$dR/dt = m \,.\, E/R,$$

or

$$R^2 - R_0^2 = m \,.\, E \,.\, t;$$

that is

$$E\left(\frac{1}{C^2} - \frac{1}{C_0^2}\right) = mt.$$

This curve has been calculated and plotted. From Table I. and fig. 2 it will be seen that the curve fits the experimentally determined points with a fair accuracy. This result obviously points strongly to the second hypothesis.

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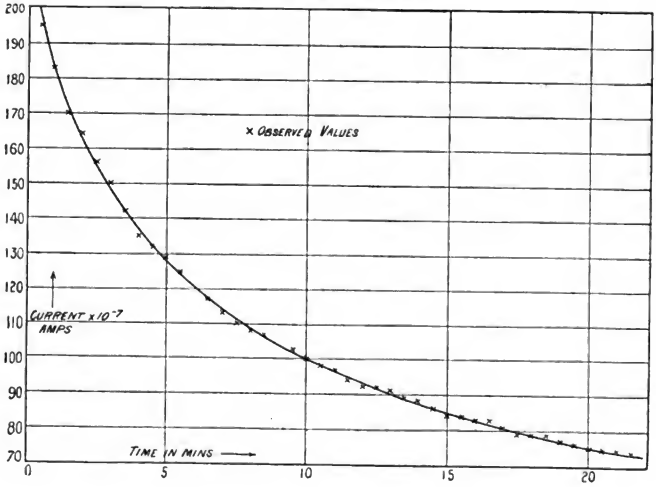
TABLE I.

Table showing Fall of Current with Time.

Time.	Deflexion.		Time.	Deflexion.		Time.	Deflexion.		Time.	Deflexion.	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
mins.			mins.			mins.			mins.		
0	210	210									
·5	195		6·0	121	120·3	11·5	94	93·8	17·0	81	80·6
1·0	183	181·4	6·5	117		12·0	92		17·5	79	
1·5	170		7·0	113	114·2	12·5	92		18·0	79	78·6
2·0	164	162·1	7·5	110		13·0	91	90·1	18·5	79	
2·5	156		8·0	108	108·7	13·5	89		19·0	77	76·8
3·0	150	149·1	8·5	107		14·0	88	87·4	19·5	76	
3·5	142		9·0	104	104·1	14·5	86		20·0	75	75·21
4·0	135	136·7	9·5	103		15·0	84	85·0	20·5	75	
4·5	132		10·0	100	100	15·5	84		21·0	75	73·5
5·0	129	127·8	10·5	98		16·0	83	82·7	21·5	74	
5·5	125		11·0	97	96·4	16·5	83		22·0	73	72·1

Fig. 2.

Fall of Current with Time.



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Variation in Value of Initial Current dependent on Hygrometric State.

Experiments were made to determine how the initial value of the current varied with the hygrometric state of the atmosphere. For this purpose a tin box was constructed in which the hygrometric state could be varied by the introduction of certain quantities of water. In this the grid was placed, and the whole was made air-tight. At a fixed temperature the hygrometric state, H, inside the box assumes a definite value: the initial current for this value of H was determined. On introducing a small quantity of water the value of H changes, and after allowing a sufficient time to elapse it becomes steady, when the value of C, the current, is again determined.

To measure H inside the box the dew-point method was used. Soldered in the top of the box was a thin hollow cylinder of polished brass through which water at varying temperatures could be passed. The temperature at which dew was formed and the temperature of the air in the box determines the dew-point. The detection of moisture on the surface of the cylinder was facilitated by means of a feather mounted on a spindle which could be manipulated from outside the box. A window was introduced for viewing this.

The introduction of moisture was effected by means of a long glass tube running from the top to the bottom of the box. A small rubber cork closed the top of this, being removed rapidly when water was introduced, and as rapidly replaced. A definite amount of water was used which ran down the glass tube into a dish inside the box.

The experiment was carried out as follows:—

A definite quantity of water was inserted in the box overnight. The next morning the dew-point was carefully determined and the initial current measured. Water was again placed in the box, and so on.

Special difficulties presented themselves owing to the determination of the dew-point having to be made in a limited space practically devoid of air-currents, so that great care had to be taken. The mode of procedure adopted was to determine the dew-point roughly with ice-cooled water, then to take water at a temperature rather greater than this dew-point and pass it through the cylinder. If no dew appeared a further quantity of water $\frac{1}{10}$ of a degree cooler was taken, and the process repeated, and so on.

The galvanometer used was adjusted to read to 10^{-7} amp. and an E.M.F. of 110 volts was used throughout the determinations.

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To get small values of H, the hygrometric state, the box was first dried with phosphorus pentoxide until no current was detected with the galvanometer adjusted as above, and then the necessary water introduced.

The results are shown in Table II. and are plotted in fig. 3. Values below H=·6, given in Table III., are omitted on

TABLE II.

Table connecting Hygrometric State and Current.

Temp. of Dew-point.	Temp. in Box.	Hygro. state, H.	Deflexion.		Mean.
			Right.	Left.	
6·7	13·8	·61	218	220	219
6·0	12·6	·64	312	318	315
6·2	13·0	·63	297	299	298
6·6	12·3	·68	110 *	110 *	1260
7·9	13·0	·71	177	179	2966
9·1	14·0	·72	220	219	3666
8·6	12·75	·75	155	150	4914
8·9	12·9	·77	200	196	6466
10·1	13·75	·78	241	239	7760
9·5	12·6	·81	160	165	13090
10·0	14·0	·80	192	190	11191
9·6	15·3	·69	21	23	1745
13·3	18·8	·71	130	130	3090
13·7	16·6	·83	200	200	27502
13·2	15·0	·83	193	192	33302
13·3	16·0	·84	205	205	32318
10·7	13·2	·85	249	233	41001
12·7	15·2	·85	280	280	44140
13·3	15·7	·86	145	143	48402

* In this and the following readings the galvanometer was shunted: the last column gives the corrected reading.

TABLE III.

Table connecting Current with Hygrometric State for Lower Values of H.

Temp. of Dew-point.	Temp. in Box.	Hygro. state. H.	Deflexion.		Mean.
			Right.	Left.	
2·5	14·5	·44	7		8
3·3	15·4	·44	10	1	10
1·5	12·8	·46	34	38	36
3·1	13·0	·51	104	104	104
5·7	14·8	·55	147	148	148
7·3	14·4	·62	250	249	250
5·4	16·1	·49	69	71	70

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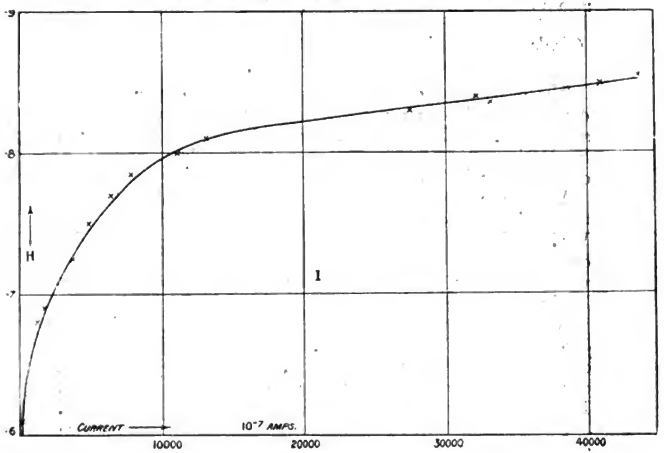
the curve as the scale is unsuited to exhibit them ; and values above H = ·86 are likewise omitted, as the trend of the curve there is obvious. Below H = ·6 the experimental points lie well on the continuation of the curve in fig. 3 when plotted on a larger scale.

Fig. 3.

Curve connecting Current and Hygrometric State.



No simple relation was found for this curve. It is possible, however, that a knowledge of the relation between the surface density of condensed moisture on glass and the hygrometric state (this is now under examination) may throw light on it.

Attempts to Optically verify Hypothesis I.

Attempts were made in two ways to detect a movement of moisture on the glass surface under the influence of the current.

The first method tried was to observe if any change occurred in the polarizing angle for light due to the film of moisture altering in thickness at the electrodes under the action of the current.

For this purpose the light from a sodium flame was reflected at the polarizing angle from the glass of the grid. This was viewed through a crossed nicol. If any motion of the water

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took place so as to bare the glass the angle of polarization should alter. No such effect was observed.

The second method depended on the relative alteration in path of two interfering beams of light. If the rays are arranged to pass through the grid one near, say, the positive electrode and the other near the negative, then on putting the current on a change in path should be observed. In this case also no effect was found to occur.

Ohm's Law examined for Surface Conductivity of Condensed Moisture on Glass.

The occurrence of the great differences in value of the currents obtained on reversal might be due, as some have supposed *, to a back-E.M.F. comparable in amount with the impressed potential-difference, or $C=(E-\epsilon)/R$, where ϵ is comparable with E.

With the view of tracing the matter out, experiments were undertaken to determine the relation between E and C. Various electromotive forces were applied to a grid and the corresponding currents observed. As much care as possible was taken not to leave the current on longer than was absolutely necessary to make a reading, so that the grid might remain as nearly as possible in the one state of sensitiveness throughout the determinations. In spite of all, variations occurred; this could not be helped, for the testing current of course must in some manner alter that state of sensitiveness. In interpreting the results this variation must be allowed for.

Two hundred small accumulator-cells were used and were arranged so that 400, 350, 300, &c. down to 50 volts could be quickly switched on to the grid. The results are shown in Table IV., and are plotted in fig. 4. The first six curves were obtained one after the other by switching on the cells always in the above order. The resistance is seen to increase slowly from curve to curve. This could not be prevented, as already mentioned. But there is also, apart from this, a greater resistance, or ratio of E/C, shown for the smaller currents along any one curve.

To further test this result, the cells were now applied in the reverse order so that, as the resistance on the whole is creeping up, the alteration in sensitiveness would act in the opposite direction to the effect of this departure from Ohm's law. Curves VII. and VIII. so obtained confirm the sign of the departure from linearity, and show that within the range of the voltages used the ratio of E/C diminishes as the current increases.

* Electrician, vol. lvi. p. 952.

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TABLE IV.

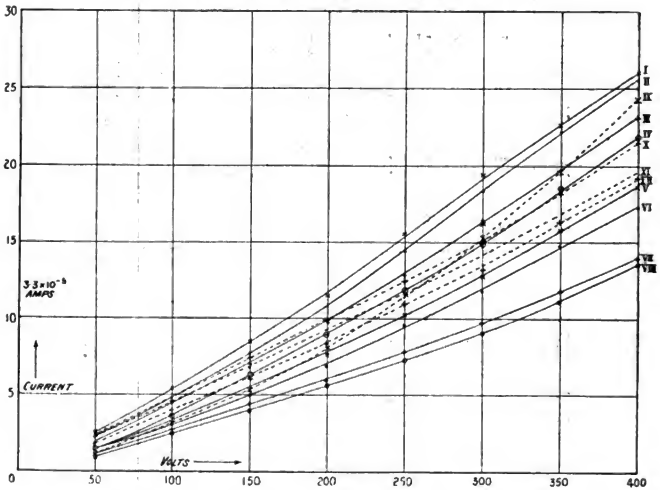
Table showing connexion between Current and Impressed E.M.F.

Volts.	Deflexion.						Volts.	Deflexion.						Volts.	Defln.
400	26·1	25·6	23·1	21·9	18·6	17·3	50	1·2	·9	Reverse current through grid.	1·2	2·4	2.3	400	19·2
350	22·6	22·4	19·6	18·5	15·6	14·9	100	2·8	2·6		3·2	5·0	4·6	350	16·4
300	19·4	18·4	16·4	14·9	12·8	11·8	150	4·5	3·8		5·3	7·5	6·9	300	13·3
250	15·6	14·5	12·9	11·8	10·3	9·6	200	6·0	5·4		8·2	10·0	9·4	250	11·2
200	11·6	10·8	9·9	9·0	7·7	6·9	250	7·8	7·4		11·7	12·5	11·7	200	8·4
150	8·6	7·7	7·1	6·4	5·6	5·0	300	9·8	9·2		15·3	15·2	14·2	150	6·2
100	5·5	4·8	4·6	3·6	3·4	3·1	350	11·8	11·2		19·7	18·2	16·9	100	4·0
50	2·6	2·4	1·9	1·5	1·5	1·5	400	14·0	13·6		24·4	21·7	19·6	50	1·9
No. of curve	I.	II.	III.	IV.	V.	VI.		VII.	VIII.		IX.	X.	XI.		XII.

Some further determinations are given, obtained with the direction of the current reversed—curves IX., X., and XI. Here the condition of sensitiveness is rapidly altering so that the departure from Ohm's law is masked to a large extent.

Fig. 4.

Curves connecting Current and Impressed E.M.F.



It will be seen, however, that the curves are all convex to the E.M.F. axis, as they should be were the resistance to alter as supposed. Curve XII. was obtained with the cells reversed, but the order of application was that originally used. It is also in general agreement with the others.

It is obvious from the trend of the curves that the supposition of a large back-E.M.F. of constant value is geometrically inapplicable, quite apart from the question of how to account chemically or otherwise for one of such a large amount as that required to explain the reversal effects, and from the fact that the usual 1·4 volts are always observable.

It thus appears to be necessary to fall back on the supposition of a variable resistance.

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Several independent effects might conspire to cause this diminution in resistance at higher voltages:—(1) If the resistance is, as above suggested, nearly all in a film of oxide on the metal, the heating by the larger current might sensibly reduce its resistance. It is true the heat generated is a very small quantity (about ·000,02 calorie per centimetre run of the metal electrode); but then the thickness of the oxide is correspondingly minute. (2) There may be an increased transference by osmosis of liquid produced by the larger currents, which flooding over the oxide causes the electrode to present a larger surface for the current. (3) Minute discharges of the nature of "sparks" may occur intermittently over the oxide * and be more pronounced with high voltages. This discharge superadded to the ordinary current would cause the apparent reduction in resistance.

Apart from these effects the experiments suggest that there may be a true want of constancy in the ohmic resistance of the surface-layers of liquids, and that it might be desirable to carefully examine for such an effect with very thin soap-films.

Conclusions.

(1) That the falling off observed in the leakage-currents is due to the formation of a film of oxide on the electrode, or in the case of platinum of gas.

(2) The alteration in the amount of the current on reversal is due to this oxide, or gas as the case may be, being reduced through the action of the reversal current itself.

(3) The initial resistance of a "grid" varies very rapidly for variations in the hygrometric state of the surrounding atmosphere for values above 80 per cent. of moisture, and suggests its applicability to measuring the hygrometric state.

(4) That the currents sent by different electromotive forces do not follow Ohm's law, but are in a greater proportion than given by it for high electromotive forces.

* We can actually see the sparks formed by discharges of this character when the grid is placed in a very damp atmosphere. A similar effect can be easily produced in the air of an ordinary room if the grid is first wiped over with a minute quantity of some slightly hygrometric material such as glycerine. Such a grid presents a very beautiful appearance under a difference of potential of even 100 volts. The whole plate spangles with minute sparks which take place at points all over the grid. The fact that sparks can under certain circumstances take place with such low voltages may be of interest from an insurance point of view.

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XLI. *The Mass and Velocity of the α particles expelled from Radium and Actinium.* By E. RUTHERFORD, *F.R.S., Macdonald Professor of Physics, McGill University, Montreal**.

[Plate V.]

THE present paper contains an account of investigations that have been made to determine, as accurately as possible, the mass and velocity of the α particles expelled from some of the products of radium and actinium. At the present stage of our knowledge of radioactivity, such measurements have an important theoretical value in throwing light on the following questions:—

1. Has the α particle expelled from all radioactive products the same mass?
2. Does the value of e/m of the α particle vary in its passage through matter?
3. What is the connexion between the velocity of the α particle and its range of ionization in air?
4. What is the connexion, if any, between the α particle and the helium atom?
5. Is the heating effect of radium or other radioactive substance due to the bombardment of the radioactive matter by the α particles expelled throughout its own mass?

In the course of these investigations, sufficient data have been accumulated, if not to answer completely all of the above questions, at least to indicate with some certainty the relations that exist between the various quantities.

The experiments outlined in this paper have been in progress for more than a year†, but publication has been delayed in order to determine the mass of the α particle from thorium and actinium as well as from radium.

The investigations on the mass of the α particle from thorium have been made in conjunction with Dr. Hahn, and are described in a following paper.

Determinations of the mass and velocity of the α particles from radium have been made by several observers. In 1902, using the electroscopic method and radium of activity 19000, I showed that the α particles from radium consisted of positively charged particles which were appreciably deflected in

* Communicated by the Author.

† A preliminary account of the measurements of the value of e/m for the particle from radium C was given before the American Physical Society, December 1905. An abstract of the results appeared in the Physical Review, Feb. 1906.

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intense magnetic and electric fields*. I deduced that the value of e/m—the ratio of the charge on the α particle to its mass—was about 6×10^3, and that the swiftest α particles emitted from radium had a velocity of about $2{\cdot}5 \times 10^9$ cms. per second. Shortly afterwards, these experiments were repeated by Des Coudres†, using the photographic method and with pure radium bromide as a source of rays. He found the value of e/m to be $6{\cdot}3 \times 10^3$, and the average velocity to be $1{\cdot}65 \times 10^9$ cms. per second.

On account of the difficulty of obtaining a sufficiently large deflexion of the α rays in passing through an electric field, the values of e/m and of the velocity of the α particles obtained by Rutherford and Des Coudres could only be considered as a first approximation to the true values.

Recently the question has again been attacked by Mackenzie‡, using the photographic method and pure radium bromide as a source of rays. Fairly large deflexions of the pencil of rays were obtained by using strong magnetic and electric fields. He showed that the α particles emitted by a thick layer of radium bromide were unequally deflected in a magnetic and electric field, and presumably consisted of α particles moving with different velocities. By assuming that the value of e/m was the same for all the α particles, he deduced that the value of e/m for the average ray was $4{\cdot}6 \times 10^3$, and that the average velocity was $1{\cdot}37 \times 10^9$ cms. per second.

It will be seen that all of these investigators have used a thick layer of radium in radioactive equilibrium as a source of rays. We know that the α particles from radium in equilibrium come from four distinct α ray products. The α particles from each of these products have different ranges of ionization in air and different velocities of projection. In addition, the α particles from each single product reach the surface from different depths of radioactive matter, and consequently have different velocities. It is thus seen that the α radiation from radium is very complex, and consists of four groups of α particles, each of which is made up of α particles which escape at widely different velocities.

On account of the dispersion of the pencil of rays in passing through an electric and magnetic field, it is difficult to interpret with certainty the deflexions observed. The difficulties which arise are clearly pointed out by Mackenzie in his paper (*loc. cit.*)

In a previous paper (Phil. Mag. July 1905) I pointed out

* Rutherford, *Phys. Zeit.* iv. p. 235 (1902); Phil. Mag. Feb. 1903.
† Des Coudres, *Phys. Zeit.* iv. p. 483 (1903).
‡ Mackenzie, Phil. Mag. Nov. 1905.

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that these difficulties would disappear if a homogeneous pencil of α rays was employed. I showed that such a homogeneous pencil could be obtained by using as a source of rays a small wire which had been made very active by exposure to the radium emanation. Fifteen minutes after removal from the emanation, radium A has been transformed, and the α particles are then emitted only from radium C.

An examination of the deflexion of the rays in a magnetic field showed that such an active wire fulfilled the conditions necessary for a homogeneous source of rays. The α particles all escaped from the thin film of radioactive matter at the same speed, and all suffered the same reduction of velocity in passing through an absorbing screen. On account of the rapid decay of the activity of the active deposit, it is necessary to employ an intensely active wire to obtain a strong photographic effect. In most of the experiments described later, the active deposit was concentrated on the wire by making it the only negatively charged surface in a vessel containing a large quantity of the radium emanation. In this way, very active wires were obtained which served as suitable sources of homogeneous α rays.

Electric Deflexion of the α Rays.

The determination of e/m and of the velocity of the α particle was made in the usual way by measuring the deflexion of a pencil of rays in passing through both a magnetic and electric field of known strength. The method employed for measuring the magnetic deflexion has already been described in a previous paper. After some preliminary experiments, the following arrangement was adopted to determine the deflexion of the α rays in passing through an electric field. The rays from the active wire W (fig. 1), after traversing a thin mica plate in the base of the brass vessel M, passed between two parallel insulated plates A and B about 4 cms. high and 0·21 mm. apart. The distance between the plates was fixed by thin strips of mica placed at the four corners, and the plates were rigidly held together by rubber bands. The terminals of a storage-battery were connected with A and B so that a strong electric field could be produced between the two plates. The pencil of rays, after emerging from the plates, fell on a photographic plate P. The latter was rigidly fixed to a ground-brass plate which fitted accurately on the top surface of the vessel. The ground surfaces were air-tight, and the photographic plate could thus easily be placed in position or removed without disturbing the rest

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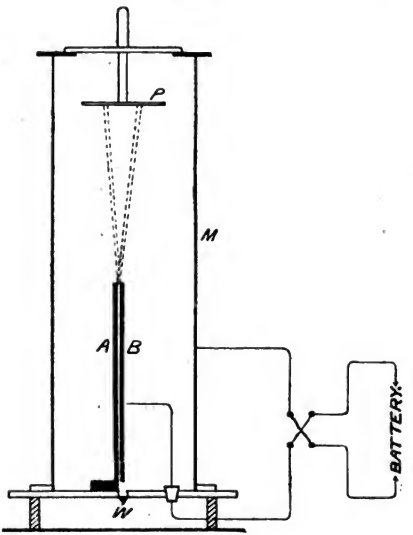
of the apparatus. The vessel was connected to a mercury pump and exhausted to a low vacuum. If necessary, the exhaustion was completed by means of a side tube filled with cocoanut charcoal and immersed in liquid air.

Fig. 1.



The plates A and B were placed close together for several reasons. In the first place, a strong electric field could be produced between the plates for a comparatively small voltage. The greatest P.D. necessary in the experiments was about 500 volts. Since the plates were about one-fifth of a millimetre apart, this voltage produced an electric field between the plates corresponding to 25000 volts per cm. One advantage of the arrangement lies in the fact that, provided the P.D. is below about 350 volts, there is no danger of a discharge between the plates, even if there is not a good vacuum. This is particularly convenient where it is found necessary to expose the photographic plate to a weak source of radiation

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for several days, for there is no necessity to continually watch the state of the vacuum.

On account of the small distance between the plates, there is no necessity to correct for the disturbance of the electric field near the ends of the plates. In addition, the parallel plates acted as a slit in order to obtain a narrow pencil of rays. In its passage through the electric field each α particle describes a parabolic path, and after emergence travels in a straight line to the photographic plate. By reversing the electric field at intervals, the direction of deflexion of the pencil of rays is reversed.

The general effect of the electric field in altering the appearance of the trace of the pencil of rays impinging on the photographic plate is shown in fig. 2 (Pl. V.) A shows the natural width of the line without an electric field, B for a P.D. of 255 volts, C for 340 volts, and D for 497 volts. These are reproduced from the actual photographs (magnification about 1·4 times). When a small P.D. is applied, the natural width of the photographic trace is broadened. Above a certain voltage, the single band breaks into two. As the voltage is further increased, the distance apart of these bands increases while the width of each band steadily narrows. The outside edge of each band is sharply defined, but it is difficult to fix with certainty the inner boundary of the bands.

Theory of the Experiment.

The theory of the experimental arrangement where the parallel plates act both as a slit and a means of applying the electric field, is more complicated than the ordinary case where a narrow pencil of α rays is made to pass between the two parallel plates of the condenser without impinging on the sides.

A diagram of the experimental arrangement is shown in fig. 3. AB and CD are the two charged parallel plates, and CE the radiant source which was of greater width than the distance between the plates. It is required to find the width of the trace on the photographic plate when a P.D. V is applied between the plates.

Let m = mass of α particle,
e = charge on α particle,
u = velocity of α particle in passing between plates,
$AB = l_1$, $CD = l_2$, $Bb = l_3$,
d = distance between plates,
D = distance between extreme edges of the photographic trace for reversal of the electric field.

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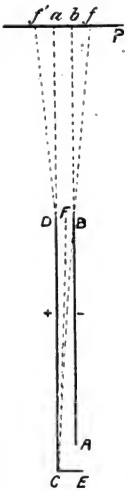
There are two cases of the theory which must be separately considered :—

Case 1, when the deflexion of the α particle in passing through the electric field is less than *d*, the distance between the plates.

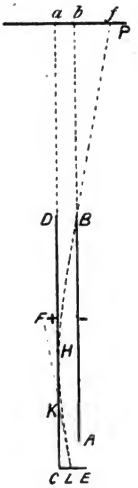
Case 2, when the deflexion is greater than *d*.

Fig. 3 A.



Case 1.

Fig. 3 B.



Case 2.

Case 1.—We shall now consider the theory for the first case. On entering the electric field at A, each α particle describes a parabolic path, and on emergence from the field moves in a straight line, the direction of which is a tangent to the parabolic path at the moment of emergence. The distance between the plates (0·21 mm.) is so small compared with the length AB (3·77 cms.), that we may assume without sensible error that the electric field is everywhere normal to the path of the rays. Suppose that the electric field is applied in such a direction that the α particle is urged in the direction of the plate AB. Some of the α particles, which before the field was applied fell on the photographic plate, are now

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stopped by the plate AB, but other α particles previously stopped by the plate CD are able to emerge.

Suppose that the α particle in passing through the electric field is deflected normally through a distance S represented by FB. All the α particles which before the application of the electric field passed through the point F now just emerge at B at grazing incidence. The α particle which forms the extreme edge of the photographic trace at f must obviously be projected initially in the direction CF, and after emergence will travel along the line Bf.

Since the normal acceleration of the α particle in passing through the electric field is $\frac{Ve}{dm}$, and the time occupied in passing between the charged plates is $\frac{l_1}{u}$, the distance

$$FB = s = \frac{Ve}{2dm} \cdot \frac{l_1^2}{u^2},$$

or

$$s = \lambda l_1^2, \quad \text{where } \lambda = \frac{e}{2m} \cdot \frac{V}{du^2}.$$

At the moment of leaving the electric field, the tangent of the angle θ, which the direction of motion makes with the initial direction of projection CF, is given by $\tan\theta = 2\lambda l_1$.

If the angle DCF $= \theta_1$, the emerging ray makes an angle $\theta + \theta_1$ with the direction of the plates Bb. The distance $bf = l_2 \tan(\theta + \theta_1)$. Since the angles θ and θ_1 are small,

$$bf = l_2(\theta + \theta_1)$$
$$= l_2\left(2\lambda l_1 + \frac{d-s}{l_2}\right).$$

In a similar way when the electric field is reversed, the corresponding distance

$$af' = l_3\left(2\lambda l_1 + \frac{d-s}{l_1}\right).$$

In this case, the α particle which is most deflected enters the electric field at grazing incidence at the point A.

The distance D between the extreme edges ff' of the photographic impression is consequently given by

$$D = bf + ab + af'$$
$$= 4\lambda l_1 l_3 + l_2(d-s)\left(\frac{1}{l_1} + \frac{1}{l_2}\right) + d.$$

Substituting the value $s = \lambda l_1^2$,

$$D = \lambda l_1 l_3\left(3 - \frac{l_1}{l_2}\right) + \left(\frac{l_3}{l_2} + \frac{l_3}{l_2} + 1\right)d.$$

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But it is easily seen that the natural width of the photographic band without the electric field is given by

$$\left(\frac{l_3}{l_2}+\frac{l_2}{l_1}+1\right)d.$$

Therefore the *increase* D_1 of the breadth of the band by the reversal of the electric field is given by

$$D_1=\lambda\, l_1\, l_3\left(3-\frac{l_1}{l_2}\right).$$

Substituting the value of λ,

$$\frac{mu^2}{e}=\frac{V\, l_1\, l_3}{2dD_1}\left(3-\frac{l_1}{l_2}\right). \quad . \quad . \quad . \quad . \quad (1)$$

This gives the formula required for determining the value of $\frac{mu^2}{e}$ for case 1.

Case 2.—In this case the electric field is supposed to be sufficiently strong to deflect the α particle in passing between the charged plates through a distance greater than d.

Suppose the electric field urges the α particle towards the plate AB (fig. 3 B). A little consideration shows that the α particle which forms the extreme edge of the photographic impression at f must touch at grazing incidence the plate CD. Let LKF be the direction of projection of such an α particle, intersecting the plate CD at K. The path of the α particle under the action of the electric field is shown by the dotted line in the figure. The path touches the plate CD at H and emerges at B at grazing incidence.

Let $DH=y$.

Then with the same notation as before, $d=\lambda y^2$.

The angle θ which the tangent to the parabola at B makes with the direction of the plate AB is given by

$$\tan\theta=2\lambda y.$$

The distance $bf=l_3\tan\theta=2\lambda y l_3$.

The total distance D between the extreme edges of the photographic impression on the plate P by reversal of the field is consequently given by

$$D=4\lambda y l_3+d.$$

Then $$(D-d)^2=16\lambda d l_3^2.$$

Substituting the value of λ as before,

$$\frac{mu^2}{e}=\frac{8Vl_3^2}{(D-d)^2}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

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It is interesting to note that this formula for the determination of $\frac{mu^2}{e}$ does not involve l_1 or l_2, and only involves the distance d to a subordinate extent. For example, in one experiment where $l_3 = 10$ cms., the value of d was only 1/15 D. This is a great advantage, as the distance d is difficult to measure with accuracy.

While the distance l_1 is not involved in the final formula, it must be remembered that the formula (2) only applies when the α particle is deflected through a distance greater than d in passing between the plates. If l_1 is made smaller, the value of V must be made correspondingly greater before the formula can be applied.

It has been mentioned that the width of two deflected traces obtained by reversal of the electric field decreases in width with increase in strength of the electric field. The reason of this can readily be shown from theoretical considerations. For example, it is seen that the inside edge of the deflected pencil (fig. 3 B) is produced by the α particles whose paths touch at grazing incidence the plate AB at A and also touch the plate CD. These conditions determine the direction of the α particle in entering the electric field at A. All α particles passing through A which make a greater angle with the plate AB than the above α particle are stopped by the plate CD. As the width of the trace is not required in the experiment, it has not been thought necessary to include here the connexion between the width of the deflected pencil and the strength of the electric field. The calculations, though a little long, are not difficult.

It is now necessary to consider how we are able to know from the photographs obtained whether the formula (1) or (2) is to be applied. The formula (1) holds provided the distance of deflexion of the α particle is not greater than d. Suppose that the α particle is deflected through a distance d in passing between the charged plates. The outside edge of the photographic impression, for example, on the right of the plate P (fig. 3) is due to the α particles which start from the point C parallel to the plate CD. With the same notation as before,

$$d = \lambda l_1^2.$$

Following the same method of calculation as for case (2), it is seen that the value of D is given by

$$D = 4\lambda\, l_1\, l_3 + d.$$

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Substituting the value $\lambda = \frac{d}{l_1^2}$,

$$D = d\left(\frac{4l_2}{l_1} + 1\right).$$

In most of the experiments to be described later,

$$l_2 = 3{\cdot}94 \text{ cms.}, \quad l_1 = 3{\cdot}77 \text{ cms.}, \quad d = {\cdot}21 \text{ mm.}$$

Consequently, $D = 1{\cdot}09$ mm.

From the data given below it will be seen that the formula (2) applies for all voltages greater than about 300, while the formula (1) applies for all values smaller than this.

Results of Experiments.

The electrostatic deflexion of the α rays from radium C was first determined for different voltages between the plates. The rays from the active wire passed through a mica plate in the base of the vessel, equivalent in stopping power to about 3·5 cms. of air. The extreme distance D between the outside edges of the photographic impressions obtained on the plate by reversal of the electric field was measured by the lantern method described in a previous paper*.

In most of the experiments, the value l_3 of the distance of the photographic plate above the parallel plates was 3·94 cms. In one experiment this distance was 10·00 cms.

$$l_1 = 3{\cdot}77 \text{ cms.}, \quad l_2 = 4{\cdot}165 \text{ cms.}, \quad d = 0{\cdot}210 \text{ mm.}$$

The values of $\frac{mu^2}{e}$ obtained for different voltages and distances of the photographic plate are tabulated below.

Volts between plates.	l_3.	D.	$\frac{mu^2}{e}$.
171	3·94 cms.	0·857 mms.	$5{\cdot}1 \times 10^{14}$
255	,, ,,	0·995 ,,	$4{\cdot}9 \times 10^{14}$
340	,, ,,	1·136 ,,	$4{\cdot}93 \times 10^{14}$
497	,, ,,	1·346 ,,	$4{\cdot}79 \times 10^{14}$
508·6	10·00 ,,	3·10 ,,	$4{\cdot}87 \times 10^{14}$

Each of the values of D given above is the mean of a large number of separate measurements which agreed closely among

* Rutherford, Phil. Mag. August 1906.

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themselves. The values for 171 and 255 volts are calculated from formula (1), the natural width of the photographic trace being 0·61 mm., and for the higher voltages from formula (2). Some of the photographs from which the measurements were made are reproduced in fig. 2, magnification about 1·4.

Two good photographs were obtained with a P.D. of 340 volts. In each case two active wires were used successively to give a strong photographic impression. On account of the greater distance, the photographic impression was not so strongly marked for the distance 10 cms.

Giving a weight 1 to the measurement of $\frac{mu^2}{e}$ for 497 volts, and a weight 2 for both 340 and 508·6 volts, the mean value is given by

$$\frac{mu^2}{e} = \underline{4{\cdot}87 \times 10^{14}} \text{ electromagnetic units.} \quad . \quad . \quad . \quad (3)$$

By measurement of the magnetic deflexion, the maximum value of $\frac{mu}{e}$ for the α rays from radium C was found to be $4{\cdot}06 \times 10^6$. The mica screen cut down the velocity of the rays to ·763 of the initial velocity, so that the value of $\frac{mu}{e}$ for the rays which passed through the electric field is given by

$$\frac{mu}{e} = 3{\cdot}10 \times 10^5. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By combining equations (3) and (4)

$$u = 1{\cdot}57 \times 10^9 \text{ cms. per second,}$$
$$e/m = \underline{5{\cdot}07 \times 10^3} \text{ electromagnetic units.}$$

I think that the values of u and e/m are certainly correct within two per cent.

The initial velocity of the α particles expelled from radium C is consequently $\underline{2{\cdot}06 \times 10^9}$ cms. per second.

Does the value of e/m *for the* α *particle vary in its passage through matter?*

In order to test this point, the value of e/m for the α particle was determined under the following conditions:—

(1) The active wire was placed on top instead of under

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the mica screen, so that the electrostatic deflexion was determined for the α particle from the unscreened wire.

(2) The α particles passed through a mica screen equivalent in stopping power to 3·5 cms. of air. The value of e/m under these conditions has been determined in the previous section.

(3) The α particles passed through a screen of mica and aluminium equivalent to about 6·5 cms. of air.

The magnetic and electrostatic deflexion were separately determined. The former gives the value of $\frac{mu}{e}$ and the latter $\frac{mu^2}{e}$. The results of the measurements are collected in the following table, where D has the same meaning as before. The value of l_3 in all cases was 3·94 cms.

Interposed absorbing screen in terms of air.	Volts.	D.	$\frac{mu}{e}$.	$\frac{mu^2}{e}$.	$\frac{e}{m}$.
0	340	0·88 mm.	$4{\cdot}06\times10^5$	$9{\cdot}4\times10^{14}$	$5{\cdot}7\times10^3$
3·5 cms.	...	...	$3{\cdot}10\times10^5$	$4{\cdot}87\times10^{14}$	$5{\cdot}07\times10^3$
6·5 cms.	340	1·62 mm.	$2{\cdot}11\times10^5$	$2{\cdot}11\times10^{14}$	$4{\cdot}8\times10^3$

The deflexion for the rays from the bare wire was small, but could be measured with fair certainty. I think the value of e/m obtained in this case, viz. $5{\cdot}7\times10^3$, is undoubtedly too high. On removing the active wire after completion of the experiment, it was noticed that its position was displaced somewhat to the side of the opening of the parallel plates. This would tend to make the observed width of the photographic trace too small, and consequently to give too great a value of e/m, when calculated from formula (1), which is based on the assumption that the active source completely covers the opening between the parallel plates.

The impression on the photographic plate due to the α particles which have passed through an absorbing screen equivalent to 6·5 cms. of air was weak but clearly defined, and admitted of fairly accurate measurement. Allowing for an error in the estimation of e/m for the α particles from the bare wire, I think the agreement of the values of e/m obtained under the different conditions is sufficiently close to prove definitely that the value of e/m for the α particle is unaltered in its passage through matter.

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Value of e/m *for the α particles from Radium* A.

In a previous paper (*loc. cit.*) I gave the results of the measurements by the photographic method of $\frac{mu}{e}$ for the α particle emitted from radium A. For the unscreened wire the value of $\frac{mu}{e} = 3{\cdot}67 \times 10^5$, and for the wire covered with a mica plate of the same thickness as that over the opening in the base of the electrostatic apparatus, I found the value of $\frac{mu}{e} = 2{\cdot}19 \times 10^5$.

I pointed out in that paper the difficulty of accurately measuring the magnetic deflexion of the α particles from such a rapidly changing product as radium A, which is half transformed in three minutes. The difficulty of obtaining a sufficiently marked photographic impression, in order to measure the electric deflexions of the rays, was still greater. It was found necessary to place twenty active wires successively in position under the base of the apparatus, in order to obtain a measurable darkening of the photographic plate. Each wire was exposed for two minutes as negative electrode in a vessel containing a large quantity of the radium emanation. For such a short exposure, the initial radiation from the wire mainly comes from radium A. The wire was rapidly removed from the emanation vessel and placed in position and left for six minutes. In that time, the α ray activity of radium A is reduced to one quarter of its initial value. The α rays from the active wire passed through the standard mica plate before entering the electric field. The electric deflexion of the α rays from radium A is considerably greater than that for the swifter α particles from radium C; so that there is no danger of confusion between the two types of rays, even though the photographic impression of the rays from radium C present on the active wire is comparable with that due to the rays from radium A.

In the experiment the voltage was 255; $l_3 = 3{\cdot}94$ cms.; and $D = 1{\cdot}30$ mm. This gave

$$\frac{mu^2}{e} = 2{\cdot}67 \times 10^{11}.$$

The value $\frac{mu}{e}$ for the α rays from radium A after traversing the standard mica was $2{\cdot}19 \times 10^5$. This gives the values

$$u = 1{\cdot}22 \times 10^9 \text{ cms. per sec.}$$
$$e/m = 5{\cdot}6 \times 10^3.$$

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It was found by experiment that the measured value of D —the distance between the extreme edges of the trace—was always underestimated for a very feeble photographic trace. An underestimate of the value of D gives too large a value for e/m. Taking this factor into consideration, the values of e/m obtained for the α particles from radium A and from radium C agree within the limit of experimental error. This shows that the α particles expelled from radium A and C have the same mass and differ only in their initial velocities of projection.

Mass of the α particle from Radium F.

A bismuth rod coated with radiotellurium was used as a source of α rays. It is now definitely established that the active constituent in both radiotellurium and polonium is the same and consists of the transformation product of radium, radium F. The active matter is deposited in the form of a thin film on the bismuth rod, and the α particles all escape from the surface at practically the same velocity. A piece of the rod was placed in position inside the electrostatic apparatus, and the photographic plate exposed for four days to the action of the α rays from the bare rod. The value $l_3 = 10{\cdot}00$ cms, the voltage $= 443$, and the observed value $D = 2{\cdot}72$ mm.

This gives a value $\frac{mu^2}{e} = 5{\cdot}63 \times 10^{14}$ for the α rays from the unscreened source. The value of $\frac{mu}{e} = 3{\cdot}2 \times 10^5$ was deduced by me in a previous paper from measurement of the range of the α particles from radium F in air. The experimental value found directly by Mackenzie (*loc. cit.*) was $3{\cdot}3 \times 10^5$. Taking the mean value $\frac{mu}{e} = 3{\cdot}25 \times 10^5$, we find that $e/m = 5{\cdot}3 \times 10^3$, and $u = 1{\cdot}73 \times 10^9$ cms. per second.

The actual photograph obtained was very weak in intensity, and, for the reasons previously mentioned, there is no doubt that the value e/m obtained is too large. We may consequently conclude that the α particle from radium F has the same mass as that expelled from radium C. Using a more active rod or a longer time of exposure, the value of e/m should be obtained with much greater precision; but there can be no doubt that it would be found identical with that observed for the α particle from radium C.

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Mass of the α particles from Actinium.

In order to obtain a homogeneous source of α rays, the active deposit of actinium was used. The active deposit was concentrated on a small copper plate by making it the negative electrode in a small vessel containing the emanating actinium compound. This active deposit consists of two products, actinium A and B, the former of which is rayless. The activity imparted to a plate, ten minutes after removal from the emanation, decays exponentially with a period of 36 minutes. The rays emitted are all of one kind and have a range in air, found by Dr. Hahn in this laboratory, of 5·5 cms.

The preparation of actinium * employed was not very active, and the activity imparted to the copper plate was too weak to produce appreciable photographic action at the distances required. With the experience gained in the previous experiments with weak radioactive sources, it was recognized that at least twenty active wires, placed successively in position, would have been required to produce a measurable photographic effect in the magnetic deflexion apparatus. Three or four times this number would have been necessary in the electrostatic experiment. In order to avoid the necessity of such a procedure, the apparatus was constructed so that the copper plate could be kept active in the position required for any length of time. The actinium compound, wrapped in thin paper, was placed round the sides of a small brass vessel, which was attached to the base of the magnetic or electric deflexion apparatus. A small insulated copper plate, with its plane slightly inclined to the vertical, was placed below a narrow slit covered with mica in the base of the apparatus and was kept negatively charged. The activity on the plate reached a maximum after three hours and then remained constant. The radiation passing through the slit into the magnetic deflexion apparatus was mainly due to the α rays from actinium B. The photographic effect of the radiations from the emanation close to the active plate was too weak to be observed. The magnetic deflexion of the pencil of the α rays was determined under identically the same conditions as in the experiments using radium C as a source of rays. The photographic plate was exposed for ten hours in a constant magnetic field which was reversed at intervals. Two fine well-defined lines were obtained on the plate. The amount of the magnetic deflexion

* I am indebted to Mr. H. Lieber of New York for his kindness in lending me the sample of actinium used in this experiment.

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was then directly compared with that due to the rays from radium C under the same conditions. For this purpose, the copper plate was removed and made active by exposure to the radium emanation. It was then placed back in its original position, and another photograph taken. The mica plate, covering the opening in the base of the magnetic apparatus, was of the same thickness as that used in the base of the electrostatic apparatus. The following numbers illustrate the results obtained :—

Distance between centres of deflected bands due to rays from actinium B = 1·85 mm.

Distance between centres of deflected bands due to rays from radium C = 1·53 mm.

We have previously shown that the value of $\frac{mu}{e}$ for the α particles of radium C after passing through the standard mica screen, absorbing-power equal to 3·5 cms. of air, is $3{\cdot}10 \times 10^5$. Consequently the value of $\frac{mu}{e}$ for the α particles from actinium B is given by

$$\frac{mu}{e} = \frac{1{\cdot}53}{1{\cdot}85} \times 3{\cdot}10 \times 10^5 = 2{\cdot}56 \times 10^5.$$

It is interesting to note that the comparative magnetic deflexions observed for the rays of actinium B and of radium C agree with those to be expected from their known ranges in air, assuming the value of e/m for the α particle to be the same in both cases.

I have shown in a previous paper that the velocity V of an α particle of range r cms. in air is given by

$$\frac{V}{V_0} = {\cdot}348\sqrt{r + 1{\cdot}25},$$

where V_0 is the maximum velocity of the rays from radium C which have a range of 7·06 cms. Now, after passing through the mica screen, the rays from actinium B have a range $5{\cdot}5 - 3{\cdot}5 = 2{\cdot}0$ cms., while those from radium C have a range $7{\cdot}06 - 3{\cdot}5 = 3{\cdot}56$ cms.

Consequently,

$$\frac{\text{velocity of rays from radium C after passage through mica}}{\text{velocity of rays from actinium B after passage through mica}} =$$

$$= \sqrt{\frac{3{\cdot}56 + 1{\cdot}25}{2{\cdot}0 + 1{\cdot}25}} = 1{\cdot}22.$$

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The experimental ratio is 1·21. From the agreement between the experimental and the theoretical ratios, it could be concluded with confidence that the mass of the α particle from actinium is the same as that from radium C. This, however has been experimentally verified by measuring the electric deflexion of the α rays from actinium B.

The apparatus for determining the electric deflexion of the α rays was the same as that used in the radium experiments. The copper plate which served as a source of α rays was kept active by an arrangement similar to that used for the magnetic deflexion. The photographic plate was exposed for six days. Through an accident in the connexions, the electric field was not acting between the plates for the first three days. The photographic plate consequently showed the undeflected trace of the rays and the deflected trace on one side of it. The distance between the centre of the undeflected trace and the outside edge of the deflected trace gives the value of $\frac{D}{2}$. The value of l_3 was 10·00 cms.; the voltage 340, and D was 3·174 cms. Consequently the value of $\frac{mu^2}{e}$ for the rays after passing through the standard mica was

$$\frac{mu^2}{e} = 3{\cdot}10 \times 10^{14}.$$

We have previously shown that

$$\frac{mu}{e} = 2{\cdot}56 \times 10^5,$$

Therefore

$$\frac{e}{m} = 4{\cdot}7 \times 10^3,$$

and

$$u = 1{\cdot}21 \times 10^9 \text{ cms. per second.}$$

We may thus conclude that the α particle from actinium has the same mass as that from radium.

Connexion of the α particle with the helium atom.

We have seen that, within the limit of experimental error, the mass of the α particle expelled from radium A, radium C, radium F, or actinium B is the same. In a later paper, in conjunction with Dr. Hahn, it will be shown that the mass of the α particle expelled from thorium C is also identical with that expelled from the radium products. We have also shown in a previous paper that the amount of the magnetic deflexion

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of the α particle from radium itself is in agreement with that to be deduced from its range in air—a result which is only to be expected if the α particle from radium has the same mass as that from radium C.

There then remains only one α ray product of radium, viz. the emanation, whose radiation has not been closely examined. There is, however, no reason to suppose that the α particles from the emanation differ in mass from those of the other products. An examination of the complex pencil of rays from a layer of radium in equilibrium shows no evidence of the presence of α rays which suffer an abnormal amount of deflexion. I think there can be no doubt that the α particles emitted from the various products of radium have an identical mass, but differ only in the initial velocities of projection. Although the mass of the α particles has been determined for only a single product of thorium and of actinium, the analogy with radium would lead us to expect that the α particle has the same mass for all the products of these substances.

We may thus reasonably conclude that the α particles expelled from the different radio-elements have the same mass in all cases. This is an important conclusion; for it shows that uranium, thorium, radium, and actinium, which behave chemically as distinct elements, have a common product of transformation. The α particle constitutes one of the fundamental units of matter of which the atoms of these elements are built up. When it is remembered that in the process of their transformation radium and thorium each expel five α particles, actinium four, and uranium one, and that radium is in all probability a transformation product of uranium, it is seen that the α particle is an important fundamental constituent of the atoms of the radio-elements proper. I have often pointed out what an important part the α particles play in radioactive transformations. In comparison, the β and γ rays play quite a secondary rôle.

It is now necessary to consider what deductions can be drawn from the observed value of e/m found for the α particle. The value of e/m for the hydrogen ion in the electrolysis of water is known to be very nearly 10^4. The hydrogen ion is supposed to be the hydrogen atom with a positive charge, so that the value of e/m for the hydrogen atom is 10^4. The observed value of e/m for the α particle is $5{\cdot}1 \times 10^3$, or, in round numbers, one half of that of the hydrogen atom. The density of helium has been found to be 1·98 times that of hydrogen, and from observations of the velocity of sound in helium, it has been deduced that helium is a monatomic gas. From this it is concluded that the helium atom has an atomic

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weight 3·96. If a helium atom carries the same charge as the hydrogen ion, the value of e/m for the helium atom should consequently be about $2·5 \times 10^3$. If we assume that the α particle carries the same charge as the hydrogen ion, the mass of the α particle is twice that of the hydrogen atom. We are here unfortunately confronted with several possibilities between which it is difficult to make a definite decision.

The value of e/m for the α particle may be explained on the assumptions that the α particle is (1) a *molecule* of hydrogen carrying the ionic charge of hydrogen, (2) a helium atom carrying *twice* the ionic charge of hydrogen, or (3) *one half* of the helium atom carrying a single ionic charge.

The hypothesis that the α particle is a molecule of hydrogen seems for many reasons improbable. If hydrogen is a constituent of radioactive matter, it is to be expected that it would be expelled in the atomic, and not in the molecular state. In addition, it seems improbable that, even if the hydrogen were initially projected in the molecular state, it would escape decomposition into its component atoms in passing through matter, for the α particle is projected at an enormous velocity, and the shock of the collisions of the α particle with the molecules of matter must be very intense, and tend to disrupt the bonds that hold the hydrogen atoms together. If the α particle is hydrogen, we should expect to find a large quantity of hydrogen present in the old radioactive minerals, which are sufficiently compact to prevent its escape. This does not appear to be the case, but, on the other hand, the comparatively large amount of helium present supports the view that the α particle is a helium atom. A strong argument in support of the view of a connexion between helium and the α particle rests on the observed facts* that helium is produced by actinium as well as by radium. The only point of identity between these two substances lies in the expulsion of α particles of the same mass. The production of helium by both substances is at once obvious if the helium is derived from the accumulated α particles, but is difficult to explain on any other hypothesis. We are thus reduced to the view that either the α particle is a helium atom carrying twice the ionic charge of hydrogen, or is half of a helium atom carrying a single ionic charge.

The latter assumption involves the conception that helium, while consisting of a monovalent atom under ordinary chemical and physical conditions, may exist in a still more elementary state as a component of the atoms of radioactive matter,

* Debierne, *C. R.* cxli. p. 383 (1905).

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and that, after expulsion, the parts of the atom lose their charge and recombine to form atoms of helium; while such a view cannot be dismissed as inherently improbable, there is as yet no direct evidence in its favour. On the other hand, the second hypothesis has the merit of greater simplicity and probability.

On this view, the α particle is in reality a helium atom which is either expelled with a double ionic charge or acquires this charge in its passage through matter. Even if the α particle were initially projected without charge, it would certainly acquire one after the first few collisions with the molecules in its path. We know that the α particle is a very efficient ionizer, and there is every reason to suppose that it would itself be ionized by its collisions with the molecules in its path, *i. e.* it would lose one or more electrons and retain a positive charge. If the α particle can remain stable with the loss of two electrons, these electrons would almost certainly be removed as a result of the intense disturbance set up by the collision of the α particle with the molecules of matter. The α particle would then have twice the normal ionic charge, and the value of e/m, as found by measurement, would be quite consistent with the view that the α particle is an atom of helium.

In a previous paper* I showed, from measurement of the charge carried by the α rays, that $6{\cdot}2 \times 10^{10}$ α particles were expelled per second from one gram of radium at its minimum activity. This was based on the assumption that each α particle carried a positive charge equal to the ionic charge of hydrogen, viz. $3{\cdot}4 \times 10^{-10}$ electrostatic units. Assuming that the α particle carries two ionic charges, the corresponding number is reduced to one half of the above, viz. $3{\cdot}1 \times 10^{10}$. This would make the calculated period of radium 2600 years instead of 1300 years (see 'Radioactivity,' second edition, 1905, p. 457). In a similar way, the calculated volume of the emanation released from one gram of radium would be 0·4 cubic mm. instead of 0·8 cubic mm. The calculated volume of helium produced per year per gram of radium would be 0·11 cubic cms. ('Radioactivity,' p. 481).

On the hypothesis that the α particle is a helium atom, the atomic weight of each product is diminished by four units, in consequence of the expulsion of an α particle. On the hypothesis that the α particle is half a helium atom carrying a single ionic charge, the atomic weight is diminished by two units instead of four. Taking the latter hypothesis, the number of α particles expelled per second from one gram of

* Phil. Mag. August 1905.

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radium at its minimum activity is $6{\cdot}2 \times 10^{10}$. The calculated volume of the emanation is 0·8 cubic mms., while the production of helium per year is 0·11 cubic cms. per gram. The two hypotheses thus lead to the same rate of production of helium by radium.

Age of Radioactive Minerals.

I have previously pointed out that the age of the radioactive minerals can be calculated from the amount of helium contained in them. The method is based on the assumption that, in a compact mineral, the greater part of the helium is mechanically imprisoned in the mineral and is unable to escape. Let us consider, for example, the mineral fergusonite, which was found by Ramsay and Travers to contain 1·81 c.c. of helium per gram of the mineral. The fergusonite contains about 7 per cent. of uranium. The amount of helium per gram of uranium is consequently 26 c.c. Now we have seen that one gram of radium produces 0·11 c.c. of helium per year. The content of radium per gram of uranium is $3{\cdot}8 \times 10^{-7}$ gram*. Supposing that uranium emits only one α particle corresponding to the five emitted by radium in equilibrium where the product radium F is present, the production of helium per year per gram of uranium is $\frac{6}{4} \times {\cdot}11 \times 3{\cdot}8 \times 10^{-7}$ or $6{\cdot}3 \times 10^{-8}$ c.c. per year. Assuming as a first approximation that the rate of production of helium has been constant since the formation of the mineral, the time required for a production of 26 c.c. of helium is about 400 million years. This is a minimum estimate, for some of the helium has probably escaped from the mineral.

As another example, consider the mineral thorianite, which contains about 72 per cent. of thorium and 10 per cent. of uranium. The evolution of helium per gram of the mineral was found by Ramsay to be 9·5 c.c. Bragg (Phil. Mag. June 1906) has shown that thorium breaks up at ·26 of the rate of uranium. This was based on measurements made with ordinary commercial thorium. Boltwood (Amer. Journ. Sci. June 1906) has, however, drawn attention to the fact that ordinary commercial thorium has in many cases only about one half of the activity obtained by direct preparation of the thorium from the radioactive minerals. This would double the rate of breaking up of thorium observed by Bragg. Remembering that a thorium atom during its transformations emits five α particles, and assuming that thorium breaks up at half the rate of uranium, it is seen that 72 per cent. of thorium in a mineral corresponds as a producer of helium to

* Rutherford and Boltwood, Amer. Journ. Sci. July 1906.

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about $\frac{5}{6} \times \frac{72}{2} = 30$ per cent. of uranium. The amount of helium corresponding to one gram of uranium or its equivalent in the mineral is consequently 24 c.c. As before, the age of the mineral works out to be about 400 million years.

Numerous other examples may be given, but these serve to illustrate the method of calculation from radioactive data of the age of some radioactive minerals, and indirectly, in some cases, of the geological strata in which they are found.

Velocity and Energy of the α particles expelled from Radium products.

If the value of e/m is the same for the α particle expelled from the various radium products, the maximum velocity of each set of α particles can be deduced from their range in air, knowing the velocity of the α particles expelled from radium C. The velocities so determined are probably more accurate than those obtained by direct measurement under difficult conditions. In the following table, the second column gives the range in air of the α particles from the radium products, found by Bragg and Kleeman; in the fourth column is given the value of $\frac{mu^2}{e}$, where u is the initial velocity of projection of the α particles.

Product.	Range of α particle in cms.	Velocity in cms. per sec.	$\frac{mu^2}{e}$.
Radium	3·50 cms.	$1{\cdot}56 \times 10^9$	$4{\cdot}78 \times 10^{14}$
Emanation.....	4·36 ,,	$1{\cdot}70 \times 10^9$	$5{\cdot}65 \times 10^{14}$
Radium A.....	4·83 ,,	$1{\cdot}77 \times 10^9$	$6{\cdot}12 \times 10^{14}$
Radium C.....	7·06 ,,	$2{\cdot}06 \times 10^9$	$8{\cdot}37 \times 10^{14}$
Radium F*	3·86 ,,	$1{\cdot}61 \times 10^9$	$5{\cdot}15 \times 10^{14}$

* The range of the rays for polonium (radium F) has been recently measured by Levin (Amer. Journ. Sci. July 1906).

Disregarding radium F, the average energy of the α particle expelled from radium in equilibrium is $3{\cdot}11 \times 10^{14}\, e$, where e is the charge carried by the α particle. Assuming that the heating effect of radium is a measure of the kinetic energy of the expelled α particles, we can at once deduce the total number of α particles expelled per second per gram of radium in equilibrium. One gram of radium in equilibrium emits 100 gram-calories of heat per hour. This rate of

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emission of energy is mechanically equivalent to $1{\cdot}16 \times 10^8$ ergs per second. Since the average energy of the expelled α particle is $3{\cdot}11 \times 10^{11}\,e$, the number of α particles expelled per second from one gram of radium in equilibrium is $\frac{3{\cdot}65}{10^9\,e}$. The number previously found by the writer by measuring the total charge carried by the α particles was $\frac{2{\cdot}82}{10^9\,e}$, *i. e.* 77 per cent. of the theoretical number. The agreement between theory and experiment is thus fairly good. In the above estimate, it is assumed that the heating effect is due entirely to the kinetic energy of the expelled α particles. It is known experimentally that the heating effect of the β and γ rays is only a small percentage of that due to the α rays. The expulsion of an α particle from an atom should lead to the recoil of the residue of the atom. Assuming that the momentum of the atom is equal and opposite to that of the α particle, the velocity of recoil of the atom can be simply calculated. Taking the mass of the α particle as 4 and of the radium atom as 225, the velocity of recoil of the disintegrated radium atom, for example, is $1/55 \times 1{\cdot}56 \times 10^9$ or $2{\cdot}8 \times 10^7$ cms. per second. The heating effect resulting from this recoil is thus only about 2 per cent. of that due to the α particle.

Assuming that each α particle carries a single ionic charge of $1{\cdot}13 \times 10^{-20}$ electromagnetic units, the number of α particles which must be expelled per second from one gram of radium in order to account entirely for the heating effect is $3{\cdot}2 \times 10^{11}$. The experimental number is $2{\cdot}5 \times 10^{11}$. If it is assumed that the α particle carries twice the usual ionic charge, each of these numbers is reduced by one half.

It is of interest to calculate the distribution of the heating effect of radium in equilibrium amongst the various α ray products. The theoretical percentages of the total heating effect are given in column 1. These are calculated from the known energy of the α particles expelled from each product. The observed percentages are deduced from the experimental numbers and curves given by Rutherford and Barnes (Phil. Mag. Feb. 1904).

Product.	Calculated heating effect.		Observed heating effect.
Radium	19·2		23
Emanation	22·7	47·2	45
Radium A	24·5		
Radium C	33·6		32

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The observed heating effects of the emanation and radium A are given together, as it is very difficult experimentally to determine their separate effects. It will be seen that there is a substantial agreement between the calculated and observed values.

Connexion between the Velocity and Amount of Ionization produced by the α particle.

Bragg (Phil. Mag. Nov. 1905) has shown that the ionization produced by a single α particle increases with the distance from the source to nearly the end of its range, when the ionization falls off very abruptly. He has shown that the ionization produced by the α particle at a distance r cms. from the end of the path is inversely proportional to $\sqrt{r+c}$, where c is a constant equal to 1·33. In a previous paper (Phil. Mag. Aug. 1906) I have shown that the velocity of an α particle at a distance r cms. from the end of its range is proportional to $\sqrt{r+d}$, where d is a constant equal to 1·25. The close agreement between these two expressions shows that the ionization produced per unit path by the α particle is inversely proportional to its velocity. This is in agreement with the theoretical views of Bragg, who supposed that the rate of expenditure of energy of the α particle in ionization at any point is inversely proportional to the energy of motion which it possesses.

A comparison of the velocities of the α particles expelled from the various products of the radio-elements, and a discussion of the connexion that exists between the velocity of expulsion of the α particle and the character of the transformation will be given in a later paper.

I desire to express my thanks to Dr. Hahn and Dr. Levin for their assistance in the measurement of the numerous photographs obtained in this investigation.

Berkeley, California, July 20, 1906.

XLII. *Mass of the α particles from Thorium.* By E. Rutherford, *F.R.S., Macdonald Professor of Physics, McGill University, Montreal, and* O. Hahn, *Ph.D.**

[Plate V. figs. 1–3.]

THE present investigation, which has involved the determination by the photographic method of the deflexion of the α rays from the active deposit of thorium in a magnetic and electric field, has been rendered possible by

* Communicated by the Authors.

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the use of the highly active preparation of radiothorium, separated from the mineral thorianite by Dr. Hahn.

Radiothorium has been shown by Hahn to produce thorium X, the thorium emanation, and the characteristic active deposit of thorium. It is a product of thorium, probably intermediate between thorium itself and thorium X. Radiothorium consequently exhibits to a very intense degree the characteristic radioactive properties of ordinary commercial thorium.

In previous papers (Phil. Mag. June and July, 1906) Hahn showed from an examination of the ionization curves that the active deposit of thorium emitted two distinct types of rays, one of which had a range 5·0 cms. in air, and the other a range of 8·6 cms. It has not so far been found possible to separate from each other the two α ray products from which these rays are emitted. Since the first product of the active deposit, viz. thorium A, is rayless, these two α ray products have been called thorium B and thorium C. It is not so far possible to decide whether the α particles of range 8·6 cms. belong to the product B or C, but, for convenience, they will be ascribed to thorium C. The α particles of thorium B thus have a range in air of 5·0 cms.

Since the active deposit of thorium can be obtained in a very thin film on a fine wire, it forms a very convenient source of α rays for a determination of the deflexion of the α rays in a magnetic and electric field. Such an active wire emits two homogeneous types of rays which have ranges in air of 5·0 and 8·6 cms. respectively. Some preliminary results of the determination of the magnetic deflexion of the rays from thorium C have been given in a previous paper by Hahn (*loc. cit.*).

The apparatus employed to determine the magnetic deflexion of the rays is the same as that described in a paper of Rutherford (Phil. Mag. *ante*, pp. 350–1). It is consequently not necessary to describe it further here. The active wire was about 1·5 cm. long, and of diameter 0·43 mm. The photographic plate was in all experiments 4 cms. above the active wire and within 2·5 or 2 cms. above the slit. After placing the active wire in position, the apparatus was exhausted of air.

The magnetic field, which was kept constant, was reversed every hour, and an experiment usually occupied ten hours. In this time, the activity of the source has decayed to about half of its initial value. The experiments in all cases were comparative, the magnetic deflexion of the α

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rays from radium C under identical conditions serving as a standard.

For this purpose, the same wire was used as a source of rays in both the thorium and radium experiments. After the wire had lost most of its activity, it was made the negative electrode in a vessel containing a large quantity of the radium emanation. The active wire, coated with a thin film of radium C, was then placed in the same position as before in the deflexion apparatus and the experiment repeated. In this case, an exposure of the plate for two hours was sufficient to obtain well-marked photographs of the traces of the deflected pencil of rays.

In order to obtain a sufficiently intense source of rays for the thorium experiment, a wire about 1·5 cm. long was made the only negative electrode in a small closed vessel containing the highly emanating preparation of radiothorium. After two days' exposure, the activity of the wire had nearly reached a maximum. In the preliminary experiments, the activity produced on the wire was not very intense and the photographic impression produced in a ten hours' exposure was very weak. By careful attention to the construction of the vessel in which the wire was made active, and by further chemical treatment of the radiothorium preparation in order to make it highly emanating, it was found possible to obtain wires sufficiently active to give a clear impression on the photographic plate under the conditions of the experiment.

Magnetic Deflexion of the α rays from Thorium B and C.

We have seen that an examination of the ionization curves of the active deposit of thorium had disclosed the existence of two distinct α ray products, thorium B and C, with different ranges in air. This conclusion was confirmed by an examination of the magnetic deflexion of the pencil of rays from the active wire. Two distinct sets of α rays were found to be present which were unequally deflected in a magnetic field. Each product emitted homogeneous rays, but the α particles from thorium B had a lower velocity than those from thorium C and were consequently more deflected by the magnetic field than the latter. The difference in the amount of deflexion of the two sets of rays was not sufficient to obtain a complete separation of the photographic bands due to them, but the amount of deflexion of each pencil of rays could be deduced in the following manner.

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Let ABC, A′B′C′ (Pl. V. fig. 1, *a*) represent the deflected bands due to the α rays from the wire coated with the active deposit of thorium. Each of these deflected bands consists of two distinct parts. The inside portions AB, A′B′ are due to the α rays from thorium C, the outer portions BC, B′C′ to the α rays from thorium B. Since the rays from the latter produce less photographic effect than those from thorium C, the outer bands BC, B′C′ are much less clearly defined than the inner bands due to the rays from thorium C. Let EF (fig. 1, *b*) represent the bands obtained under identical conditions using the same wire coated with radium C. The width D of the bands E, F represents the normal width of the photographic trace due to a single homogeneous source of α rays.

Let d_1, d_2, d_3 represent twice the normal magnetic deflexions of the pencil of α rays emitted by thorium B, thorium C, and radium C respectively. The distances CC′, AA′, are measured on the photograph in the usual way.

Then

$$d_1 = \mathrm{CC'} - \mathrm{D}$$
$$d_2 = \mathrm{AA'} + \mathrm{D},$$
$$d_3 = \mathrm{EF},$$

where E and F are the centres of the radium bands. Now the amount of deflexion of the pencil of α rays is inversely proportional to the value $\frac{mu}{e}$, where m is the mass of the α particle, e its charge, and u its velocity. Knowing the value of $\frac{mu}{e}$ for the α rays from radium C under the conditions of the experiments, the value of $\frac{mu}{e}$ for the rays from thorium B and C can at once be determined. A number of experiments were made on the magnetic deflexions of the α rays from the active thorium wire when covered with different absorbing layers to cut down the velocity of the α rays. We have seen in a previous paper that the velocity of an α particle which has a range r cms. in air is proportional to $\sqrt{r+1{\cdot}25}$. This relation should hold for the α particles from thorium as well as from radium if the α particles in all cases have the same value of e/m. It is of interest to compare the observed deflexions of the α rays from thorium B and C with those to be expected from the known range of their α particles in air.

Experiment 1.—The magnetic deflexions of the α rays from thorium B and C were first determined with an unscreened

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source of radiation. The following relative values were observed:—

$$\frac{d_1}{d_3} = 1{\cdot}09\ ;\ \frac{d_2}{d_3} = {\cdot}91\ ;\ \frac{d_1}{d_2} = 1{\cdot}20.$$

Remembering that the ranges in air of the α rays from thorium B, thorium C, and radium C are 5·0 cms., 8·6 cms., and 7·06 cms. respectively, and that the amount of deflexion of a pencil of rays is inversely proportional to the velocity of the rays, we should expect the following values of the ratios:—

$$\frac{d_1}{d_3} = \sqrt{\frac{7{\cdot}06 + 1{\cdot}25}{5{\cdot}0 + 1{\cdot}25}} = 1{\cdot}15,$$

$$\frac{d_2}{d_3} = \sqrt{\frac{7{\cdot}06 + 1{\cdot}25}{8{\cdot}6 + 1{\cdot}25}} = {\cdot}92,$$

$$\frac{d_1}{d_2} = \sqrt{\frac{8{\cdot}6 + 1{\cdot}25}{5{\cdot}0 + 1{\cdot}25}} = 1{\cdot}255.$$

Taking into consideration the difficulty of determining the edge of the weak photographic impression due to the rays from thorium B, the relative velocities observed experimentally substantially agree with those deduced from the known ranges of the α particles. The α rays from thorium C are projected with a velocity about 9 per cent. greater than those from radium C, while the α particles from thorium B are projected with a velocity about 13 per cent. less than those from radium C.

Experiment 2.—The experiments were repeated with a thickness of aluminium over the active wires equivalent in stopping-power to about 2·5 cms. of air. A good photograph was obtained which showed clearly the bands due to the rays from thorium C, bounded on the edge by the weaker bands due to the rays from thorium B. The effect of covering both the thorium and radium wire with the screen is to bring out more clearly the difference in velocities of the three types of rays. The actual photographs obtained are reproduced in fig. 2 A & B. (Pl. V.) Fig. 2 A shows the thorium bands. The well-defined dark band due to the α rays from thorium C is seen to be bounded on the outer edge by a hazy band due to the α rays from thorium B. The original negative showed the two sets of rays very clearly, but much of the definition is lost in the reproduction, which is magnified about 1·4 times. It will be seen that the corresponding radium photograph (fig. 2 B) shows two clear well-defined bands.

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The results of the measurements are given below, using the same notation as before. The values of d_1, d_2, d_3 in this case refer to the deflexions observed for the rays after passing through the aluminium screen:—

$$\frac{d_1}{d_3} = 1{\cdot}32\,;\ \frac{d_2}{d_3} = {\cdot}85\,;\ \frac{d_1}{d_2} = 1{\cdot}55.$$

Remembering that the ranges of the three sets of rays are each reduced 2·5 cms. in their passage through the aluminium screen, the emerging pencils of rays from thorium B, thorium C, and radium C have ranges of 2·5 cms., 6·1 cms., and 4·56 cms. respectively. The ratios, calculated as before, are

$$\frac{d_1}{d_3} = 1{\cdot}245\,;\ \frac{d_2}{d_3} = {\cdot}89\,;\ \frac{d_1}{d_2} = 1{\cdot}38.$$

The calculated and experimental ratios are not in as good agreement as would be expected from the excellence of the photographs. It is not unlikely that a part of the difference is due to an accidental shift of the photographic plate in arranging the apparatus for the radium experiment.

Experiment 3.—A layer of mica equal in stopping-power to about 3·5 cms. of air was placed over the active wires. The photographic effect of the α rays from thorium B was in this case too weak for accurate measurement. The photographic intensity of an α particle appears to be approximately proportional to its range in air. The range of the α rays from thorium B in this experiment was only 1·5 cm., while the rays from thorium C had a range 5·1 cm. The photographic effect of the rays from thorium B was consequently less than one-third of the rays from thorium C. The observed value of $\frac{d_2}{d_3}$ was 1·12 while the ratio calculated from the ranges is 1·15.

Experiment 4.—A layer of aluminium equivalent in stopping-power to 1·5 cm. of air was placed on top of the mica. The interposed screens consequently corresponded to 5 cms. of air and completely cut off the rays from thorium B. The photographs obtained are shown in fig. 3, magnification about 1·4. Fig. 3 A shows the bands due to thorium C and fig. 3 B the bands due to radium C. The much greater deflexion of the rays from radium C is obvious from an inspection of the photographs. As we should expect, there

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is no blurring of the edges of the bands due to the rays from thorium B, such as is shown in fig. 2 A.

The observed ratio $\frac{d_2}{d_3} = \cdot 78$, while the calculated ratio is ·82.

Values of $\frac{mu}{e}$.

Considering the difficulty of accurately measuring up some of the weak photographs obtained with the thorium rays, the observed deflexions are found to agree fairly closely with those calculated from the known ranges of the α particles. The calculated ratios are probably more accurate than the experimental values, and the former will consequently be used to determine the value of $\frac{mu}{e}$ for the rays from thorium B and C. The value of $\frac{mu}{e}$ for the unscreened rays from radium C has been shown in a previous paper to be $4 \cdot 06 \times 10^5$.

The value of $\frac{mu}{e}$ for the rays from thorium B is consequently given by

$$\frac{mu}{e} = \sqrt{\frac{5 \cdot 0 + 1 \cdot 25}{7 \cdot 06 + 1 \cdot 25}} \times 4 \cdot 06 \,.\, 10^5 = 3 \cdot 52 \times 10^5.$$

The corresponding value for the rays from thorium C is given by

$$\frac{mu}{e} = \sqrt{\frac{8 \cdot 6 + 1 \cdot 25}{7 \cdot 06 + 1 \cdot 25}} \times 4 \cdot 06 \,.\, 10^5 = 4 \cdot 42 \times 10^5.$$

The value of $\frac{mu}{e}$ for the α rays from thorium C after their passage through the standard thickness of mica, equal to 3·5 cms. of air, is given by

$$\frac{mu}{e} = 3 \cdot 53 \times 10^5.$$

Electrostatic Deflexion of the α rays.

The same apparatus was used as in the radium experiments. The rays from the active wire passed through the standard mica screen and the base of the vessel. Two wires were used to obtain a measurable photographic effect, each wire

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being left two days in position in the base of the apparatus. The rays from both thorium B and C were present, but the photographic effect of the former was small compared with the latter, and could not be distinguished on the plate.

The electrostatic deflexion was obtained in two distinct experiments. In one case, the photographic plate was 3·94 cms. above the parallel plates and the P.D. applied was 337 volts; in the other, the distance of the photographic plate from the parallel plates was 10 cms. and the voltage 423·5. The formulæ employed in the calculations have already been given in the accompanying paper by Rutherford.

In the first experiment the value of $\frac{mu^2}{e}$ was found to be $6{\cdot}7 \times 10^{14}$, and in the second $7{\cdot}3 \times 10^{14}$. The mean value

$$\frac{mu^2}{e} = 7{\cdot}0 \times 10^{14}.$$

From the magnetic experiments,

$$\frac{mu}{e} = 3{\cdot}53 \times 10^{5}.$$

This gives the values

$$u = 1{\cdot}98 \times 10^{9} \text{ cms. per second.}$$

$$e/m = 5{\cdot}6 \times 10^{3}.$$

It has been shown in a previous paper that the value of e/m for the α particle from radium is $5{\cdot}07 \times 10^{3}$. Remembering that too high a value of e/m is to be expected when the electrostatic deflexion of the rays is determined from a weak photograph, the agreement between the values of e/m is sufficiently close to indicate that the α particle expelled from thorium C has the same mass as the α particle expelled from radium, and is identical with it.

The significance of this result has been discussed in the companion paper dealing with the mass of the α particle expelled from radium and actinium.

Berkeley, Cal.
July 30, 1906.

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XLIII. *On the Coefficient of Absorption of the β Rays from Uranium.* By J. Arnold Crowther, *B.A., Hutchinson Student of St. John's College, Cambridge, and Open Research Student of Emmanuel College, Cambridge*.*

Introduction.

THE β rays of radioactive substances consist of small negatively charged corpuscles travelling with a high velocity. On entering any material substance they are more or less rapidly absorbed, the intensity of the radiation decreasing, as the thickness of matter traversed is increased, very nearly in accordance with an exponential law. Thus if I_0 is the initial intensity of the radiation before entering the substance, the intensity I after passing through a thickness d of the material is given by the equation

$$I = I_0 e^{-\lambda d},$$

where λ is the coefficient of absorption of the given substance for the particular β radiation employed.

We may regard the absorption of the β rays by matter as being due to collisions between the swarm of corpuscles, constituting the rays, and the particles composing the substance. From the minute size of the corpuscles, however, and their high velocity we must consider them as colliding, not with the atom or molecule of the absorbing substance as a whole, but rather as penetrating the atom, and coming into collision with the individual corpuscles of which the atom is constructed†. The absorption of the β rays is thus due to collision between the β corpuscles and similar corpuscles in the atom of the absorbing medium.

On this view, therefore, the fundamental quantity in the absorption of the β radiation is the stopping-power of the individual corpuscles in the atom, and this quantity can be immediately deduced from the coefficient of absorption. For if ρ is the density of the absorbing medium, M its atomic weight, and n the number of corpuscles in the atom, then the number of corpuscles N in unit volume is given by the equation

$$N = \frac{n}{M} \cdot \rho.$$

But n/M is a constant, since the number of corpuscles in an atom is simply proportional to the atomic weight, and thus

$$N \propto \rho.$$

Hence the absorption per corpuscle is simply proportional to λ/ρ.

* Communicated by Prof. J. J. Thomson, F.R.S.

† J. J. Thomson, 'Conduction of Electricity through Gases,' p. 312 (1903).

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If the stopping-power of the corpuscle were the same for all substances, and independent of the mode of grouping of the corpuscles in the atom, we should have the ratio of the coefficient of absorption to the density of the absorbing medium a constant for all substances.

This is the "density law" of absorption, and was first propounded by Lenard* as the result of some experiments on the absorption of the cathode rays. Although these experiments included substances ranging in density from $3{\cdot}6 \times 10^{-7}$ for hydrogen at 3 mm. pressure, to 19·3 for gold, and gave values of λ varying between ·00149 for the former and 55,600 for the latter, the extreme limits for the ratio λ/ρ were only 2070 and 5610; thus showing that although the ratio might not be absolutely constant, it was at any rate a relation of fundamental importance.

Similar experiments were performed by Strutt † using the β rays from radium, with similar results, the values obtained for λ/ρ varying from 3·84 for card, to 7·34 for platinum. The β radiation of radium, however, is complex, and consists of rays travelling with very different velocities. As each component ray has its own coefficient of absorption, the absorption of the bundle as a whole is not represented by any simple law, and the values obtained were therefore somewhat indeterminate.

Further experiments were therefore performed by Rutherford ‡ using uranium as the source of radiation. The β rays from this substance are practically homogeneous, and are therefore not open to the same objection as the radium rays. The results were as follows:—

Substance.	λ/ρ.
Glass	5·7
Mica	5·1
Ebonite	5·7
Wood	5·4
Cardboard	5·3
Iron	5·6
Aluminium	5·4
Copper	7·0
Silver	7·1
Lead	10·8
Tin	13·2

* Lenard, Wied. *Ann.* lvi. p. 255 (1895).
† Strutt, 'Nature,' lxi. p. 539 (1900).
‡ Rutherford, 'Radioactivity,' p. 114 (1904).

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An examination of this table will show that though the ratio λ/ρ is not subject to large variations, yet there is a decided alteration as we pass from one element to another; and that consequently the stopping-power of a corpuscle is not the same in atoms of different substances.

On theoretical grounds also we might expect the nature of the arrangement of the corpuscles within the atom to have some effect upon the absorbing power. Thus Prof. J. J. Thomson * has shown that the absorption of an atom in which all the corpuscles were rigidly connected, would be twice that of an atom containing the same number of freely moving corpuscles. It is also quite possible that the presence of so many charged corpuscles within the space of the atom may produce a constraint in the æther analogous to specific inductive capacity. This would affect the magnitude of the forces between the fixed and the β corpuscles, and the extent of the alteration would depend again upon the number and arrangement of the corpuscles within the atom.

Thus differences in the value of the ratio λ/ρ for different elements were to be expected, and it was considered desirable to make an extended series of observations upon as many elements as possible, with the object of investigating these differences and their relation to the atomic weight.

Experimental.

Uranium oxide was used as the source of the β radiation, the homogeneity of the rays and their fairly penetrating character more than compensating for the smallness of their amount as compared with the radiation from preparations of radium. The relative intensity of the radiation before and after passing through the material under investigation was measured by the relative ionization produced in the air of a test-vessel. This is a measure of the energy of the rays, and depends not only upon the number of corpuscles but also upon their velocity. Lenard, however, has shown that there is no appreciable change in the velocity of a β corpuscle during its passage through matter, at any rate for small thicknesses of material; and this result is supported by results to be described later in this paper. It has been shown theoretically † (and the result is borne out by a comparison of the values obtained for the absorption-coefficient for rays of different velocities) that the penetrating-power of the β rays varies as the fourth power of their velocity. Thus if there were any appreciable diminution in the velocity of the rays

* Phil. Mag. vol. xi. p. 779 (June 1906).

† J. J. Thomson, 'Conduction of Electricity through Gases,' p. 315.

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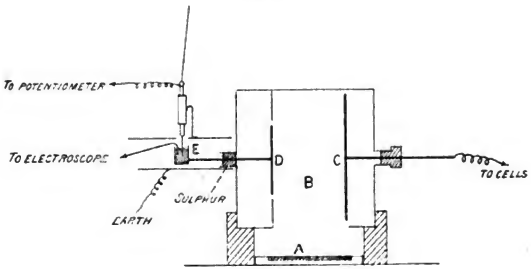
on passing through matter, there should be a marked increase in the coefficient of absorption as the thickness of matter traversed was increased. No such effect was observed, and hence we may assume that, at any rate until the energy of the rays is greatly reduced, there is no appreciable change in their velocity.

As the plan of the experiments included the investigation of elements, many of which could only be obtained in small quantities, the apparatus had to be kept quite small. Its final form was as follows :—The uranium oxide was contained in a depression, 2 mms. deep and 4·5 cms. square, in a lead plate A (fig. 1), and covered in with a sheet of aluminium, 0·1 mm. in thickness, in order to cut off all the α radiation.

Fig. 1.



The rays then passed up into the chamber B, ionizing the air between two plates C and D, 4·2 cms. apart. C was connected to a battery of small storage-cells, and raised to a sufficiently high potential (360 volts) to produce the saturation current through the vessel when the radiation was a maximum. D, 4 cms. in diameter, and surrounded by a guard-ring, was connected to a Wilson inclined gold-leaf electroscope through a mercury key E, which could be operated from a distance and by means of which the gold-leaf system could be earthed, left insulated, or charged to a definite potential. The insulations in the gold-leaf system were of sulphur, and the whole was screened from induction effects by earthed metal tubes. In its sensitive position the electroscope was capable of giving 30 scale-divisions per volt, and could be read to one-fifth of a division.

The rate of leak of the electroscope measured the ionization

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in the chamber B, and this was in turn a measure of the intensity of the radiation entering the chamber, and thus the ratio of the leak of the electroscope when the plate A was uncovered (except for the thin aluminium plate used to cut off the α rays) to the leak when the plate A was covered with a thin sheet or layer of the substance under investigation, enabled the coefficient of absorption, λ, to be calculated, when the thickness of the absorbing material was known.

In almost every case experiments were made with several different thicknesses of the substance under observation, in order to ascertain whether the absorption of the substance was a simple exponential function of the thickness. The logarithms of the intensities of radiation were plotted against the thicknesses of material through which the radiation had passed, and specimen curves so obtained are shown in fig. 2. In the large majority of cases the curve was a straight line, as illustrated by the curves for aluminium and potassium; thus showing that for these substances, within the limits of experimental error, the law of absorption was expressed by the relation

$$I/I_o = \epsilon^{-\lambda d}.$$

In the case of a few of the heavier metals, namely, lead, gold, platinum, tin, silver, and palladium, there was a more or less marked departure from the exponential law, for very small thicknesses of material, as illustrated by the curve for tinfoil (fig. 2, p. 384). McClelland * has shown that this would be the case if an appreciable amount of secondary radiation were produced by the action of the incident β rays; and experiments made later upon the above metals showed that there was a considerable amount of return radiation from them. We shall return to this question later. For the present we may state that the value of λ employed is that calculated from the straight portion of the curve after λ has become constant.

Whenever possible, the element was procured in the form of thin foil. Many, however, were only obtainable as powders. The lighter powders were formed into layers by spreading them out evenly on shallow cardboard trays of different depths; the heavier ones were finely powdered, suspended in chloroform, and allowed to deposit on thin mica sheets. The absorption due to the tray or sheet was of course measured and allowed for.

The thickness of the layer was in every case measured by finding the weight of a known area. (In the case of the

* Scientific Trans. Roy. Soc. Dublin, 1906.

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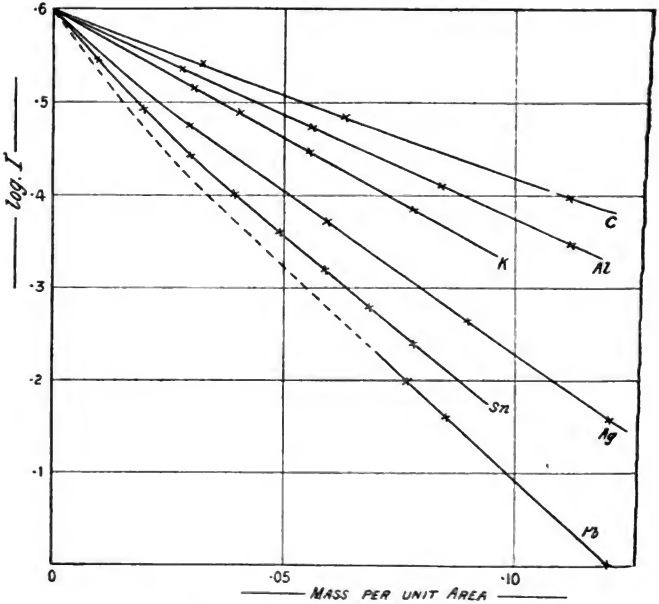
powders the area was that of the tray or mica sheet.) This method had the advantage of giving the ratio λ/ρ directly.

Fig. 2.—Absorption-Curves.



For if x is the thickness of the layer,

ω its mass, and

α its area,

then $\omega = \alpha\rho x$,

$$\therefore \quad x = \frac{\omega}{\alpha \cdot \rho}$$

and $$I/I_0 = \epsilon^{-\lambda x} = \epsilon^{-\frac{\lambda}{\rho} \cdot \frac{\omega}{\alpha}}.$$

An accurate knowledge of the density of the particular specimen of the element used was thus rendered unnecessary. For convenience in comparison, thickness in fig. 2 is represented by mass per unit area.

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Absorption of the Elements.

Over thirty elements have now been determined, with the results shown in the following table :—

Substance.	At. Wt.	λ/ρ.	Substance.	At. Wt.	λ/ρ.
Boron	11	4·65	Arsenic	75	8·2
Carbon	12	4·4	Selenium	79	8·65
Sodium	23	4·95	[Strontium]	87·5	8·5
Magnesium	24·4	5·1	Zirconium	90·7	8·3
Aluminium	27	5·26	Palladium	106	8·0
Silicon	28	5·5	Silver	108	8·3
Phosphorus	31	6·1	Tin	118	9·46
Sulphur	32	6·6	Antimony	120	9·8
Potassium	39	6·53	Tellurium	126	10·8
Calcium	40	6·47	Iodine	126	10·8
Titanium	48	6·2	[Barium]	137	8·8
Chromium	52	6·25	Platinum	195	9·4
Iron	56	6·4	Gold	197	9·5
Cobalt	59	6·48	Lead	207	10·8
Copper	63·3	6·8	[Uranium]	240	10·1
Zinc	65·5	6·95			

The whole of the above values were obtained from the elements themselves, except in the case of strontium, barium, and uranium, values for which were calculated from the absorption of their oxides, in a way which will be subsequently described. A comparison of these results with those of Rutherford shows on the whole a satisfactory agreement, except in the case of tin. The discrepancy here is probably due to the fact that the absorption for tin, as explained above, does not follow a simple exponential law. A calculation of λ from the first few points on the curve for tinfoil gives a value very near that of Rutherford.

If now we plot the values of λ/ρ given in the above table against the atomic weights, we obtain the curve shown in fig. 3 (p. 386). This curve presents several points of interest. It will be seen at once that the elements arrange themselves very definitely along a series of similar curves. Moreover, these curves correspond closely in extent with the divisions of the Periodic Classification of Chemistry. Boron and carbon stand alone as the only representatives of the first short period. The second short period is represented by a distinct portion of the curve rising from sodium to sulphur. Potassium, the first member of the first long chemical period, is the first point on a fresh portion of the curve, which extends through the whole of the first long period to selenium. Another distinct section of the curve runs from strontium to iodine, thus

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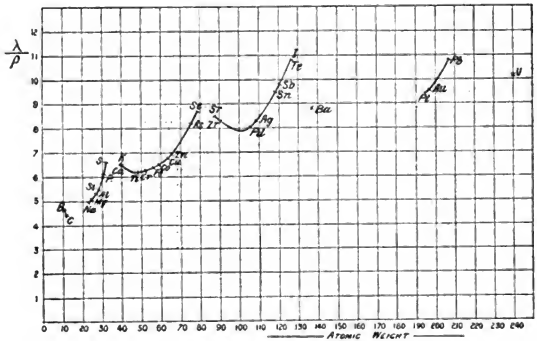
coinciding with the second long period; while barium, obviously on still another division of the curve, is the only

Fig. 3.



representative of the third. The fourth long period is somewhat poorly represented by platinum, gold, and lead, but sufficient is shown of the curve to prove that this section is distinct from and similar to the previous sections. Uranium is the only representative of the fifth long period, and a fifth portion of the curve. It is interesting to note in passing, that the absorption by uranium of the uranium β rays is in no way exceptional, the coefficient of absorption being quite what might be expected from an element of its atomic weight.

Again, the different sections of the curve are very similar in appearance, if we exclude the first short period, which is also somewhat exceptional in character chemically. Each commences with a nearly horizontal portion, and then rises sharply up to a maximum. Practically the whole of the increase occurs in the second half of the long periods.

It will be noticed also that similar elements occur on similar portions of the curve. Thus the alkali and alkali earth metals come at the commencement of the different sections; the metals of the eighth group come in similar positions near the minima; while the sulphur, selenium, and tellurium group, and the halogens form the maxima.

In brief, it appears evident that the ratio λ/ρ, measuring the absorption per corpuscle, is not a constant, but is a periodic function of the atomic weight; the periods corresponding exactly with those of the chemical classification.

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In addition to this periodic variation there is also an increase in the value of the ratio λ/ρ with increase of atomic weight. This is clearly brought out by considering groups of similar elements. Thus we have:—

	$\frac{\lambda}{\rho}$		$\frac{\lambda}{\rho}$
Phosphorus.....	6·1	Copper.....	6·8
Arsenic	8·2	Silver	8·3
Antimony	9·8	Gold	9·5

	$\frac{\lambda}{\rho}$
Sulphur	6·6
Selenium.....	8·65
Tellurium	10·8

It will thus be seen that passing down any group from elements of lower to elements of higher atomic weight, there is a continuous increase in the value of λ/ρ. The rate of increase, however, seems to fall off as the atomic weight increases, and is distinctly less rapid after passing the second long period.

Allotropy.

Several elements occur in two or more distinct forms, differing among themselves in density, appearance, and other physical and chemical properties. It was thought that these different varieties might give slightly different values for the ratio λ/ρ.

Accordingly a careful comparison was made between pure graphite ($\rho = 2·25$) and pure charcoal ($\rho = 1·85$) prepared from grape-sugar; but no difference whatever could be detected between the values of λ/ρ in the two cases. Similar experiments were made with the different modifications of sulphur, with a similar result. It appears therefore that, whatever the nature of allotropy may be, it does not affect the absorbing power of the atom.

Absorption of Compounds.

The variation of the absorption per corpuscle with atomic weight having been established, it remained to decide whether the absorption depended upon the state of molecular combination; or, in other words, if the forces which link together the atoms of a compound influence in any way the absorption of the constituent atoms. A considerable number of compounds have been tested with a view to determining this point.

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If we assume that combination does not influence the absorption of the combining atoms, we can easily calculate the value of the ratio λ/ρ for a compound from the knowledge of its value for the constituent elements. A comparison between the values so calculated and those obtained by direct experiment will then enable us to test the truth of our assumption.

Let X and Y be two elements of atomic weight m_1 and m_2 respectively. Since the number of corpuscles in the atom is proportional to the atomic weight, the number of corpuscles in the atoms of X and Y respectively will be km_1 and km_2. Since the absorption per corpuscle is, as previously explained, equal to the ratio λ/ρ, the absorption per atom of the element X will be $\frac{\lambda_1}{\rho_1} km_1$, and that of Y, $\frac{\lambda_2}{\rho_2} km_2$.

If, now, the compound is represented by the formula X_aY_b, the absorption per molecule will on our assumption be equal to the sum of the absorptions of the individual atoms, and must therefore equal

$$a\frac{\lambda_1}{\rho_1} km_1 + b\frac{\lambda_2}{\rho_2} km_2.$$

But the absorption per molecule of a compound will also equal the product of the average absorption per corpuscle multiplied by the number of corpuscles in the molecule; that is, it will equal $\frac{\lambda}{\rho}(akm_1 + bkm_2)$, where λ and ρ are the values for the compound. Thus equating the two expressions we have

$$\frac{\lambda}{\rho}_{\text{(compound)}} = \frac{a \cdot \frac{\lambda_1}{\rho_1} m_1 + b \cdot \frac{\lambda_2}{\rho_2} m_2}{am_1 + bm_2},$$

which is known when the values of the absorption per corpuscle for the component elements are known.

The following table shows the nature of the agreement between the values calculated from the above formula and the direct experimental values, for fourteen different compounds.

The value of the absorption of oxygen could not be directly determined. The value used in calculating the results in the above table was calculated from the observed absorption of the different oxides, by means of the formula obtained above.

On the whole, the agreement between the observed and the

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Substance.	Formula.	λ/ρ.	
		Calculated.	Observed.
Magnesium Oxide.....	MgO	5·6	5·5
Aluminium Oxide.....	Al_2O_3	5·8	5·9
Ferric Oxide.....	Fe_2O_3	6·4	6·4
Zinc Oxide.....	ZnO	6·8	7·3
Cuprous Oxide	Cu_2O	6·8	7·1
Cupric Oxide.....	CuO	6·7	6·9
Arsenious Oxide	As_2O_3	7·7	7·5
Ferrous Sulphide	FeS	6·7	6·5
Zinc Sulphide	ZnS	6·8	6·8
Arsenious Sulphide ...	As_2S_3	7·5	7·5
Stannous Sulphide ...	SnS	8·9	8·9
Antimony Sulphide ...	Sb_2S_3	8·6	8·5
Cuprous Iodide.....	Cu_2I_2	9·4	9·1
Lead Iodide	PbI_2	10·8	10·8

calculated values is good; in the majority of cases the difference does not exceed the probable experimental error. Hence we may fairly assume that the absorption per molecule is equal to the sum of the absorptions of the constituent atoms, or in other words that the absorption per corpuscle of an element is not altered when the element enters into a state of combination.

This result was used to calculate the values of the ratio λ/ρ for strontium, barium, and uranium. In these cases, where the element to be determined forms a large proportion of the compound, the proportional error introduced is not great. It is clear from the formula, however, that if an element forms only a small proportion by weight of the compound, its absorption will appear as the small difference between two large quantities, and the proportionate error will be greatly increased. Hence it was not possible to use this method to calculate values for the elements of small atomic weight.

Secondary Radiation.

As mentioned before, McClelland* has shown that even with homogeneous primary β rays there would be a departure from the simple exponential law of absorption if any appreciable amount of secondary radiation were produced in the absorbing substance. He also showed that using radium rays the secondary radiation was very considerable. However, as previously explained, in the case of the large majority

* McClelland, Scientific Trans. Roy. Soc. Dublin, 1906.

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of substances whose absorption was measured no deviation from the simple exponential law could be detected. This seemed to show that the secondary radiation produced by the β rays from uranium was much less in amount than that set up by the radium rays. It was considered desirable to have some direct evidence upon this point.

Owing to the much smaller intensity of the radiation from uranium as compared with that from radium, it was impossible to use any method similar to that of McClelland. The method employed consisted in measuring the ionization between a charged plate and a parallel wire-gauze electrode, due to the uranium radiation; and finding the increase in the ionization when the gauze was backed by a thick plate of the metal under observation. The first reading was a measure of the intensity of the incident radiation I_0; the second was proportional to the sum of the incident and the return radiation, that is to I_0+R, if R is the amount of radiation sent back from the plate. From these two readings therefore we can calculate the value of R/I_0.

The uranium oxide was contained in a circular depression 2 mms. deep and 4 cms. in diameter, in the centre of a plate A, 8 cms. in diameter, which was kept at a high potential by means of a battery of small storage-cells. The uranium was covered in as before with thin aluminium to cut off all α radiation. The gauze electrode, 8 cms. in diameter, was placed parallel to the plate A, and connected to the electroscope through a mercury key, in the same manner as in the absorption experiments.

A small amount of primary radiation may escape sideways without striking the upper plate. This, though reckoned in the incident radiation, does not in reality add its quota to the secondary radiation. However, by placing the electrodes sufficiently close together, we can make this error as small as we please. The measurements were actually made with the plates only 5 mms. apart, and it was found that there was very little alteration in the value obtained for the ratio R/I_0 even if the distance between the plates was increased to twice that amount.

The results obtained showed that, as expected from the absorption-curves, the return radiation was much smaller than that obtained by McClelland with radium rays, and only those substances whose absorption-curves departed from the exponential law gave very appreciable amounts. The following brief table gives a comparison of the results obtained by this method for uranium rays, with McClelland's results for radium.

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Substance.	$R/I_0 \times 100$.	
	Radium.	Uranium.
Lead	49	15
Gold	...	14·5
Platinum	47	14·5
Tin	42·5	12
Copper	36	4
Aluminium	24	very small.

It will be at once seen that the values for uranium are considerably smaller than those for radium rays, and the ratio also seems to decrease more rapidly with the atomic weight for the rays from the former substance.

These differences may possibly be due in part to the presence in the radium radiation of comparatively slowly moving β rays. Becquerel has shown that the amount of secondary radiation produced decreases rapidly as the velocity of the primary rays is increased. It seems possible therefore that a large proportion of the secondary radiation due to the radium rays may be set up by the more slowly moving portions of the rays, while the more penetrating portion corresponding to the uranium rays in velocity may produce comparatively little.

Whatever the cause, both the absorption-curves and the direct experiments seem to show that the secondary radiation set up by the uranium rays is distinctly less in amount than that produced by radium radiation.

No attempt has been made to calculate what McClelland has called the true coefficient of absorption for the rays.

McClelland* has shown that, if secondary radiation is set up, the true coefficient of absorption μ of the rays is connected with the coefficient λ as previously defined, by the equation

$$\lambda = \mu\sqrt{1-k},$$

where k is the proportion of absorbed energy which is restored to the stream of rays as secondary radiation. He further shows that this quantity k is connected with the total return

* McClelland, *loc. cit.*

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radiation from a thick plate of the substance by the relation

$$R/I_0 = \frac{1-\frac{1}{2}k-\sqrt{1-k}}{\frac{1}{2}k},$$

or if we denote R/I_0 by the letter p

$$k = \frac{4p}{(p+1)^2}.$$

We are thus enabled to calculate μ when p and λ are known.

The secondary radiation in the case of uranium rays was, however, so small, that the errors of observation were so great as to outweigh any theoretical advantage there might possibly be in the use of the "true" coefficient μ, instead of the coefficient λ. The return radiation R was obtained as the difference of two quantities, I_0 and I_0+R, which in most cases were nearly equal. The proportionate error in R therefore was very great. Thus even in the case of lead, an error of only 1 per cent. in measuring the intensity of I_0, or I_0+R, would produce an error of 7 per cent. in the value of p, while in the case of copper the error would be as much as 25 per cent. As, owing to the smallness of the radiation from uranium, it was certainly not possible to ensure an accuracy greater than 1 per cent. in the measurement of the intensities of the rays, it was considered preferable to retain the directly measured coefficient λ.

No great amount of error is introduced in this way, and the general results of the experiment are quite unaltered. In the majority of cases the secondary radiation was almost inappreciable, and λ therefore very nearly equal to μ. Substituting the values of p actually obtained, we have for aluminium, μ is equal to λ; for copper, μ equals $1{\cdot}08\,\lambda$; while for tin, gold, platinum, and lead, which show the maximum effect, μ is equal to $1{\cdot}3\,\lambda$. The effect, moreover, is roughly proportional to the atomic weight.

The effect of substituting μ for λ in the curve of fig. 3 would therefore not affect the general appearance of the curve, nor the general results deduced therefrom. Its only effect would be to slightly increase the slope of the later portions, and thus somewhat increase the rate of increase of the absorption with atomic weight.

In conclusion I wish to express my best thanks to Prof. J. J. Thomson for his kindness in suggesting the subject of this research, and also for much kindly interest during the course of the experiments.

Cavendish Laboratory, Cambridge,
August 3rd, 1906.

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XLIV. *Some New Methods for Separating Uranium X from Uranium.* By RICHARD B. MOORE, *B.Sc., Professor of Chemistry, Butler College, Indianapolis, U.S.A., and* HERMAN SCHLUNDT, *Ph.D., Assistant Professor of Physical Chemistry, University of Missouri, U.S.A.**

IN 1900, by adding ammonium carbonate to solutions of uranium salts until the precipitate which first formed redissolved, Sir William Crookes† was able to separate that portion of the uranium salt which was active to a photographic plate. He showed that the minute undissolved residue, which was mostly due to impurities in the salt, was one hundred times more active, weight for weight, than the uranium from which it had been separated. He also showed that a partial separation could be made by dissolving crystallized uranium nitrate in ether. Under such conditions the uranium divides itself between the ether and water in unequal proportions. That portion in the water layer was intensely active to the photographic plate, whilst that in the ether, although its activity had been only slightly reduced when measured by the electrical method, was practically inactive photographically. The explanation for these results was given by Soddy‡, and by Rutherford and Grier§, who showed that a radio-active constituent, to which Crookes gave the name uranium X, had been separated by the above processes from the parent uranium, and that this Ur X gave rise to all of the β rays but to none of the α rays.

Becquerel‖ obtained similar results by another method. He found that when barium sulphate was precipitated in a uranium nitrate solution a portion of the Ur X was carried down with the precipitate. By a number of precipitations the uranium ultimately became almost inactive photographically.

Recently we have confirmed the above methods of separation of Ur X from radium, and in addition have obtained several new methods for making the separation, which give still better results. The details of the experiments follow.

Five grams of crystallized uranium nitrate (Kahlbaum's best ¶) was dissolved in about 60 c.c. of acetone. The solution was slightly cloudy, probably in consequence of the presence of

* Communicated by the Authors.
† Proc. Roy. Soc. lxvi. p. 409 (1900).
‡ Trans. Chem. Soc. lxxxi. p. 460 (1902).
§ Phil. Mag. Sept. 1902.
‖ *C. R.* cxxxi. p. 137 (1900); cxxxiii. p. 977 (1901).
¶ McCoy has shown that this does not contain radium. Phil. Mag. Jan. 1906.

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a small amount of basic nitrate. This residue was filtered off, washed three times with acetone, and tested by the photographic method. It was found to be exceedingly active, an excellent radiograph being obtained in sixty hours. On testing the uranium nitrate in the filtrate it proved to be only slightly active to a photographic plate. Several samples of uranium nitrate of different degrees of purity were tested in the same way. Generally speaking, the larger the amount of insoluble residue the better the separation. If, after the insoluble residue is filtered off, a small amount of moist freshly precipitated ferric hydroxide be stirred into the clear filtrate, the uranium nitrate is practically freed from Ur X. The ratio of the amounts of Ur X contained in the insoluble residue and in the ferric hydroxide varies to a certain extent with the sample of uranium nitrate used. In one experiment the ratio between the activities, as measured by a Dolezalek quadrant-electrometer and the usual testing vessel, in the insoluble residue and in the ferric hydroxide, was as 9 to 11. In a similar experiment the insoluble residue was more active, the ratio being almost 2 to 1.

Another 5 gram sample of the nitrate was dissolved in acetone, and, without filtering, moist ferric hydroxide was stirred into the liquid. On filtering the solution and washing the residue with acetone, the latter was found to contain all of the β activity of the original sample, the uranium obtained from the filtrate giving no effect on a photographic plate after an exposure of five days.

Ur X can be separated from uranium in the same manner by using the following solvents instead of acetone:—Methyl acetate, ethyl acetate, methyl, ethyl, and amyl alcohols, and aceto-acetic-ether. In all cases the separated Ur X was not only recognized by the photographic method, but the curves of decay were also taken by the electrical method, the β activity decaying to half value in about 22 days. The same variation in the amount of activity removed, with the sample of nitrate used, noticed in the case of acetone, was also found to hold good with these solvents. In no case was all of the Ur X removed by filtering off the small residue which remained insoluble in each liquid. The addition of moist ferric hydroxide, or in the case of a liquid immiscible with water, dry barium sulphate, completes the separation. The barium sulphate, however, does not answer as well as the ferric hydroxide.

The Ur X, no matter which solvent was used to make the separation, always gave off α as well as β rays. Rutherford* and Grier found that in the separation by means of ammonium

* Phil. Mag. Sept. 1902.

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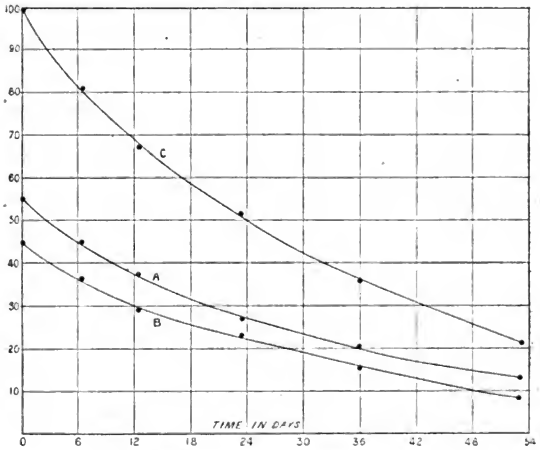
carbonate the α rays constituted about one-third of the total activity at the start. It apparently has been assumed that this small amount of α activity belongs to traces of uranium, for Ur X is generally accredited with giving only β rays. We found that in the Ur X separated by the above methods, not only did the β radiation decay according to an exponential law, falling to half value in about 22 days, but the α radiation also decayed and at the same rate. With acetone the proportion of α activity was usually about 50 per cent. of the total activity, although the amount depended to a certain extent upon the sample of nitrate used, and the manner in which the residue to be tested was spread upon the plate. With the other solvents the ratio varied with the solvent and also with the sample of nitrate used.

Fig. 1.



A = Decay curve of α radiation.
B = " " β "
C = " " total activity.

The best curves showing the relation between the rates of decay of the α and β radiations were obtained by first filtering off from the insoluble residue and then stirring in a little ferric hydroxide. By washing with the solvent the hydroxide could be practically freed from uranium, the presence of which would affect the decay rate of the α radiation. Fig. 1 represents the curves of decay obtained in this manner

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by means of acetone. The β activity was measured by covering the sample with two thicknesses of paper which had previously been tested and shown to cut off all α radiation. The activity due to the α rays was obtained by subtracting the β activity from the total activity.

The separation by means of ammonium carbonate does not work well unless there is a considerable amount of undissolved impurity. The presence of ferric hydroxide, either added to or precipitated in the solution, materially aids the separation. The results obtained by us with ammonium carbonate, however, were not nearly so satisfactory as those with acetone and the other solvents mentioned.

In order to see whether the decay curves of Ur X, obtained by means of ammonium carbonate, were similar to those of Ur X obtained by means of acetone &c., the following experiment was performed:—12 grms. of Kahlbaum's uranium nitrate was dissolved in 60 c.c. of water. A few drops of dilute ferric chloride solution were then added, followed by ammonium carbonate solution in excess. After filtering, the precipitate was washed with ammonium carbonate solution. It was then redissolved in dilute nitric acid, and reprecipitated with excess of ammonium carbonate in order to get rid of traces of uranium. A few drops of ferric chloride were again added to the first uranium filtrate, and the precipitate thus obtained filtered off and washed with ammonium carbonate. The filter-papers with their precipitates were then incinerated together, and the ferric oxide and ash ground up in alcohol and spread in a very thin layer on a metal tray. The alcohol was burnt off, and after cooling the total activity, as well as that due to the α and β radiations, was measured at intervals of several days in the manner described above. The three curves representing these activities fell to half value in from 21 to 22 days, and were in every way comparable to those obtained by means of acetone.

We have thus shown, first, that Ur X may be separated from uranium by means of a number of solvents in which Ur X is insoluble, these new methods being more satisfactory than those already in use, and secondly that Ur X emits not only β but also α rays.

Butler College, Indianapolis, Ind.
University of Missouri, Columbia, Mo.

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XLV. *Chemical and Electrical Changes induced by Ultra-violet Light.* *By* Sir WILLIAM RAMSAY, *K.C.B., F.R.S., and* J. F. SPENCER, *M.Sc., Ph.D.**

[Plates VI. & VII.]

Theoretical Introduction.

IN order to interpret the experiments of which an account will be given in the following pages, some hypotheses are necessary. The now almost universally acknowledged theory of corpuscles or electrons must of necessity be adopted. It is, however, immaterial, so far as the present work is considered, whether or no it be assumed that what is commonly termed "matter" consists wholly of congeries of electrons. It will prove sufficient for the purpose if the existence of atoms of "matter" be postulated to which electrons have the power of attaching themselves, or of entering into combination. This suggestion was first made by Nernst, we believe, in a lecture delivered before the Göttingen Academy. A positive ion, according to Nernst, may be conceived as an atom of an element which has lost one or more electrons, according to its valency; a negative ion, as an atom which has gained one or more electrons. Thus Ag· (where · means a + charge) is an ion of silver, *i. e.*, an atom of silver from which an electron has escaped; and Cl′ (where ′ denotes a − charge) as an atom of a "stuff" which we may term chlorine, but which becomes a chlorion by combining with an electron. The equation $AgNO_3Aq + NaClAq = AgCl + NaNO_3Aq$ may be written thus, where ⊖ signifies an electron:—$AgAq + NO_3\ominus Aq + NaAq + Cl\ominus Aq = AgCl + NaAq + NO_3\ominus Aq$. But it will here be noticed that AgCl is written as if electrically neutral. What has become of the electron attached to the chlorine? Is it not associated with the AgCl molecules? When metallic sodium and gaseous chlorine combine to form sodium chloride, the distinctive properties of each element disappear. It is of course open to say that much energy has been lost. True; but that is no "explanation" of the change of character of both elements: it is merely a concomitant; no doubt of the greatest importance, but unless we endow "energy" with a personality, a quasi-substantiality, and regard what we term "sodium" and "chlorine" as compounds of energy with "chlorine-stuff" and "sodium-stuff" (which is undoubtedly true in a sense) we fail to secure an "explanation." For the word explanation involves a mental picture in mechanical terms;

* Communicated by the Authors.

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and the doctrine of energetics, strictly applied, as for instance by Ostwald, ignores the "mechanical" nature of energy, and refuses to picture it. Hence, at present at least, almost all physicists and chemists are agreed rather to assign a mechanical or depictable nature to the electron, and to attempt an explanation of natural phenomena by its means. It may be remarked parenthetically that such considerations closely touch metaphysical speculations; they differ, however, inasmuch as they afford a mental picture and a partial explanation of what is actually observed. It is obvious to all, then, that by interaction of atomic sodium and molecular chlorine, common salt is produced. And it is now generally acknowledged that when dissolved in water, the salt is "ionized," that is the sodium atom has acquired a positive charge, or in terms of electrons, has lost an electron, while the chlorine atom, with its negative charge, is now a chlorion, viz., an atom of chlorine which has gained an electron. But between the physical properties of salt and of its solution there is merely a quantitative difference. The refractive index of salt is lessened by admixture with water; but a solution of salt has little if any difference in refractive index from the mean of the indices of the mixture of salt and water, taken in the proportion in which they are present in solution. Compounds which show absorption spectra show the same, or nearly the same spectra in solution: the specific heats of solutions do not greatly differ from that of the mean of the constituents, and so with other physical properties. But most of the properties of compounds differ radically from those of the elements which yield them. It appears probable, therefore, that a molecule of NaCl has been formed from 2Na and Cl_2 by displacement of an electron thus:—$2Na\ominus + Cl_2 = 2NaCl\ominus$; the electron is now more intimately attached to the chlorine atom; and if opportunity be given, by the presence of mobile molecules of water, these ions will part from each other into $2NaAq + 2Cl\ominus Aq$. It is not necessary here to discuss the protective nature of the water and the possible combination between the ions given above and molecules of the solvent.

It will somewhat aid these conceptions if the action of a simple galvanic cell be considered. Suppose a plate of zinc and a plate of copper dipping in dilute hydrochloric acid. When connected by a wire the zinc "dissolves as chloride," and a current passes through the wire. It has been suggested by Nernst, with useful results, that the zinc possesses a higher "solution pressure" than the copper. That the phenomenon can be "explained" on this hypothesis is obvious. But it conduces to clearness if we regard the sequence of events in

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the following way:—The zinc plate consists of atoms of zinc; each of these consists of "zinc stuff" combined with two electrons. The hydrochloric acid, in its turn, consists of ions of "hydrogen stuff," *i. e.* of atoms of hydrogen, each of which has lost an electron; and of ions of chlorine, generally termed chlorions: these may be viewed as "chlorine stuff" (what we generally term chlorine) each atom of which is in combination with an electron. It may also be postulated that when zinc goes into solution each atom loses two electrons; and also that if electrons be delivered to one end of a wire, a displacement of electrons occurs throughout the wire, and unless electrons escape at the other end, the wire will be charged, or, more exactly expressed, overcharged with electrons. Now a zinc plate and a copper wire are in a sense semi-permeable membranes, that is, they are permeable to electrons, but impervious to "matter":—to hydrogen, chlorine, or zinc. Hence the solution of zinc in HCl involves each zinc atom on the exterior of the plate parting with two electrons; these enter the zinc plate, displace others, which in their turn displace those in the copper wire, and finally in the copper plate, until at last, the external atoms of the copper plate are over-charged with two electrons. Each atom of the hydrogen in the ionic state present in the solution of hydrochloric acid, however, is capable of combining with one electron; and when thus combined it becomes ordinary hydrogen, and is evolved as gas. Thus the flow of electricity through a wire, and the electric pressure thus caused, may be regarded as analogous to the flow of water through a tube, if that water enter a solution containing a compound which is prevented from escape by a semipermeable membrane. It may be remarked that there remains a difficulty, namely, the union of two atoms of hydrogen to form a molecule; that fact should not be disregarded, and the "explanation" given above does not cover it.

Atoms of "matter" and electrons may be associated in another manner; and here, too, an analogy may be brought forward, although a somewhat imperfect one. An object may be "charged with electricity." What does this mean? On the corpuscular hypothesis we must consider it as covered with a layer of electrons greater or fewer in number. The quasi-analogy consists in the wetting of a solid object with a film of liquid. Here the liquid distributes itself over the surface and is attached to the solid by virtue of the fact, as usually stated, that the attraction of the liquid for the solid is greater than that of the liquid for itself. The phenomenon

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is of the nature of surface tension. Here the analogy fails; for the reason that electric corpuscles accumulate on the surface and not in the interior, is probably due to the repulsion of the corpuscles for each other. Yet, as it is advisable to make some kind of mental picture, this imperfect analogy must serve. And it may be conceived that the long-known process of electrification by friction may consist in the rubbing off or the rubbing on of electrons. We are accustomed to speak of the "electrification of a rod of sealing-wax." In this case it may be supposed that the surface layer of the sealing-wax acquires electrons (*i. e.* a negative charge) from the wool or fur with which it is rubbed. When glass is rubbed with silk, on the other hand, the process may be viewed as the removal of electrons from the surface of the glass by friction with a silk cloth. These actions relate mainly, if not wholly, to the surface layer; but it is not impossible that ionic charges may be influenced in a minor degree.

Lastly, it is not merely conceivable, but highly probable, that the atom of a metal may lose not only its "metallic corpuscle" (*i. e.* the corpuscle the loss of which changes it to an ion) but also other electrons. It is true that only in one single instance has this been certainly proved. Ramsay and Collie investigated the spectrum of the gas (emanation) evolved continuously from salts of radium, and found it to be definite and characteristic. Its volume was measured by Ramsay and Soddy, and it was found to be a gas, resembling other gases in conforming with Boyle's law; it was investigated originally by Rutherford and Soddy, who showed that like other gases it can be solidified by cold. They also showed that it is certainly an elementary gas, very like the gases of the argon group, for it is unaffected by treatment with any reagent. When freshly prepared its spectrum shows no trace of the very characteristic lines of helium. Yet Ramsay and Soddy showed that when kept it changes largely into helium indubitably recognizable by its spectrum. The volume of helium produced was measured and amounts to between three and four times that of the emanation. At the same time the Curies and Rutherford have shown that this spontaneous change is attended by the loss of relatively enormous amounts of energy, and that during the change β rays, or electrons, are shot off from it with enormous velocity. Such electrons are characterized by their ability to pass through screens impermeable to molecules of "ordinary matter." Elster and Geitel and others have proved that similar corpuscles are shot off from many forms of ordinary matter; and it is only reasonable to draw the conclusion that the

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phenomenon also denotes a disruption or disintegration of the atoms of such matter; although up to the present our knowledge as to the products is purely conjectural. The inference of transmutation, however, is justifiable; and the following experiments constitute a first attempt to ascertain the extent and the manner in which ordinary forms of matter disintegrate, whether they be elementary or compound. The order of affinity of the atoms of elements for electrons, which they part with during the process of disruption or disintegration, will be seen to be the same, in the main, as that which they manifest towards electrons, the loss of which converts them into ions.

If it turns out to be true, as Soddy claims to have shown ('Nature,' Aug. 2nd, 1906), that a disintegrating element, which parts with β-rays or electrons, leaves behind it matter not associated with a positive charge; and if it be also true that such "disintegration" implies transmutation into some other form of "elementary" matter, then it may be that the phenomena, of which a description is given in the following pages, refer to cases of transmutation. When zinc, for example, illuminated by ultraviolet light, parts with corpuscles, it may be that the residual matter, the zinc minus electrons, is no longer zinc, but some other form or forms of quasi-elementary matter.

If these views be adopted, it would appear that electrons are capable of associating themselves with matter in three ways: First, as a surface-film. When electrons are removed by friction their absence is termed "positive electrification." Secondly, in a condition analogous to that of one element in a chemical compound: an ion of chlorine, for example, is "chlorine-stuff" plus an electron; common salt is a compound of "sodium-stuff" with "chlorine-stuff," and an electron. A "positive ion" would then consist of an elementary "stuff," unassociated with an electron or with any other kind of atom. And lastly, it may be conceived that if an atom of element loses one or more electrons without leaving the remainder positively charged, that "elementary" matter will change its nature: it will cease to be the original element, but will be transmuted into atoms of lower atomic weight; or, it may be, several such "residues" may associate themselves to build up an atom of higher atomic weight.

These, as already remarked, are observations of a speculative character; but in carrying out research it is necessary to form some kind of hypothesis in order to guide experiment Should the hypothesis not be substantiated by observation, a new one will become necessary.

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1. The work communicated in this paper was undertaken with the object of repeating some experiments of Le Bon, described in several papers published in the *Comptes rendus*, and afterwards in greater detail in his treatise *L'évolution de la Matière*. It will be remembered that Le Bon, by allowing ultraviolet light to fall upon clean metallic surfaces raised to a high potential, caused them to give up their charges. He also showed that when ultraviolet light fell on a metal plate placed above, and at an angle of 45° to the cap of a positively charged electroscope, it caused the discharge of the electroscope. This he found was also the case when the electroscope cap was completely covered by an earth-connected aluminium screen. The discharge likewise takes place when the electroscope is charged negatively, although with greatly diminished rapidity. Le Bon concluded, from these experiments, that disintegration is effected by the action of light on the surface of the metal, and that this consists in the expulsion of charged particles capable of penetrating metallic screens. It is not proposed to discuss here the literature on this subject, but it may be mentioned that previous to Le Bon's publications some of his results had been anticipated, among others by Elster and Geitel, Hoor, Stoletow, Wiedemann and Ebert, Hallwachs, Righi, and Branley.

Fig. 1.

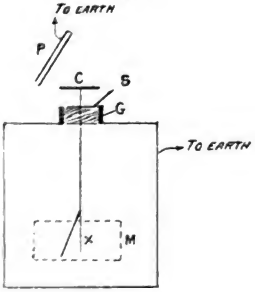
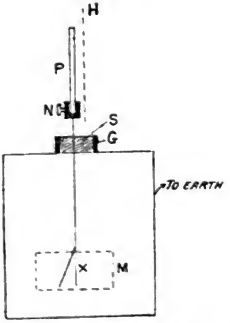


Fig. 2.



2. In our first experiments an iron arc served as the source of ultraviolet light, but it was discarded because of the great difficulty experienced in maintaining its intensity constant. An electroscope of very small capacity was used as the measuring instrument in all the experiments to be described. It consisted of a very thin brass plate X (figs. 1 and 2) connected to a thin rod which also carried the cap of

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the electroscope C. The leaf used was made of aluminium. This was enclosed in a metal box with mica windows, and insulated by a sulphur plug S. The sulphur was surrounded with an earth-connected guard-ring G. In some experiments the cap of the electroscope was a flat circular plate C (fig. 1), and in others a screw-clamp N (fig. 2). The position of the leaf was read by a microscope with a scale in the eyepiece. Before use, the electroscope was calibrated both for negative and positive charges, against a battery of small cells. The metals examined were made into plates 6 cms. wide, 9 cms. long, and about 5 mms. thick. It was found that when a plate was placed so as to reflect rays on to the cap of the electroscope, as indicated in fig. 1, that the electroscope was discharged irrespective of the nature of its charge. The circular plate C (fig. 1) was covered by an earth-connected box with aluminium sides and having an open top which could be covered with any material. Whether positively or negatively charged, the electroscope was discharged when the top of the box was covered over with either a single gold leaf, two gold leaves, or with silver leaf 0·00021 millimetre thick; but with very thin paper, tinfoil, or thin guttapercha no discharge could be observed, whilst in the case of aluminium leaf 0·0005 mm. in thickness a difference in the rate of leak was observed, but so small as to make the result doubtful. The figures in Table I. give the values obtained with the various screens; these figures, in spite of the fact that all possible precautions were taken to keep the intensity of the arc from varying, cannot be correct to more than 5 per cent.

TABLE I.

Radiating plate.	Charge on electroscope.	Screen.	Blank discharge for 10 divs.	Discharge for 10 divs.
Zn amalgam ...	Positive.	Au leaf.	130 mins.	1 min. 15 secs.
Zn amalgam ...	Negative.	Au leaf.	130 mins.	4 mins.
Zn amalgam ...	Positive.	Ag leaf.	183 mins.	16 mins.
Zn amalgam ...	Negative.	Ag leaf.	183 mins.	96 mins.
Sn amalgam ...	Positive.	Al leaf.	266 mins.	210 mins.

The rays shot off in this manner were deflectable by a magnet; this was proved by allowing the radiation to pass through a pair of fine slits (S) placed horizontally, one above

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the other, between which a powerful magnet was inserted, as indicated in fig. 3. The values obtained are given in Table II.; similar values were obtained by using a Soddy's grating instead of the slits.

Fig. 3.

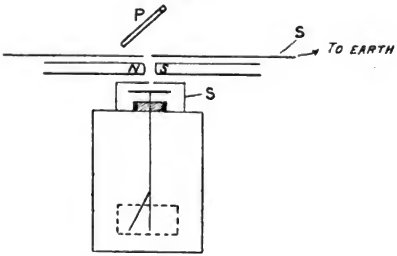

TABLE II.

Zinc amalgam plate. Electroscope positively charged.

	Time of discharge.
Without magnet	3 divs. in 37 mins.
With magnet	3 divs. in 96 mins.

By the method illustrated in fig. 2, the electroscope was discharged whether excited positively or negatively; the negative charge, however, was much more quickly dissipated than the positive, and both discharges were more rapid than in the former case. Some experiments were made by this method, using plates coated with sulphides of the metals, and here it was generally found that the discharge due to the sulphide was more rapid than that due to the metal itself.

3. At this stage in the research, owing to difficulty in maintaining constant the source of illumination, the arc was abandoned in favour of a Cooper-Hewitt mercury lamp, made by Schott and Genossen of Jena, using their Uviol glass, which is transparent to the ultraviolet spectrum of mercury vapour. The light intensity of this lamp is constant for a steady current; this in all subsequent experiments was maintained at three ampères.

A series of measurements was made both with the plates

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of metal connected directly to the electroscope, as in fig. 2, and with the plates in the position of a reflector at an angle of 45°, as in fig. 1. The electroscope was charged negatively in the former case, and positively in the latter; on reversing the charges, we were unable to get a measurable discharge. It was found impossible to discharge the electroscope if any of the metallic shields were used. This fact may easily account for our being unable to discharge the electroscope rapidly through an aluminium leaf, as Le Bon did; for he used very high tension sparks as his source of ultraviolet light, and these are much richer in ultraviolet rays than the iron arc. Elster and Geitel were, probably for the same reason, unable to obtain any discharge with tin, cadmium, lead, copper, iron, carbon, and platinum when illuminating with daylight. All the measurements were made against a standard plate of copper amalgam. It was found that plates of zinc amalgam and tin amalgam gave discharges much too rapid for accurate measurement; but copper amalgam was found to give constant and reproducible values when prepared by placing a clean piece of copper into a solution of mercurous nitrate for a few moments, then removing, washing and drying; the plate immediately before the measurement was polished with fine emery-paper and finally cleaned with a dry piece of linen. Such standard values were recovered for every substance measured. For plates giving a very slow rate of discharge, the electroscope was moved nearer the source of light, and a new standard value taken. The constancy of the standard plate was demonstrated by comparing it with a tin plate under various conditions; the measurements are given in Table III.

TABLE III.

Positive charge 800 volts.

Rate of discharge of copper amalgam.	Rate of discharge of tin plate.	Ratio Sn/Cu amalgam.
26·8 secs.	22·0 secs.	0·82
26·2 secs.	22·0 secs.	0·84
33·4 secs.	28·0 secs.	0·835
29·0 secs.	24·4 secs.	0·841
22·4 secs.	18·0 secs.	0·82
		Mean 0·83

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To show that the effect of altering the relative positions of the light and the electroscope made only a relative change in the rate of discharge, measurements were made with tin and the standard at two distances; the relation between the discharge at the two distances was found to be the same for both plates, as seen in Table IV.

TABLE IV.

Negative charge 800 volts.

Plate.	Distance d' 66 cms.	Distance d'' 26 cms.	Ratio d'/d''.
Tin	37 secs.	6·2 secs.	5·96 } 6·00
Copper amalgam ...	29 secs.	4·8 secs.	6·04 }

All rates of discharge in these and subsequent experiments are for 10 scale-divisions. In all experiments the initial potential of the electroscope was 800 volts, and the final potential, *i. e.* after the leaf had moved over 10 scale-divisions, was 680 volts; this potential was found to be well above that required to ensure a saturation current in the case of the most rapid discharge.

Table V. (p. 408) gives the relative rates of discharge for a series of elements. Each element, when enough was obtainable, was fashioned into a plate 9 cms. long and 6·1 cms. wide, and compared with a plate of copper amalgam of the same size; moreover, each measurement was corrected for the natural leak of the electroscope. The value of the natural leak was obtained in every case by making blank experiments before and after the measurement of each plate. Where it was impossible to obtain plates of the above dimensions, smaller plates were compared with smaller standards. The ratio between the rates of discharge of the large and small standards was also determined, and from these values the figures obtained were brought into a uniform relative series. In the case of mercury a special arrangement was made as illustrated in fig. 4. The electroscope consisted as before of a metal box fitted with the necessary earth-connected guard-rings. The arrangement of the leaves was as indicated in the diagram, the charging disk C was insulated in the same manner as the leaves by a sulphur plug. The mercury was placed in an insulating dish D of sulphur, which also served as an insulating support for the standard plate. Series A

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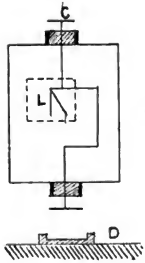
was made by the method already described, in which the plate was clamped to the electroscope. The measurements in Series B were carried out by the reflexion method with the plate earth-connected.

Fig. 4.



Considered generally the rate of discharge of the elements as given in the table are in the same order as their electro-potentials. This fact was noticed by Elster and Geitel* in the case of the alkali metals; these were found to have a much more rapid rate of discharge than zinc, so that they would naturally take their position at the top of Table V. There are, however, notable exceptions to the general regularity in the above series, amongst which may be noted manganese, iron, chromium, gold, nickel, and cobalt. It is a noteworthy fact that so many of these exceptions should be elements which very readily take up the passive form. When in this form they do not give their correct potentials, so that we may here be dealing with metals which are not giving their normal rate of discharge, but one consequent upon their being in the passive condition. It was found impossible to obtain a quicker rate of discharge by reconverting them into the active condition; this may be due to the ultraviolet light converting them into the passive form immediately it falls upon them.

Prof. J. J. Thomson † has shown that sodium, even

* Elster and Geitel, Wied. *Annalen*, xliii. p. 225.

We should like to express our thanks to Messrs. Johnson & Matthey for the loan of plates of Au, Ir, Pd, Ag, and Pt; to Messrs. Siemens & Halske for their kindness in presenting us with a plate of tantalum; and to Messrs. Mond & Co. for a plate of nickel, and the Thermit Co. for a plate of pure chromium.

† Thomson, Phil. Mag. vol. x. p. 584 (Nov. 1905).

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TABLE V.

Element.	Relative rate of discharge. Series A. Electroscope charged negatively.	Series B. Electroscope charged positively.	Remarks.
Magnesium ...	1·5	1·4	Contains trace Al.
Aluminium 1...	1·2	1·8	
Aluminium 2...	...	1·8	
Zinc 1	2·4	2·1	
Zinc 2	2·4	2·1	
Cadmium	13·3	8·1	
Tin.....	12·8	11·9	
Indium	10·0	15·8	
Lead 1	14·6	29·6	Commercial.
Lead 2	16·9	23·6	Kahlbaum's.
Antimony	21·8	31·8	
Bismuth.....	31·2	37·4	Trace Mn & Fe.
Palladium	31·8	61·1	Charged with O.
Palladium	41·6	71·3	Charged with H.
Gold	45·7	73·9	
Cobalt	81·4	103	Electrolytic.
Nickel	64·2	127	
Copper 1	76·6	172	Commercial.
Copper 2	100·9	196	Electrolytic.
Silver.....	108·0	217	
Cobalt	121	218	Commercial.
Palladium	127·8	394	
Iron 1	421	446	Electrolytic.
Tantalum	288	639	
Platinum	135	670	
Iron 2	356	781	Mild steel.
Iridium	680	896	
Iron 3	318	1218	Wrought iron.
Iron 4	1018	2251	Cast iron.
Uranium	35·2	The majority of these elements gave values which were too near the blank value to be of any use. Sulphur and selenium gave values which were slightly slower than the blank experiment.	
Mercury.....	216		
Tellurium	1450		
Molybdenum...	1520		
Manganese ...	1783		
Chromium	2920		
Carbon	3560		
Tungsten	3766		
Selenium	7540		
Silicon	867		
Arsenic	11595		
Sulphur.....	Could not be measured because of its insulating properties.		

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in the dark, emits charged corpuscles deflectable by a magnet; the amount of disintegration is very small, even in the case of so active an element as sodium: so that one would scarcely expect to be able to detect it in the case of the less electropositive element. Nevertheless, in all probability the ultraviolet light, as suggested by Thomson, acts as a detonator, bringing about the disintegration, otherwise immeasurably slow, at an increased rate. The ultraviolet light would therefore seem to act as an accelerator in the breaking up of atoms. This idea seems to be confirmed by the experiments of Strutt*, on the radioactivity of ordinary materials. He obtained small discharges from plates of various metals under ordinary conditions. These values, as

TABLE VI.

Element.	Leak in divs. per hour.
Al	1·4
Zn	1·2
Pb	2·2
Cu	2·3
Pt	(3·9—2·0)

will be seen from Table VI., run parallel with the electro potential of the elements; aluminium is the only element which is an exception to the series. Silver was also measured, and gave a result which does not come into the series; but the silver was deposited on glass, and this may have modified the result. Clearly, then, the action in our experiments and those of Strutt is the same, the only difference being that we have increased a small natural radiation by the use of ultraviolet light. The experiments of Bumstead†, although they are in direct opposition to ours, do not of necessity prove very much: they have in the first place only been carried out for two elements, and in the second place, as pointed out by the author himself, there are a great many sources of error. When palladium was charged with either hydrogen or oxygen very remarkable results were obtained: the value in both Series A and B, Table V., places them between lead and copper, a position in accordance with their place in electro-

* Strutt, Phil. Mag. June 1903, p. 680.
† Bumstead, Phil. Mag. Feb. 1906, p. 292.

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potential series. This also disproves any suggestion that the discharges are brought about by oxidation of the surface of the plate, for it is inconceivable that palladium and hydrogen should oxidize at the same rate. Measurements made with various specimens of iron show that the carbon contained has a retarding action, for the more carbon the iron contains the less rapid is the rate of discharge. It will be noticed that, with the exception of the first few elements, the rate of discharge in Series A is much higher than that of Series B. It is very possible that even in the first few elements this is also the case, for the intervals of time were so short that an increase of one-fifth of a second would reverse the position. The reason for the difference in the rate of discharge may possibly be explained by the hypothesis, as stated in the introduction, that a metallic element is a compound of an ion with one or more electrons. The action which takes place under the influence of ultraviolet light might be conventionally depicted by the equation

$$\text{Element} = \textit{ion} + n.\ \text{Electrons}.$$

This action will proceed at a perfectly definite rate if all other conditions remain constant. Suppose that such an element is charged to a given potential, or, in other words, a greater number of electrons distributed on its surface. Now an element in its normal condition is able to retain a certain number of electrons, but if a great many more are added it will naturally hold them with very little force; and, consequently, when a detonator in the shape of ultraviolet light is applied, they will be given off at a much greater rate than when the plate is uncharged. An explanation for the charging of the electroscope when sulphur and selenium were measured, is probably to be found in the case with which these elements take up a charge themselves; this is rendered more probable by the fact that in several experiments the sulphur was proved to be charged. Hallwachs and Le Bon proved that an insulated plate takes up a positive charge when placed in ultraviolet light. We repeated this experiment, using a piece of magnesium attached to a positively charged electroscope, and found that the rate of the natural leak of the electroscope was retarded. The surfaces of all the elements measured tire with extreme rapidity, so much so in some cases that the first and most rapid discharge could only be recovered after the plate had remained for several days in the dark and had then again been polished. The tiring of the plates and a possible explanation thereof will be referred to later.

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4. Our next series of experiments dealt with compounds of several elements. The sulphides and iodides, so far as they were investigated, have a perfectly definite rate of discharge for negative electricity; this rate is much slower than in the case of the metals, and presents a phenomenon similar to that observed with the various specimens of iron, which may be regarded as compounds in a state of more or less purity. The sulphides measured were prepared in two different ways. For Series I. the plates of the metals were placed either in a solution of sulphuretted hydrogen or ammonium sulphide until they were completely coated with a film of the sulphide. Then they were removed, washed, and thoroughly dried with filter-paper. For Series II. the plates were made by precipitating the sulphide from a solution of its salt, and after it had been washed and dried it was made into a paste with a starch solution and coated on plates of the requisite size and dried at 80° C. In either case the plate was clamped on to the electroscope and measured as shown in fig. 2. The two series of measurements ran parallel, as will be seen from Table VII., although the values of the same sulphide differ

TABLE VII.

Sulphide of	Series 1. Rate of discharge.	Series 2. Rate of discharge.
Antimony, Sb_2S_3 ...	3361	5060
Copper, CuS	338	663
Lead, PbS	413	1732
Tin, SnS.....	7500	12750
Iron, FeS	3138	...
Cadmium, CdS	7650	7610

in the two series. This is not of much importance, because the starch in the composition of the plates must have some influence on the rate of discharge. The figures given in Table VII. are corrected for the natural leak, and calculated to the same arbitrary standard adopted in Table V. It will be noticed that with the exceptions of antimony and tin sulphides, the rates of discharge run in a series in the opposite direction to the electropotentials of the metals. It would have been interesting to have made experiments with the metals with a more rapid discharge, but their sulphides are not obtainable in a condition suitable for measurement. A possible reason for the non-agreement of antimony lies in the

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fact that it was present as a trisulphide, whereas all the other elements were present as monosulphides; the additional amount of sulphur probably acting like the carbon in cast iron. Tin sulphide was very probably mixed with tin oxide and stannic sulphide, both of which would influence the value obtained. We are proposing, at a later date, to make a more thorough study of the action of ultraviolet light upon compounds, both from the chemical and physical points of view. Knoblauch made a series of measurements with sulphides: from his results he attempted to prove that the whole action is due to oxidation of the sulphides to sulphates, because he could obtain no discharge with sulphates. He uses the unipolar nature of the discharge to show that it must be a case of oxidation; against his theory is the fact that these discharges occur equally well in an atmosphere of hydrogen. The most oxidizable sulphide in Table VII. is probably that of iron; and therefore, according to Knoblauch*, it ought to give the most rapid discharge, which is not the case. Again, if we take the iodides, Table VIII., they are not at all easily oxidizable, yet copper and silver iodides give quicker rates of discharge than iron sulphide, and it is difficult to imagine that they are more easily oxidized than any one of the sulphides. The iodides are not in the same order as either the sulphides or the metals; they were prepared by suspending plates of the metals over a dish of iodine until the surface was coated with iodide. The plate was then allowed to stand for some time, in order that we might have a simple iodide, and not a solution of iodine in an iodide, Many iodides could not be measured on account of their deliquescence. The rate of discharge is calculated to the same standard as in Tables V. and VII.

TABLE VIII.

Iodide of	Rate of discharge.	per cent. I in the compound.
Copper, CuI_2.....	1240	81 per cent.
Silver, AgI.....	1363	56 ,,
Cadmium, CdI_2.....	4145	69 ,,
Tin, SnI_2	6280	68 ,,
Lead, PbI_2.....	7960	55 ,,

With the exception of silver iodide, the only moniodide in the series, the rates of discharge increase with increasing percentage of iodine in the salt. The above experiments on sulphides and iodides, together with those of Knoblauch, lead to the conclusion that there is an emission of corpuscles from

* Knoblauch, *Zeitschr. f. Physikalische Chemie*, vol. xxix. p. 527.

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compounds in the same way as from metals. This means that here also a slow disintegration is taking place, which is accelerated by the light, and which in compounds depends on the nature and quantity of each constituent. The effect of alloying one metal with another does not seem to give the mean of the two values, as is shown in the case of some amalgams, Table IX.

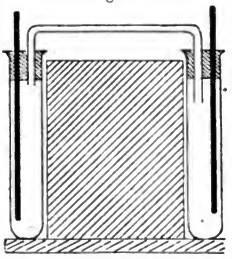
TABLE IX.

Element.	Rate of discharge.
Tin	12·8 secs.
Zinc	2·4 secs.
Mercury	216 secs.
Copper	100·9 secs.
Tin amalgam	0·6 secs.
Zinc amalgam	0·7 secs.
Copper amalgam	10·0 secs.

Saturated amalgams, cleaned by the method described for the standard, were used for these experiments.

5 Certain cells only show an electromotive force when one electrode is exposed to the light, whilst the other is kept in the dark. Such differences of potential could easily be explained by the hypothesis used throughout this paper. If this hypothesis is applicable to the case of such an element then a cell consisting of two similar electrodes of the same metal in the same solution should have an electromotive force when one electrode is placed in ultraviolet light, and

Fig. 5.



the other is covered up. To investigate this theorem cells were constructed of uviol-glass (fig. 5); one limb could be darkened by slipping a blackened tube over it, whilst the other was exposed to the light. The metals used in these

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cells were gold, copper, and cadmium; in every case a change of potential was observed when one of the electrodes was exposed to the light. The changes were small, but quite distinct, and of the order of magnitude of ten millivolts, although they were somewhat variable. In these experiments sunlight was found to be quite as active as the mercury lamp. Additional confirmation was afforded by the fact that if a cell with one of its electrodes uncovered was left exposed to light for some days, a deposit appeared and accumulated upon the uncovered electrode, whilst no change occurred on the other. In the case of copper there was a green deposit, with gold the deposit was orange. The change on the exposed metal can be explained on the disintegration hypothesis: the light falling on to the electrode causes it to lose electrons; *i. e.* ions of the metal are formed which combine, as rapidly as they are formed, with the hydroxyl ions of the water forming hydroxides. All chemical actions influenced by light come under this category, for the light will obviously increase ionization, and this fact is well known to be a controlling factor in the velocity of reactions.

6. Reference has already been made to the tiring of metals when exposed to ultraviolet light. It was found that when a plate was measured several times, without re-cleaning, that each successive measurement was slower than the preceding one. In some cases, more especially those with a rapid rate of discharge, the most rapid value could be regained by re-polishing the plate; whilst in others, *e. g.* palladium and platinum, the quickest value could not be regained until after several days had elapsed. To investigate this point a cleaned plate was clamped on the electroscope, and its rate of discharge was measured immediately, and at intervals over a period of twenty-four hours. Such series of measurements were made for plates of magnesium, zinc, tin, and aluminium; the rates of discharge were then plotted against the times of exposure. The curves obtained showed a number of breaks which was different for each metal, but with the exception of aluminium, the numbers corresponded with the number of valencies of the metals. When an element combines with another, it expels a number of electrons representing its valency; these electrons are more lightly held in the atom than the other electrons which constitute its mass. This would mean that the lightly bound electrons are expelled by the ultraviolet light in definite stages, represented by the breaks in the curves. Aluminium shows at least five or six breaks which may be explained by a still further splitting off of electrons after those, corresponding to the valencies, have been expelled.

When a metal, *e. g.* Mg, has been subjected to one series of

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measurements it gives the same initial value on recleaning, but the subsequent rate of tiring differs from that of the previous experiment; the breaks are obtained but they are much less persistent, and the rate of tiring is very much accelerated. The increase in the rate of tiring is more noticeable in each successive series of experiments. Six series were made with a magnesium plate at various intervals of time, and these show very well the breaks and the increasing rate of retardation of the discharge (Curves I., II., III., IV., Pl. VI.). The two series between II. and III. were spoiled. The numerical results are given in Table X. It will

TABLE X.—Magnesium.

I.		II.		III.		IV.	
18 May.		13 June.		21 June.		18 July.	
Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.
0	0	0	0	0	0	1·7	0·6
2·2	1·0	1·0	0·4	1·7	0·2	4·2	1·6
2·6	1·6	1·8	1·2	2·8	1·2	6·8	3·0
4·2	2·4	3·2	2·6	3·6	2·4	8·2	4·4
5·4	4·2	4·8	3·6	5·3	3·8	12·9	5·4
7·3	6·2	6·6	5·4	7·4	6·6	13·2	7·0
8·5	9·2	8·5	7·8	7·4	9·2	17·1	9·0
10·0	13·0	10·0	10·2	9·8	12·0	18·0	11·8
10·8	19·2	11·0	13·4	10·6	16·0	19·1	15·0
11·7	26·4	12·5	18·4	12·5	21·0	19·7	17·8
12·5	36·0	13·0	23·8	13·0	25·2	21·1	20·4
13·6	46·6	14·8	32·0	13·6	30·2	22·4	25·0
13·4	61·4	15·5	44·2	14·1	37·0	20·4	29·4
14·1	85	15·4	58·0	15·0	45·4	22·4	34·4
13·5	104·8	15·4	71·6	14·8	53·0	22·1	39·8
13·8	123·6	15·4	88·6	17·1	65·4	22·4	46·6
14·0	145·9	16·1	114	19·2	79·0	22·4	55·6
16·0	200	17·6	164	23·0	92·0	23·1	66·4
16·8	233	19·0	211	21·8	110	25·2	75·4
19·4	258	19·8	247	21·8	146	25·2	83·0
20·0	319	20·3	297	31·6	190	25·5	97·0
19·8	397	20·2	336	36·4	243	28·2	122
19·4	458	22·4	388	45·2	288	30·6	149
19·4	533	27·2	461	73·3	330	37·4	176
23·0	656	23·0	526	87·6	381	41·5	202
19·3	720	38·8	563	108	428	46·0	229
76·0	1440	40·6	627	131	470	60·0	259
		47·2	689	141	525	75·0	298
		76·2	828	292	615	89·0	339
		231	1428	439	692	126	379
				665	758	166	455
				981	1418	207	538
						269	580
						375	642
						584	798
						1424	1450

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be noticed that, whereas the Curve I. only shows a discharge rate of 20 seconds after ten hours exposure, Curves II., III., and IV. show rates of 38, 270, and 290 seconds respectively. The same phenomenon was observed for tin, aluminium, and zinc (Table XI.: Curves VII., VIII., IX., X., Pl. VII.).

TABLE XI.

Aluminium.		Zinc.		Tin.			
X.		IX.		VII.		VIII.	
21 May.		14 June.		24 May.		15 June.	
Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.	Leak in secs.	Exposure in mins.
1·2	0·2	0	0	6·4	0	9·6	1·6
1·8	2·0	2·0	0·4	8·6	1·8	11·2	2·6
1·8	3·0	2·2	1·6	10·5	3·5	12·4	4·4
2·1	6·0	2·5	3·2	11·0	5·4	13·1	6·2
2·7	7·2	2·6	5·0	11·5	7·8	15·0	9·2
2·7	9·0	2·7	7·0	12·2	10·2	16·6	12·6
2·6	11·2	2·6	9·6	13·2	13·1	17·0	16·0
3·0	14·6	2·8	12·0	13·8	17·8	18·8	19·8
2·9	17·6	3·2	15·0	15·0	23·4	20·8	24·2
3·8	21·6	2·8	18·8	16·6	32·6	21·5	30·2
4·0	28·4	3·4	23·0	17·8	45·6	22·7	36·4
3·8	34·0	3·4	28·0	18·8	65·0	22·8	43·4
4·0	38·3	3·4	32·2	30·2	90·0	25·7	58·0
4·5	44·0	3·7	36·4	62·6	156	29·8	72·8
4·5	50·0	4·2	43·2	66·8	181	32·4	87·6
4·5	56	4·8	61·0	71·0	225	39·6	115·6
4·9	67	5·4	77·2	79·2	284	41·4	161
5·6	73	6·4	94·2	104	368	49	224
6·0	86	7·4	115·6	102	450	45·8	271
6·4	99	8·6	155	115·4	522	57·4	332
6·7	117	10·0	190	134	626	70·6	387
7·2	140	10·3	216	139	730	78·2	468
7·8	163	12·0	256	166	1420	85·0	538
8·8	193	13·8	307			100	630
9·0	244	14·0	361			104	747
9·4	275	16·0	477			98	821
10·0	327	18·0	572			146	1554
11·8	384	18·6	630			152	1634
11·8	438	18·6	747				
12·0	506	18·8	821				
14·3	569	23·0	1421				
14·7	681						
16·2	768						

Magnesium which has once been subjected to ultraviolet light does not seem to regain its first rate of firing even after standing for several days. A second piece of magnesium, which had been

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used for several experiments of short duration, gave a "tiring" Curve V., which lies between II. and III. Similar "tiring curves" were obtained for zinc, using the reflexion method (fig. 2), in which no charge was put on the plate. To explain the foregoing experimental results we must recall the idea, postulated in the introduction, that a metal is composed of a large number of electrons, of which a few, previously termed by us "metallic corpuscles," and corresponding to its valency, differ from the residue. The explanation will be clearer if we consider the experiments as made by the reflexion method, in which the plate was not charged. When ultraviolet light falls upon a plate of metal, *e. g.* Mg, it effects, as before stated, an expulsion of electrons; this splitting off proceeds at a continually decreasing rate, until a certain point indicated in the curve is reached: at this point a constant rate of expulsion of electrons is obtained which must be attributed to the loss of the first of the "metallic corpuscles," during which we have a constant electric pressure, just as we have the constant vapour pressure of various hydrates in the case of dehydration of certain crystallized salts, *e. g.* $CuSO_4,5H_2O$. The steady rate of discharge continues until the metallic electron has been expelled from the whole of the atoms at or near the surface of the plate. Then we get a further splitting off of the other electrons until we come to the second break in the curve where the second metallic electron is expelled, the constant rate of discharge being explained as before. We obtain repetitions of these breaks so long as there are metallic corpuscles still combined in the atoms; when the last of these has been expelled we have a curve which exhibits a continuously decreasing rate of discharge, this means that the metal is undergoing a further disintegration at a slowly decreasing rate. Aluminium, with its five or six breaks, does not seem to fall into line with the above hypothesis; the additional breaks, however, may be due to the formation of definite electron complexes, less, or even more, complicated than the complex we know as aluminium. If we now consider the successive curves obtained for magnesium, we have here the breaks approximately in the same places, but they are of much shorter duration, and there is a very much increased rate of tiring in each successive curve. The increased rate of tiring can only be explained by the fact that the magnesium has become permanently changed, at least on the surface. The persistence and decreasing length of the delays in the successive curves show that, although the metallic corpuscles are removed from the surface layer, yet there is a slow diffusion of electrons from

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the inner layers to the surface sufficiently large to give the delays. If we now revert to the cases where the exposed plate receives an electric charge, as was the method of procedure in the majority of the experiments, it is obvious that a metal will hold a charge with a greater force the more electrons it has lost, and this being the case we shall have rates of discharge and curves of the same nature in both series of experiments. We suggest that the experiments described furnish a step in the determination of the rate at which the various forms of matter are undergoing disintegration under normal conditions.

Chemical Laboratory,
University College, London,
July 1906.

XLVI. *The Effect of Combined Stresses on the Elastic Properties of Iron and Steel.* By E. L. HANCOCK, *Assistant Professor of Applied Mechanics, Purdue University, La Fayette, Indiana* *.

[Plate VIII.]

THE tests reported below were carried out as a part of the "programme of work" outlined by the writer, and presented to the American Society for Testing Materials in a "Preliminary Report of the Effect of Combined Stresses on the Elastic Properties of Iron and Steel" (see Proceedings of the Society, 1905). This report gave the results of tests on certain specimens of low carbon-steel and nickel-steel. The specimens were tested in tension while under different torsional stresses, *i. e.* in tension while under torsion to one-third, two-thirds, and the full elastic limit in torsion, respectively. The results showed that the torsional stresses lowered the elastic limit in tension.

The additional tests reported in this paper were made, for the most part, on steel tubing, the method of testing and general procedure being about the same as that reported for the nickel- and carbon-steel solid rounds. Some tests were also made on low carbon-steel solid rounds in compression while under torsion, the latter being the beginning of a series of tests in compression-torsion. Only a part of the results of these tests is included in this report. The tests were all made under the direction of the writer in the Laboratory for Testing Materials of Purdue University. Acknowledgment is here

* Communicated by the Author, having been read before the American Society for Testing Materials, June 1906.

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made of the help given by Messrs. Carl Wagner, H. N. Hubbard, N. L. Arbuckle, O. L. Arbuckle, R. Elsea, W. Thornton, and J. T. Voshell, senior students in engineering, in connexion with the tests.

Materials.

It is known that, when a solid bar is subjected to torsion, the outer fibres are stressed to the elastic limit first, and that the stress varies linearly to zero at the centre. It was evident that, in the case when a solid round was tested in tension while under torsion to, or beyond, its elastic limit in torsion, considerable tensional elasticity would be shown, since in such a case there would be a certain body of fibres in the interior of the piece that had not been stressed to their elastic limit. It was thought that by doing away with this interior core and using hollow tubes, the true effect of combined tension and torsion would be more nearly shown. It will be seen in what follows that the results of tests on thin tubing do not differ materially from those obtained from the tests of thick tubing and solid rounds, except as regards unit elongation at the elastic limit (see Plate VIII. fig. 6). It is the opinion of the writer at this time that nothing new would be learned by using a thinner tubing than was used in these tests.

The steel tubing used in the torsion-tension tests had an outside diameter of 1 inch. Three different thicknesses were used as follows: 0·05 inch, 0·075 inch, and 0·25 inch, giving an inside diameter of 0·90 inch, 0·85 inch, and 0·50 inch respectively. The tubes were thoroughly annealed before using and were, as nearly as possible, of uniform thickness. About forty torsion-tension tests were made on this material.

The material used in the torsion-compression tests was low carbon-steel solid rounds:—The test pieces were 8 inches long over all, and turned down to $\frac{3}{4}$ inch with a gauge portion turned down to $\frac{1}{2}$ inch diameter. This material was from the same shipment as the low carbon-steel used in the torsion-tension tests.

Method of Test.

The apparatus used in making the tests on steel tubing was the same as that used in testing the nickel and carbon steel solid rounds (see Preliminary Report *).

To make the torsion-compression tests, however, one ball-bearing head of the apparatus was placed on the platform of the machine and the other attached to the moving head. With the specimen inserted and under the desired torsional

* Phil. Mag. Feb. 1906.

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load, a compression test was made by lowering the moving head. The amount of compression was measured by means of an Olsen Compressometer. The details of these torsion-compression tests will be given in a subsequent report.

Results.

The lowering of the elastic limit in tension due to the presence of various torsional stresses is shown in Plate VIII. figs. 1, 4, and 6 (3) (for steel tubing 0·85 inch inside diameter). In this case the amount of torsional load applied was sufficient to stress the piece to $\frac{1}{3}$, $\frac{2}{3}$, and full elastic limit in torsion respectively. Fig. 2 shows the lowering of the elastic limit in tension of steel tubing 0·90 inch inside diameter, the torsion being applied in the same way as in the case of the pieces represented in fig. 1. Figs. 4 and 6 also show this same lowering of the elastic limit. In each of the cases A and B the curves represent an average of two or more tests. Fig. 3 shows the effect of torsion on the elastic limit in tension of steel tubing 0·50 inch inside diameter. In this case the torsional load was applied and the piece tested in tension as indicated on the curves. Each curve represents an average of three or more tests. The lowering of the elastic limit in tension is also shown for this case by figs. 4 and 6 (3).

Fig. 6 (3) shows graphically the lowering of the elastic limit in tension for various degrees of torsion, and includes, in addition to the steel tubing, the results obtained from the nickel- and carbon-steel solid rounds (see Preliminary Report). The ordinates represent the percentage of lowering of the elastic limit and the abscissæ the amount of torsional stress applied. A similar statement is true for the rest of the diagrams in fig. 6 as well as for those in figs. 4 and 5. It is seen from this representation on fig. 6 that the variation in the elastic limit is a linear one, and might be represented by a straight line. Fig. 4 shows this straight line with the individual points represented. The equation of the line, referred to the horizontal and vertical lines as axes, might be written

$$y = (p/2p_s)(2p_s - x),$$

where p and p_s represent unit simple tension and unit simple torsion respectively. This same lowering of the elastic limit is shown in Tables I. and II. Table I. also shows the lowering of the elastic limit of low carbon-steel in compression due to different torsional stresses. These latter results are shown graphically in comparison with those of the

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torsion-tension tests in fig. 5. This compression line, however, only holds, so far as known, for low carbon-steel.

The variation of the unit elongation at the elastic limit for different torsional stresses is shown graphically in fig. 6 (2). It is seen that, with the exception of the cases A and B (*i. e.* steel tubing 0·85 and 0·90 inch inside diameter), the torsional stress lowers the unit elongation at the elastic limit. Fig. 6 (1) also shows the change in the modulus of elasticity in tension caused by the different torsional loads. There seems to be no regularity in this case, although in most cases the modulus of elasticity was lowered slightly by the torsional stress.

The results of the tests, as shown by the curves, might be summed up in the statement, that *the presence of a torsional stress lowers the unit stress and unit strain at the elastic limit in tension and also lowers the modulus of elasticity, somewhat.* These same facts are also shown by Table I.

TABLE I.

Stress Applied.	Inside Diameter in inches.	Elastic Limit in pounds per sq. inch.	Modulus of Elasticity in pounds per sq. inch.	Unit Elongation at Elastic Limit in inches.
Steel Tubing.				
Simple Tension.....	0·85	17,000	27,400,000	·00062
1/3 Tors.-El. Ten....	0·85	16,000	26,600,000	·00060
2/3 Tors.-El. Ten....	0·85	12,000	27,200,000	·00044
3/3 Tors.-El. Ten....	0·85	9,000	17,000,000	·00053
Simple Tension.....	0·90	28,000	29,700,000	·00094
1/3 Tors.-El. Ten....	0·90	24,000	27,800,000	·00086
2/3 Tors.-El. Ten....	0·90	17,000	25,000,000	·00068
3/3 Tors.-El. Ten....	0·90	14,000	16,200,000	·00086
Simple Tension.....	0·50	20,000	33,300,000	·00080
1/6 Tors.-El. Ten. ..	0·50	18,000	27,200,000	·00066
2/6 Tors.-El. Ten....	0·50	16,000	29,600,000	·00054
3/6 Tors.-El. Ten....	0·50	14,000	28,000,000	·00050
4/6 Tors.-El. Ten....	0·50	12,000	26,000,000	·00046
5/6 Tors.-El. Ten....	0·50	10,000	30,300,000	·00033
6/6 Tors.-El. Ten....	0·50	8,000	26,600,000	·00030
Nickel-Steel Solid Rounds.				
Simple Tension.....	.....	56,000	35,000,000	·0016
1/3 Tors.-El. Ten....	.....	52,000	29,800,000	·00174
2/3 Tors.-El. Ten....	.....	44,000	28,800,000	·00152
3/3 Tors.-El. Ten....	.....	20,000	28,800,000	·0067
Low Carbon-Steel Solid Rounds.				
Simple Tension.....	.....	34,000	30,800,000	·0011
1/3 Tors.-El. Ten....	.....	32,000	30,500,000	·00105
2/3 Tors.-El. Ten....	.....	24,000	25,800,000	·00083
3/3 Tors.-El. Ten....	.....	16,000	25,200,000	·00063

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It is interesting at this time to see the results obtained by substituting the results obtained from these tests in the various formulæ used for determining the unit stresses for the case of combined stresses. The formulæ generally used in designing parts subjected to combined flexure and torsion, tension and torsion, and compression and torsion, are as follows:—

$$q_1 = (1/2)[p + \sqrt{p^2 + 4p_s^2}], \quad . \quad . \quad . \quad . \quad (1)$$

$$q_s = (1/2)\sqrt{p^2 + 4p_s^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where q_1 represents the greatest unit tension on any internal plane, q_s the greatest unit shear on any internal plane, p the unit simple tension, and p_s the unit simple shear. The formulæ give the apparent unit tension and shear acting on any particle of the body. To obtain the true maximum unit stresses on any internal plane it is necessary to take account of the lateral contraction as well as the elongation, *i. e.* it is necessary to use Poisson's ratio. Taking this ratio as 1/4 (Johnson, 'Materials of Construction'), the formulæ giving the maximum unit tension and the maximum unit shear on any internal plane are

$$T_1 = (3/8)p + (5/8)\sqrt{(p^2 + 4p_s^2)}, \quad . \quad . \quad . \quad (3)$$

$$T_s = (5/8)(\sqrt{p^2 + 4p_s^2}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Or using Poisson's ratio as 1/3 (Merriman, 'Mechanics of Materials,' 1905), the maximum unit stresses are

$$T_1' = (1/3)p + (2/3)\sqrt{p^2 + 4p_s^2}, \quad . \quad . \quad . \quad (5)$$

$$T_s' = (2/3)\sqrt{p^2 + 4p_s^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Formulæ (1), (2), (3), (4), (5), and (6) have been used in constructing Table II. For this purpose p was taken as the unit stress at the elastic limit in tension and p_s the unit torsional stress on the specimen. The unit stresses thus obtained, whether apparent or real, which are greater than the simple unit stress, are underlined. For example, low carbon-steel shows a unit stress in simple tension at the elastic limit of 34,000 and a unit shear at the elastic limit in torsion of 30,000, when held in torsion at one-third the elastic limit (corresponding to a fibre stress of 12,540), and tested in tension

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the elastic limit was found to be 32,000; taking p as 32,000 and p_s as 12,540, the apparent and true stresses given in the same line have been calculated. This same procedure was followed in constructing the entire table. It is seen that the apparent maximum unit shear is generally less than the shearing strength of the material, and that the true maximum unit shear is generally greater than the shearing strength of the material (here shearing strength means the elastic strength as determined from torsion tests), higher values being given for the case where Poisson's ratio was taken as 1/3. In only a few cases in the case of the solid rounds and the thick tubing did the apparent maximum tension exceed the tensile strength of the material (tensile strength meaning unit tensile stress at the elastic limit in simple tension); the number of cases where the true maximum tensile stresses, for solid rounds and thick tubes, exceed the tensile strength of the material is greater than the corresponding number for the apparent stresses. In the case of the thin tubing, however, it is seen that in no case did the apparent maximum unit stress or the true maximum unit stress in tension reach the elastic strength of the material; but in the case of maximum unit shear the true stresses are greater than the elastic strength of the material (not true for apparent maximum shear, except in one case). In general, it is seen from this table that the formulæ for true unit stresses with Poisson's ratio as 1/3 give the highest values, those with Poisson's ratio as 1/4 the next highest, and the formulæ for apparent stresses the lowest. This is shown graphically for the carbon and nickel steel solid rounds by fig. 7.

The conclusion to be drawn from Table II. is that in the design of parts subjected to combined stresses the true shearing strength should be used. Mr. J. J. Guest, whose investigation was referred to in the Preliminary Report, arrived at the same conclusion as the result of his investigations. The entire uniformity of the results as shown by the thin tubing, where the effect of the combined stresses can be more exactly determined, gives the confidence in this conclusion, in spite of the fact that the maximum tension is usually supposed to govern the design of parts subjected to combined stresses.

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TABLE II.—Unit Stresses for Steel in Tension-Torsion.

Kind of Stress.	Apparent Stresses.		True Stresses. $\lambda=1/4$.		True Stresses. $\lambda=1/3$.		Simple Stresses.	
	q_1.	q_s.	T_1.	T_s.	T_1'.	T_s'.	p.	p_s.
Nickel-Steel Solid Rounds.								
Simple Tension.....	56000	28000	56000	35000	56000	37300	56000	00000
Simple Torsion.....	36000	36000	45000	45000	48000	48000	00000	36000
1/3 Tors.-El. Ten.....	55400	29400	56300	36800	56600	39270	52000	13700
2/3 Tors.-El. Ten.....	57700	35700	61100	44600	62200	47600	44000	28100
3/3 Tors.-El. Ten.....	52500	42500	60600	53125	63390	56660	20000	41300
Low Carbon Steel Solid Rounds.								
Simple Tension.....	34000	17000	34000	21250	34000	22560	34000	00000
Simple Torsion.....	30000	30000	37500	37500	40000	40000	00000	30000
1/3 Tors.-El. Ten.....	36325	20325	37430	25430	37760	27100	32000	12540
2/3 Tors.-El. Ten.....	35930	23930	38920	29920	39912	31912	24000	20710
3/3 Tors.-El. Ten.....	39225	31225	45000	39000	46965	41632	16000	30170
Steel Tubing 1 inch, outside diameter 0·075 inch thickness.								
Simple Tension.....	17000	8500	17000	10625	17000	11330	17000	00000
Simple Torsion.....	11520	11520	14400	14400	15366	15366	00000	11520
1/3 Tors.-El. Ten	17170	9170	17460	11460	17550	12226	16000	4480
2/3 Tors.-El. Ten.....	15190	9190	15987	11487	16250	12250	12000	6960
3/3 Tors.-El. Ten.....	15685	11185	17355	13980	17910	14910	9000	10240

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Steel Tubing 1 inch outside diameter, 0·05 inch thickness.								
Simple Tension.....	28000	14000	28000	17500	28000	18670	28000	00000
Simple Torsion.....	16020	16020	20025	20025	21360	21360	00000	16020
1/3 Tors.-El. Ten.....	25955	13955	26445	17445	26610	18610	24000	7100
2/3 Tors.-El. Ten.....	22963	13963	24200	17453	24620	18620	18000	10680
3/3 Tors.-El. Ten.....	22080	15080	24095	18845	24767	20100	14000	13350
Steel Tubing 1 inch outside diameter, 0·25 inch thickness.								
Simple Tension.....	20000	10000	20000	12500	20000	13333	20000	00000
Simple Torsion.....	12000	12000	13000	15000	16000	16000	00000	12000
1/6 Tors.-El. Ten.....	18222	9222	18275	11525	18295	12295	18000	2000
2/6 Tors.-El. Ten.....	16947	8947	17180	11180	17360	11930	16000	4000
3/6 Tors.-El. Ten.....	16220	9220	16775	11525	16967	12300	14000	6000
4/6 Tors.-El. Ten.....	16000	10000	17000	12500	17333	13333	12000	8000
5/6 Tors.-El. Ten.....	16183	11183	17725	13975	18243	14910	10000	10000
6/6 Tors.-El. Ten.....	16650	12650	18815	15815	19537	16870	8000	12000

Unit Stresses for Low Carbon-Steel in Compression-Torsion.

Simple Compression....	37000	18500	37000	23125	37000	24667	37000	00000
Simple Torsion..... ..	26000	26000	32500	32500	34667	34667	00000	26000
1/6 Tors.-El. Comp....	35535	18035	35665	22540	35712	24045	35000	4333
2/6 Tors.-El. Comp....	34200	18200	34750	22750	34934	24267	32000	8666
3/6 Tors.-El. Comp....	34330	18630	35562	24537	35970	26170	29400	12000
4/6 Tors.-El. Comp....	35315	21915	37440	27390	38160	29230	26800	17333
6/6 Tors.-El. Comp....	35613	24780	39970	30970	42040	33040	24000	21666

El stands for Elastic Limit.

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XLVII. *Tests of Metals in Reverse Torsion.* By E. L. HANCOCK, *Assistant Professor of Applied Mechanics, Purdue University, La Fayette, Ind.**

[Plates IX. & X.]

IT is known generally that, in a cylindrical bar acted upon by a torsional couple, the outer fibres are subjected to the greatest stress, and the stress on the inner fibres varies as the distance from the centre of the bar. If the torsional couple be sufficiently increased the outer fibres are stressed to the elastic limit of the material, and any further increase causes an overstrain of these fibres, the depth of overstrain depending upon the amount that the couple has been increased beyond the elastic limit. If, now, the couple be released, the bar does not return to its original condition, but shows a set, and in its new condition possesses properties quite different from those possessed originally. Its strength to resist torsion is now greater in the direction of the original couple than in the opposite direction. A simple analysis shows, that *the bar is twice as strong in one direction as in the other* (see paper by James Thomson, Cambridge & Dublin Math. Journ., Nov. 1848, and Lord Kelvin, Elasticity, *Encyc. Britannica*). This means that in its new condition the bar has two limits of elasticity—one higher than the elastic limit before overstrain, and one lower. The absence of experimental data on this point led the writer to undertake a series of tests (about 150 in all) to determine the truth of the theoretical determinations as applied to iron and steel, and to determine whether or not the known phenomena shown by metals when overstrained in tension or compression, held also for torsion.

Materials.—The materials tested were: low nickel-steel, low carbon-steel, common wrought-iron, and staybolt wrought-iron. All specimens were about 20 inches long and tested on a gauge-length of 10 inches. The diameters varied from 0·5 inch to 1·0 inch. None of the specimens were turned down or annealed.

Method of Testing.—The tests were made in the laboratory for testing materials at Purdue University, on a Riehle torsion machine of 60,000 inch-pounds capacity (see Pl. IX. fig. 1). The suspended head of this machine allowed the specimen to shorten without introducing external tensile forces, and gave more accurate results than could have been obtained by using a machine where no such adaptation was possible. In

* Communicated by the Author, having been read before the American Society for Testing Materials, June 1906.

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order to test the materials in the reverse direction, it was necessary to devise some special attachment for the machine. This finally took the form of a lever attached to the suspended head, as shown in fig. 2. The lever carried a bucket at the free end, to provide for the insertion of the weights. By reversing the jaws of the machine and also the motion, it was possible to give the specimen the desired reverse twist.

After the specimen was properly inserted and the lever and bucket in place, the beam of the testing machine was balanced, eliminating from consideration the weight of the parts attached. The distance from the centre of the test piece to the point of attachment of the bucket being known (60 inches), made it possible to add weights to produce any desired number of inch-pounds of moment. The loads thus added caused the scale-beam to rise. By running the machine in the reverse direction, the load was gradually taken up by the test piece, the balancing of the scale-beam indicating when all the load had been so taken up. Deformations were measured by means of an Olsen troptometer, as seen in fig. 2, and the routine testing, in the reverse direction, consisted of adding the desired weights, running the machine backward until the specimen took up the load, and then measuring the corresponding deformation. To test the accuracy of the apparatus, series of ordinary torsion tests were made, first with the arm attachment and then without; *i. e.*, specimens were tested in simple torsion using the arm attachment, and then others were tested in torsion in the same direction without using the arm attachment (in each case the specimens had not previously been tested). The results were precisely the same, showing the accuracy of the arm attachment.

Outline of Work.—As before stated, the tests were made to determine the effect of overstrain, of the outer fibres in torsion, on the elastic strength of these fibres in the opposite direction, *i. e.*, against stress of the opposite sign, or, it might be stated, the effect of overstrain in tension on the elastic properties in compression. To accomplish this the following plan of work was carried out:—

(*a*) Bars tested in torsion to some stress below the elastic limit (proportional limit of bar) and the direction of torsion reversed immediately.

(*b*) Bars tested in torsion to the elastic limit and the direction of torsion reversed, (1) immediately, (2) after a certain time.

(*c*) Bars tested in torsion to the yield-point (of the bar) and then the direction of torsion reversed, (1) immediately, (2) after a certain time.

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The elastic limit, as used in this paper, is the proportional limit of the bar, and the yield-point, the yield-point of the bar. This latter for torsion is not so definitely marked as in the case of tension or compression, and might, more properly, be called a *yield region*; it is to be looked for, in ductile materials, a little beyond the elastic limit where very great deformations take place for small increase of torque. The bars were considered overstrained when stress ceased to be proportional to strain.

Results.

The results of the tests outlined are shown by the following plates and tables:—

Plate X. fig. 1, shows the results of tests made on nickel-steel specimen (1 C).

Test No. 1: to elastic limit.

Test No. 2: reversed after 24 hours to a point a little beyond the elastic limit.

Test No. 3: direct to yield-point (not shown on curve) after 151 days.

Test No. 4: reversed to yield-point after 6 days (yield-point not shown on curve).

Test No. 5: direct test to rupture after 6 days.

Test No. 6: portion of No. 5 to rupture—reverse—after 2 days.

(Note: curves are drawn only a little beyond the elastic limit in most cases.)

Fig. 2 shows the results of tests made on nickel-steel specimen (1 D). (The curves as numbered run from left to right.)

Test No. 1: direct to elastic limit (set curve shown).

Test No. 2: load of No. 1 held for 15 hours, then taken off and reversed a little beyond the elastic limit.

Test No. 3: direct to yield-point after 158 days.

Test No. 4: reversed to yield-point after 5 days.

(Note: yield-point in 3 and 4 not shown.)

Fig. 3 shows the results of tests of a nickel-steel specimen (2 C) (3). (The curves as numbered run from left to right.)

Test No. 1: direct to yield-point.

Test No. 2: reversed to yield-point after 30 days.

Test No. 3: direct to yield-point after 151 days.

Test No. 4: reversed to yield-point after 113 days.

Test No. 5: direct to yield-point after 4 days.

(Note: No attempt has been made to show the yield-point in fig. 3). Figs. (4), (5), (6), (7), and (8) are self-explanatory.

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Results of series (*a*), where the torsion was not carried to the elastic limit in the first test, show that this test had no appreciable effect upon the elastic properties of the material in the opposite direction. But when the stress in the first test (direct test) is carried to, or beyond, the elastic limit, the elastic properties are changed. From fig. 1 it is seen that the elastic limit in the second test was changed very little by the first test to the elastic limit, but the elastic limit in the 4th test, due to the overstrain in the 3rd, is much lower (about one-third as much), even after a rest of 6 days. The 5th test gave a higher elastic limit than the 4th, due probably to the fact that the stress was in the same direction as the original. All curves are nearly parallel, showing that there was little, if any, change in the modulus of elasticity. Fig. 2 shows that the first stress to the elastic limit had little, if any, effect upon the elastic limit in the reverse direction. A slight overstrain is here indicated by the set curve (4th curve from left to right). The overstrain in this test, even after a rest of 158 days, caused a considerable lowering of the elastic limit. The overstrain in the third test reduced the elastic limit in the fourth test almost one-half. In this case the stress was reversed after five days.

When the piece was stressed to the yield-point, as shown in fig. 3, the elastic limit in the reverse direction is less than one-half of what it was at first, even after a rest of 30 days. Overstrained in this direction, and allowed to rest 151 days, the specimen shows an elastic limit somewhat higher than in the second test. The third overstrain, with a period of rest of 113 days, shows the elastic limit raised to about what it was in the third test. The fourth overstrain in the reverse direction after 4 days shows a lowering of about one-third in the elastic limit. The behaviour of several pieces of nickel-steel is shown by the table on p. 430.

(*Note.*—The abbreviations, e.l., y.p., and rup. refer to the elastic limit, yield-point, and rupture respectively.)

The effect on the elastic limit of immediately reversing the stress after overstrain to the yield-point is shown for certain pieces of nickel-steel in fig. 4. The overstrain destroyed the elasticity of these pieces. Low carbon-steel stressed to the yield-point and reversed after seven days, shows an elastic limit of less than one-third the original; when reversed immediately no elasticity is exhibited (see fig. 5). When the original stress is carried to, or a little beyond, the elastic limit, the specimens show some elasticity when reversed immediately (see fig. 6). Wrought iron stressed to the yield-point and reversed exhibits the same properties as steel (see figs. 7 and 8).

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Specimen.	Stressed to	Reversed after — days to	Direct after — days to	Reversed after — days to	Direct after — days to	Reversed after — days to
1 D	e.l.	held.	158 y.p.	5 y.p.		
1	e.l.	7 e.l.	173 y.p.			
1 C	e.l.	1 e.l.	151 y.p.	6 y.p.	6 rupture.	2 rupture.
1 E	¾ e.l.	69, ¾ e.l.	113 y.p.	113 y.p.	4 y.p.	
1 B	e.l.	0 e.l.	164 y.p.	6 rup.		
2 C	y.p.	30 y.p.	151 y.p.	113 y.p.	4 y.p.	71
Stress in pounds per sq. inch at the elastic limit for above cases.						
1 D	32,600	30,500	25,000	14,000		
1	30,500	14,400	31,200			
1 C	36,000	35,100	34,400	13,200	14,000	13,600
1 E	not	reached.	31,800	18,800	14,000	
1 B	32,500	26,500	30,400	14,000		
2 C	33,000	18,450	24,000	24,000	15,900	25,350
Average...	32,900					

Conclusions.

The curves show that when wrought-iron or mild-steel is stressed in torsion below its elastic limit (proportional limit), its elastic properties in the opposite direction are not impaired. If the original stress be below the yield-point but above the elastic limit, the elastic limit in the reverse direction is lowered but does not disappear even when the stress is reversed immediately. If the original stress be above the yield-point, the elasticity in the reverse direction is destroyed but reappears, gradually, after some rest. The modulus of elasticity is unchanged when the stress in the first test is below the yield-point. It is not constant, for reverse tests, when the original stress is carried beyond the yield-point, but comes back to its normal value after a period of rest.

The results obtained in these tests are in accord with those already worked out for tension and compression ; and, indeed, the present case of torsion may be considered as a unique way of changing from compression to tension, or from tension to compression, as is easily seen by considering the particles of the cylindrical test-piece on the 45 degree helix.

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XLVIII. *Notices respecting New Books.*

The Theory of Determinants in the Historical Order of Development. By THOMAS MUIR, *M.A., LL.D., F.R.S.* Second Edition. London: Macmillan & Co., Ltd. 1906.

IN August 1890, the first edition of this work was reviewed in these pages. The present work is nearly twice the size of the earlier volume; but that is almost entirely because of the new matter added under Part II., which treats of the development of special determinants down to the year 1841. Part I. gives an account of general determinants down to the same date, and is, with a few additions, essentially the same as the first edition. A suggestion made at the end of the former review that the names and dates of authors being discussed should be mentioned in the head-lines of the pages has been followed, greatly increasing ease of reference. The different chapters of Part II. deal with Axisymmetric Determinants, Alternants, Jacobians, Skew Determinants, Orthogonants, etc., all of which received attention at the hands of mathematicians before the specific name had been invented. This indicates one great merit of Dr. Muir's labours, which—to quote his own words—show "the extent to which the study of special forms of determinants had been carried prior to 1841, an extent probably hitherto unsuspected." In these earlier days authors had a tendency to use their own particular notations, which in certain cases differed markedly from the notations now familiar; and here again mathematicians cannot be too grateful to Dr. Muir for the masterly way in which he has brought the whole literature into line. He quotes important parts of the original papers, explains the notations, and elucidates the results obtained in many cases by translating them into modern notation. The broad features are clearly brought out in three short chapters termed 'Retrospects.' In recent papers communicated to the Royal Society of Edinburgh, Dr. Muir has continued his historical studies down to a later date than 1841; but there is no hint in the present volume that he contemplates a discussion of the more modern developments in the same systematic and thorough manner. The closing sentence of the book states that "The fruitful era of nomenclature, but not of that alone, was ushered in by Sylvester shortly after the date with which" the present volume closes. It may be doubted whether an historic sketch, concise yet complete, of these later developments can be given; but if it can, then no one could do it better than Dr. Muir.

Lectures on the Method of Science. Edited by T. B. STRONG. Oxford: at the Clarendon Press. 1906. Pp. viii+249.

THE present volume is the outcome of a new plan adopted by the Delegates for the Extension of University Teaching during the Summer Meeting at Oxford, in August 1905. Instead of providing a course on some special branch of science, it was suggested

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to illustrate the methods of scientific enquiry by explaining their application to problems in various departments of research. The course consisted of the nine lectures embodied in the volume before us. The first two are of a general nature, "Scientific Method as a Mental Operation," by Professor T. Case, and "On Some Aspects of the Scientific Method," by Professor Gotch. The remaining lectures serve as illustrations of the working of scientific methods in connexion with various actual problems. Professor Sherrington writes on "Physiology; its Scope and Method"; Professor Weldon on "Inheritance in Animals and Plants"; Mr. McDougall on "Psycho-Physical Method"; Dr. Fison on "The Evolution of Double Stars"; Sir Richard Temple on "Anthropology: The Evolution of Currency and Coinage"; Professor Flinders Petrie on "Archæological Evidence"; and the General Editor on "Scientific Method as Applied to History." It will be seen that a fairly large number of different departments of scientific enquiry are represented. The work is intended for the general reader, and the interesting manner in which most of the problems are dealt with cannot fail to render the book highly attractive to the mass of the well-educated public.

Moteurs à Collecteur à Courants Alternatifs. Par le Dr. F. Niethammer, Professeur à l'École Technique Supérieure de Brünn (Autriche). Paris: edité par "L'Éclairage Électrique." 1906. Pp. 132.

Few subjects have, within the last three years, attracted more attention or excited keener interest in electrotechnical circles than the single-phase commutator motor, whose advent at a time when the problem of main-line electric traction was being discussed opened up new possibilities of development. The unremitting efforts of all the large manufacturing companies to perfect the single-phase motor afford ample testimony to the importance attached to the solution of this problem. The monograph under review comes at an opportune moment, and will be welcomed by all who have been following the evolution of the commutator motor. Although the author includes in his treatment polyphase as well as single-phase motors, the former type is of far less technical importance, nor is it likely to find any extended use in the future. Much of the matter included in the present monograph has already made its appearance in the form of articles in various technical periodicals, but most readers will be glad to have the author's contributions to this subject arranged systematically in the form of a monograph. The booklet is divided into three chapters. In the first, the author deals with generalities, including a brief historical sketch. The second chapter is devoted to the theory of commutation and the construction of exact diagrams to represent the behaviour of the motor. The concluding chapter contains descriptions of leading types of single-phase commutator motors, and a useful outline of the principles underlying the design of such motors.

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THE

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[SIXTH SERIES.]

NOVEMBER 1906.

XLIX. *Experiments on the Propagation of Longitudinal Waves of Magnetic Flux along Iron Wires and Rods.* By

THOMAS R. LYLE, *M.A., Sc.D., Professor of Natural Philosophy in the University of Melbourne, and* J. M. BALDWIN, *M.A., B.Sc.**

1. THE following experiments were undertaken in order to investigate the changes in the Fourier characteristics of waves of magnetic flux as they travel along iron wires and rods. In all cases these flux-waves were started at the middle points of the rods or wires by sending alternating currents through short coaxial solenoids placed there.

The subject has already been attacked in different ways by several experimenters, and from the difference in phase of the flux oscillations at two points a so-called "velocity of magnetization" has been obtained from the formula,

$$v = \frac{2\pi l}{\phi T},$$

where v = the "velocity of magnetization,"
T = the period of the oscillations,
ϕ = the difference in phase at two points l centimetres apart.

Oberbeck † worked with rods and bundles of wires, in some cases 40 cm., in others 1 metre long, and by means of a

* Communicated by the Physical Society: read January 26, 1906.
† A. Oberbeck, Wied. *Ann.* vol. xxii. p. 73 (1884).

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dynamometer measured the phase-difference of the currents induced in two short solenoids placed at various distances apart on the rod. This was equal to the phase-difference of the two resultant fluxes at the points, provided the time-constants of the circuits of the two solenoids were negligible.

The "velocity" deduced varied from 232,300 cm./sec. for 64 steel wires each 1·8 mm. diam., to 4410 cm./sec. for a steel rod 12 mm. diam., a result which indicates the great influence of eddy currents. He found that the "velocity" was not directly influenced by the character of the iron of the rod

The amplitude F_x of the resultant flux at different points he found to be given by the formula

$$F_x = F_0 e^{-\lambda x},$$

where λ is to a first approximation a constant depending on the material of the bar, but independent of the area of its cross-section.

The values he found for λ when x, the distance from the centre of the magnetizing solenoid, is in centimetres, were

·1027, ·1017, ·1007 for soft iron,
·148 for hard iron,
·1451, ·1616, ·1637 for steel.

One frequency only was used.

Trouton * attempted to obtain stationary flux-waves by the interference of two trains of waves travelling in opposite directions round a ring of iron wires, the two trains being produced by an alternating current of known frequency circulating in a short solenoid looped on the ring. He concluded, however, that the effects which he observed were not due to interference but to some peculiarity of the ring.

Zenneck †, assuming the permeability μ and the leakage coefficient $\lambda = -dF/Fdx$ constants, where F is the flux at x, obtained from theory expressions for the flux-waves at different points along iron cylinders in terms of the initial flux where $x=0$, and to verify his theoretical conclusions used a Braun tube to determine the relative amplitudes and phases of the flux oscillations at any two points.

For thick wires (some millimetres in diameter) he found that the "velocity" and the leakage coefficient λ increased with frequency, while for wires less than 1 mm. in diam.

* F. T. Trouton, 'Nature,' vol. xlv. p. 42 (1891).

† J. Zenneck, *Ann. der Phys.* vol. ix. p. 497 (1902), and vol. x. p. 845 (1903).

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they were, to a first approximation, independent of the frequency.

More recently Perkins*, working at a frequency of 60, has measured by means of a quadrant-electrometer the amplitude and phase, at different points along an iron bar, of the flux oscillation which had been started in the usual way at the centre. The bar used was 1 metre long, 2·83 cm.^{2} in cross-section, and observations were made over a range of 25 cm. from its centre.

The phase was found to change less rapidly as the distance from the magnetizing-coil increased, and the "velocity of magnetization" obtained varied from

7500 cm./sec. at the centre

to 24000 cm./sec. at 24 cm. from the centre.

The curve he obtained showing the relation between the lag in phase of the flux at any point and the distance of the point from the magnetizing-coil agrees in general character with the corresponding small portions of the curves obtained by us.

The same problem has been theoretically discussed by Thomson †. He assumes that the permeability is constant for all values of the induction, and takes no account of the lag in phase of the induction behind the magnetizing force. As the flux-density or induction diminishes with great rapidity as we pass down the bar, and as both the permeability and the lag in phase of the induction vary within wide limits for different values of the latter, the theoretical conclusions arrived at on the assumption that one of these two quantities is constant and the other negligible, can scarcely be expected to be realized in practice.

2. In all the experimental investigations hitherto carried out on this subject, the range along the rod over which it was possible to determine even roughly the amplitude and phase of the flux oscillations at different points was limited, and the apparent behaviour over a short range afforded a very doubtful and misleading description of the whole phenomenon. No attempt, moreover, was made to determine the change of form of the flux-waves as they passed along the bar.

For this investigation the wave-tracer ‡ designed by one of us is peculiarly well suited, as it enables us to determine

* H. A. Perkins, Amer. Jour. Sci. vol. xviii. p. 165 (1904).

† J. J. Thomson, 'Recent Researches,' p. 392.

‡ T. R. Lyle, "Wave-Tracer and Analyser," Phil. Mag. vol. vi. p. 549 (1903).

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completely, not only current and E.M.F. waves, but also flux-waves over a wide range of amplitudes.

For instance, by its means can be accurately determined flux-waves whose amplitudes range from very large values down to a fraction of a single magnetic line; and by subjecting its readings to harmonic analysis we have been able to obtain the Fourier characteristics of the resultant flux oscillations at all points along rods of iron or steel, $\frac{1}{4}$ in. in diam. and 10 feet long, which have been magnetized by an alternating current circulating in a short solenoid placed at their middle points, when the amplitude of the flux-density at the centre was as low as 10,000.

3. The experiments, of which the results will be given below, were made on

A. A long rod of Lowmoor iron $\frac{1}{4}$ in. in diameter.
B. A long rod of silver steel $\frac{1}{4}$ in. in diameter.
C. A long iron wire $\frac{1}{6}$ in. in diameter.
D. A bundle of 185 charcoal iron wires, each ·079 cm. in diameter, 370 cm. long, tightly taped together so as to form a circular cylinder.
d_1. A bundle of 46 of the wires from D.
d_2. A bundle of 12 of the wires from D.

In each case at the middle point of the rod or bundle a short coaxial solenoid was placed, and through it an alternating current obtained from a rotary converter supplied with direct current from storage-cells was sent.

The magnetizing-current wave and the resultant flux-waves, crossing the sections of the bar or bundle at different points along its length, were quantitatively determined by the wave-tracer using the galvanometer method * described in the paper already quoted. The wave forms so obtained were subjected to harmonic analysis, the results of which are given in the Tables below. Some of the more interesting of the results of the investigation are also exhibited by means of curves.

The arrangement of the apparatus, the method of experiment, and the reduction of the observations were the same in all essential features as that fully described in a former paper † by one of us. Instead of the fixed secondary coil of 5 or 10 turns wound on the iron rings used in the experiments just quoted, three small search-coils were used. One of these, wound close to the rod or bundle, was fixed at the

* T. R. Lyle, "Wave-Tracer and Analyser," Phil. Mag. vol. vi. p. 549 (1903).

† T. R. Lyle, "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. vol. ix. p. 102 (1905).

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centre of the magnetizing solenoid; while the other two, of 166 and 805 turns respectively, could be moved from point to point along the rod, the one with the larger number of turns being used for exploring the rod at points near its ends.

During each series of experiments the magnetizing current and the rate of alternation were kept as constant as possible, and a continuous record was made by a chronograph of the times taken for every 200 periods.

The method of procedure in any one series of experiments was as follows:—The speed of the rotary converter having been adjusted to the desired value by means of resistances in the field circuit, or, if necessary, by changing the number of storage-cells used on its direct-current side, the magnetizing current drawn from its alternating side was adjusted by means of resistances to the desired value, and allowed to run until all conductors had attained a steady temperature.

By means of the wave-tracer equi-spaced ordinates of the magnetizing current wave sufficient in number to determine it were now taken. Then the corresponding ordinates of the flux-waves at the centre and at different points along the rod were obtained, and finally the ordinates of the current wave were redetermined to serve as a check on the constancy of the magnetizing force during the run.

Each set of ordinates was now subjected to harmonic analysis and the results affected by their proper factors * to reduce them either to current or flux, as the case might be, in absolute measure.

Three frequencies were used with specimens A and C, and two with specimen D, the central or initial flux being approximately the same for the different frequencies in any one specimen. With A and D additional series were obtained at one of the frequencies already used, but with different initial fluxes. For each of the other specimens B, d_1, and d_2, the results of one series of experiments only are given below.

For all the specimens but d_1 and d_2, by means of a ballistic galvanometer, using the method of reversals, the fluxes at different points along their lengths, due to a continuous magnetizing current in the central solenoid, were determined when the continuous current was such as produced at the centre a flux approximately equal to the amplitude of the central alternating flux previously used.

In addition the statical permeability of the material of each of the specimens for different values of the flux density was obtained by the method of reversals, a long solenoid being used to carry the magnetizing current.

* T. R. Lyle, "Wave-Tracer and Analyser," Phil. Mag. vol. vi. p. 549 (1903).

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The specific resistance of each specimen was also measured.

In the case of specimen A. after all the observations that were deemed necessary for its full length had been taken, it was shortened by successive steps by cutting off each time equal lengths from both ends; each reduced length was then investigated for one frequency and one central flux.

4. We now proceed to give the results of the different experiments.

For every series the Fourier characteristics of the magnetizing current wave and of the resultant flux wave at different distances from the centrally placed magnetizing solenoid will be given in tabular form, and curves will be given showing how the phase, and the natural logarithm of the amplitude, of the first harmonic of the resultant flux varies as we pass along the specimen from the centre to the end. In the tables will also be given for different points the values of the leakage coefficient λ_1 of the amplitude f_1 of the first harmonic of the flux, where

$$\lambda_1 = -\frac{1}{f_1}\frac{df_1}{dx} = -\frac{d}{dx}(\log_e f_1)$$

$$= \frac{(\log_e f_1)_x - (\log_e f_1)_{x'}}{x' - x} \text{ (q. p.)}$$

at the point $\frac{1}{2}(x+x')$, where x and x' are two adjacent points at which f_1 has been determined. By means of the last expression the values of λ_1 have been calculated, and they are placed in the tables between the two adjacent rows of characteristics from which they have been obtained.

In the tables T will represent the period of the alternating current and flux in any one experiment, x the distance from the centre of the specimen (and of the magnetizing solenoid) of the section across which the resultant flux F (in absolute measure) passes, and C the magnetizing current used during one series (also in absolute measure).

No harmonics higher than the 5th will be given, though they were determined in all cases, unless their amplitudes were below one per cent. of the amplitude of the corresponding first harmonic.

The galvanometer circuit of the wave-tracer when search-coil No. 3 was used had a very small time constant which was allowed for.

[*Note.*—The figures given in the tables are the results from the individual experiments. No smoothing of values has been resorted to, though a small error in the setting of the search-coil would make a large error in the result.]

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5. *Details of Specimen* A.

A straight cylindrical rod of Lowmoor iron.
Length = 267 cms.
Diameter = ·644 cm. Section = ·3260 sq. cm.
Length of magnetizing solenoid ..... 2·54 cm.
Radial depth of „ „ ..... 2·40 cm.
Mean radius of „ „ ..... 1·70 cm.
No. of turns on „ „ ..... 240.

Specific resistance of the iron = $1{\cdot}140 \times 10^4$.

Statical Permeability for different Inductions.

B ...	25	50	100	250	500	750	1000	2000	3000	5000	7000	9000	11000	13000	15000
μ ...	290	325	370	451	550	650	750	1160	1490	1960	2215	2300	2165	1720	900

The analysed results of the alternating current experiments with this specimen are given in the following tables (I. to IV.), and the values of the flux and leakage coefficients at different points due to a continuous magnetizing force at the centre of the rod are given in Table V.

TABLE I.—Specimen A. T=·053 q.p.

$C = {\cdot}1371 \sin \omega t + {\cdot}00642 \sin 3(\omega t - 55{\cdot}28) + {\cdot}00057 \sin 5(\omega t - 68{\cdot}23)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	T.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0...	·0537	3798	51·7	23·3	30·64	93·35	31·85	0	·0744
5...	·0536	2618	70·1	14·0	41·42	92·04	32·19	10·78	·0852
10...	·0534	1710	54·8	7·97	53·59	92·35	29·17	22·95	·0826
15...	·0531	1131	37·6	5·54	64·41	94·68	31·68	33·77	·0855
20...	·0532	738	23·4	1·85	74·46	98·09	31·03	43·82	·1009
30...	·0518	269·1	6·83	...	93·21	106·06	...	62·57	·1167
40...	·0528	83·8	1·50	0·49	105·42	109·52	38·67	74·78	·1134
50...	·0530	27·0	0·28	...	107·27	...	...	76·63	·1042
60...	·0526	9·51	...	...	101·97	...	...	71·33	·0775
70...	·0515	4·383	...	...	92·31	...	...	61·67	·0529
80...	·0526	2·583	...	...	84·80	...	...	54·16	·0394
100...	·0527	1·175	...	...	78·00	...	...	47·36	·0333
120...	·0528	0·604	...	...	74·61	...	...	43·97	·0626
130...	·0528	0·323	·0058	·0035	69·64	133·3	...	39·00	

TABLE II.—Specimen A. T=·053 q.p.

$C = \cdot 2154 \sin \omega t + \cdot 01077 \sin 3(\omega t - 55 \cdot 82) + \cdot 00072 \sin 5(\omega t - 72)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	T.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0...	·0530	5009	125·6	43·05	30·72	96·80	37·6	0	
10...	·0530	2232	109·3	23·7	54·80	88·69	25·2	24·08	·0809
20...	·0530	985·1	40·5	4·38	76·18	89·84	28·0	45·46	·0818
30...	·0532	393·4	9·46	...	94·87	95·80	...	64·15	·0918
40...	·0532	132·1	1·64	...	109·27	106·73	...	78·55	·1091
50...	·0532	42·0	...	...	113·64	...	...	82·92	·1146
60...	·0532	14·32	0·15	...	109·13	158·4	...	78·41	·1076
70...	·0531	6·16	0·10	...	98·23	177·2	...	67·51	·0843
80...	·0527	3·48	0·06	...	89·37	186·6	...	58·65	·0571
100...	·0525	1·572	0·027	...	80·84	195·9	...	50·12	·0397
120...	·0526	0·797	0·017	...	75·94	196·8	...	45·22	·0340
130...	·0526	0·428	0·009	...	71·04	203·0	...	40·32	·0622

TABLE III.—Specimen A. T=·032 q.p.

$C = \cdot 1649 \sin \omega t + \cdot 00655 \sin 3(\omega t - 58 \cdot 61) + \cdot 00096 \sin 5(\omega t - 36 \cdot 1)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	T.	f_1.	f_3.	f_5.	θ.	β_3	β_5.	$\theta_x - \theta_0$.	λ_1.
0...	·0320	3730	78·14	26·8	33·04	89·39	32·02	0	
10...	·0320	1530	66·2	14·2	59·70	86·88	24·30	26·66	·0891
20...	·0319	596·6	17·1	...	84·89	91·19	...	51·85	·0942
30...	·0320	192·0	3·6	...	107·21	104·68	...	74·17	·1134
40...	·0320	52·9	0·83	...	122·74	110·30	...	89·70	·1289
50...	·0321	15·14	0·12	...	121·88	136·54	...	88·84	·1251
60...	·0321	5·42	0·056	...	111·21	161·53	...	78·17	·1027
70...	·0321	2·783	0·029	...	99·47	178·23	...	66·43	·0667
80...	·0322	1·740	0·019	...	93·06	182·94	...	60·02	·0470
100...	·0322	0·862	...	...	88·20	..	...	55·16	·0351
120...	·0323	0·450	...	...	84·40	...	...	51·36	·0325
130...	·0322	0·237	...	...	77·3	...	...	44·3	·0641

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TABLE IV.—Specimen A. T=·0216 q.p.

$$C = \cdot 1776 \sin \omega t + \cdot 00728 \sin 3(\omega t - 58 \cdot 3) + \cdot 00087 \sin 5(\omega t - 30 \cdot 55).$$
$$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$$

x.	T.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ.
0...	·0216	3537	90·6	37·7	33·75	88·57	30·68	0	
5...	·0216	2220	96·3	25·6	48·04	85·30	25·96	14·29	·0932
15...	·0216	805·5	29·3	3·4	77·30	85·64	22·53	43·55	·1014
25...	·0216	251·9	4·35		104·37	99·48		70·62	·1162
35...	·0217	62·05	0·67		125·69	111·39		91·94	·1401
45...	·0217	14·90	0·20		132·64	124·01		98·89	·1426
55...	·0217	5·02	0·06		117·00	158·35		83·25	·1088
65...	·0217	2·437	0·038		103·53	169·67		69·78	·0723
80...	·0217	1·236	0·015		94·73	178·70		60·98	·0453
100...	·0217	0·597	0·009		90·59	180·57		56·84	·0364
120...	·0216	0·336	0·006		87·13	183·43		53·38	·0287
130...	·0217	0·194	0·005		79·47	179·73		45·72	·0547

TABLE V.—Specimen A. T = ∞.
(Static Leak.)

x.	C.	F.	λ.	x.	C.	F.	λ.
0 ...	·0751	3863		40...	·0749	261·3	
5 ...	·0750	2992	·0511	50...	·0752	100·8	·0953
10 ...	·0752	2254	·0566	60...	·0748	38·7	·0955
20 ...	·0751	1237	·0600	80...	·0750	7·44	·0825
30 ...	·0751	609·4	·0708	100...	·0750	2·60	·0525
40 ...	·0749	261·3	·0847	110...	·0750	1·49	·0557

6. In order to exhibit graphically some of the more striking results contained in the preceding tables, the curves in figs. 1 and 2 have been plotted. In fig. 1 the abscissæ are distances x from the origin or magnetizing solenoid and the ordinates of one set of curves are the natural logarithms of the amplitudes of the first harmonic of the flux at different distances, while the ordinates of the other set of curves are the retardations in phase of the first harmonic of the flux at different distances behind that of the initial flux where $x=0$.

It will be seen that the $\log f_1$, x, curves are far from being straight lines as Zenneck assumed for the basis of his theoretical work. This is still more clearly shown in fig. 2,

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Fig. 1.

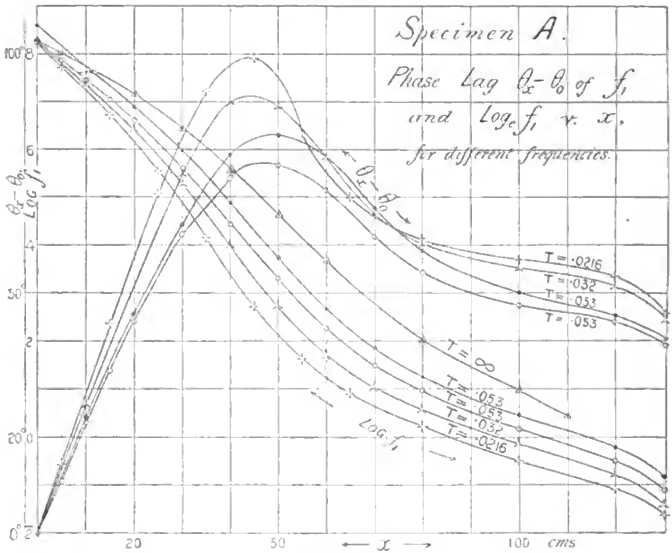
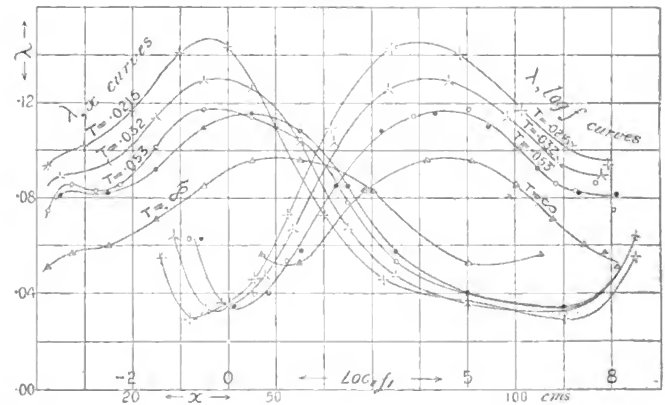


Fig. 2.



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where the leakage coefficients $\lambda_1 = -d(\log f_1)/dx$ are plotted against the distances from the origin at which they occur, as well as against $\log f_1$, the logarithm of the corresponding flux. Thus in all cases, including the statical, as we move away from the magnetizing coil, λ_1 first increases, attains a maximum, and then diminishes until near the end of the rod when it again increases. For the same initial flux both the points at which max. λ_1 occurs and the corresponding fluxes are different for different frequencies.

Figs. 1 and 2 show the effects of the end of the rod on the characteristics of the fluxes in its neighbourhood. Thus in fig. 1 the downward turn of the flux and phase curves due to the end effect can be recognized 30 cm. from the end of the rod, and the corresponding rise in the λ curves in fig. 2 is very marked. The end effect always manifests itself in this way, and in very long rods this change in the direction of curvature does not appear.

From the λ_1, $\log f_1$ curves in fig. 2 we see that for the same value of f_1, λ_1 increases with the frequency. In the same set it will be noticed that one curve can be drawn (q.p.) through the two series of points for λ_1 when $T = \cdot 053$ arising from two different initial fluxes. This would suggest that λ_1 is independent of x and is a function of f_1 only. In the sequel it will be seen that λ_1 does depend on x as well as on f_1, and the coincidence of the above two curves is due to the fact that with the initial values used (Tables I. and II.) there is never much difference between the abscissæ at which equal fluxes occur in the two series.

Returning to fig. 1 we see how, for different frequencies, the phase of the first harmonic of the flux at any point x lags behind that of the initial central flux at $x = 0$.

Thus as the flux moves away from the magnetizing coil its phase is at first retarded at a fairly regular but diminishing rate for any one frequency, while for the higher frequencies the rate of retardation $d\theta/dx$ is higher. To this part of the phenomenon the observations of Oberbeck, Zenneck, and Perkins have been limited, and following them we would deduce (see § 1) from our results the following mean values for the "velocity of magnetization" over the first 30 cms. of the rod in question :—

3380 cm./sec.	when	$T = \cdot 053$ sec.
4790 cm./sec.	,,	$T = \cdot 032$,,
6230 cm./sec.	,,	$T = \cdot 0216$,,

Continuing along the rod, however, we see that the space-rate of retardation $(d\theta/dx)$ of the phase becomes zero, and

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the retardation itself a maximum at distances between 40 and 50 cms. from the starting-point, depending upon frequency and initial flux. After this the retardation diminishes, that is, the phase of the flux advances for the remainder of the length of the rod, and near the end the rate of advance $(-d\theta/dx)$ is greatly increased, this latter being due to the end effect.

From this it would seem that the deduction by Oberbeck and others of a "velocity of magnetization" from the rate of retardation $(d\theta/dx)$ of the phase of the flux close to the source by means of the formula

$$v=\frac{2\pi}{T\frac{d\theta}{dx}}$$

can hardly be legitimate.

For if so, we should have to allow that the "velocity" is infinite where the phase is stationary and $d\theta/dx=0$, and negative beyond this point, where the phase advances and $d\theta/dx$ is negative.

It is interesting to note that the points at which maximum phase retardation and maximum leakage coefficient occur are always situated very near each other.

7. It was thought that perhaps for the same frequency the phenomena at points in the rod beyond x might depend in great part only on the flux at x; and if that were so, by making the initial flux as small as that at which maximum retardation occurred in any one of the preceding series, we should get an advance of phase right from the start at the magnetizing coil. To test this a rod of $\frac{1}{4}$ in. Lowmoor iron 12 ft. long, taken from the same stock as specimen A, was used *.

The phase of the flux-wave at any point was taken as given directly by the divided-circle readings of the wave-tracer when the ordinate was zero, and the amplitude was obtained by applying the proper factor to the maximum ordinate of the same flux-wave. This was sufficiently accurate for the purpose in hand. The results obtained are exhibited graphically in figs. 3, 4, and 5. The initial values of the fluxes used were 69, 163, 288, 687, 1732, 2994, and 4100, while the period was ·052 sec. (q.p.) in all cases. In fig. 3 the logarithms (nat.) of the fluxes are plotted as ordinates against the corresponding distances from the origin as abscissæ, the points pertaining to any one initial flux being

* When we thought of this, test specimen A had been cut up for the purpose of investigating change of length.

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Fig. 3.

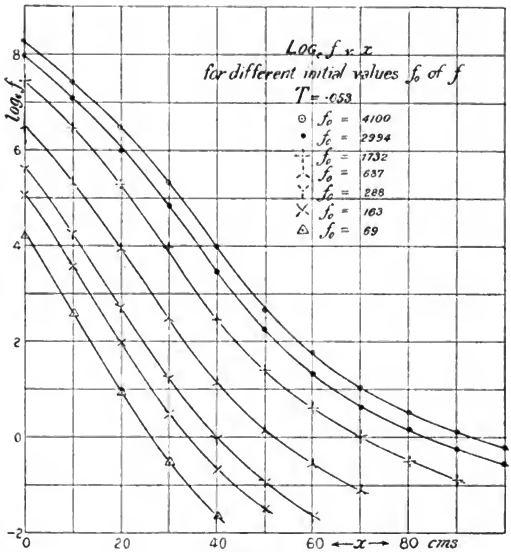
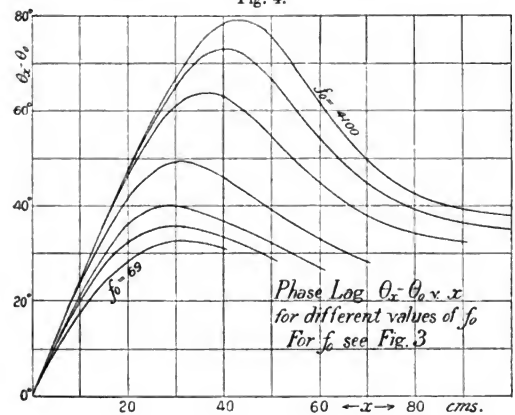


Fig. 4.



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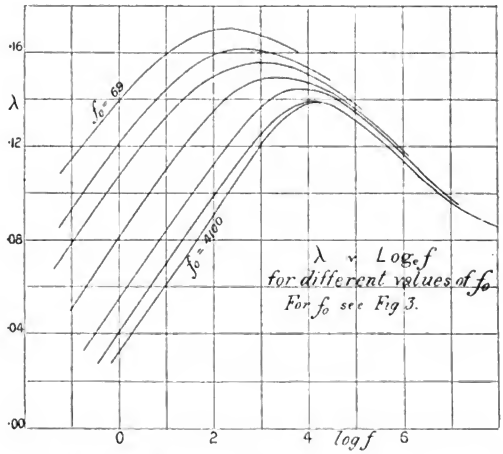
joined (as also in figs. 4 and 5) by a continuous curve. In fig. 4 against the same abscissæ are plotted the retardation in phase of the flux at any point behind that of the initial central flux; while in fig. 5 the leakage coefficients at different points in the several series are plotted as ordinates against the logarithms of their corresponding fluxes.

Fig. 5.



It will be seen that in all cases over the wide range of initial fluxes used, the general nature of the phenomena is the same as that exhibited by figs. 1 and 2 and detailed in Tables I.–IV. Always as the flux moves away from the origin its phase is at first retarded, the retardation attains a maximum, and then the phase advances and keeps advancing (as the rod is long) until the flux is completely dissipated. The leakage coefficient also always begins by increasing, attains a maximum and then diminishes; and we found in this case as in all others that the points on the rod at which maximum retardation and maximum leakage coefficient occurred were practically coincident. A glance at figs. 4 and 5 will show the effect of change of initial flux in the same specimen at the same frequency on the retardations of phase and the leakage coefficients. Thus increase of initial flux increases the maximum retardation, diminishes the

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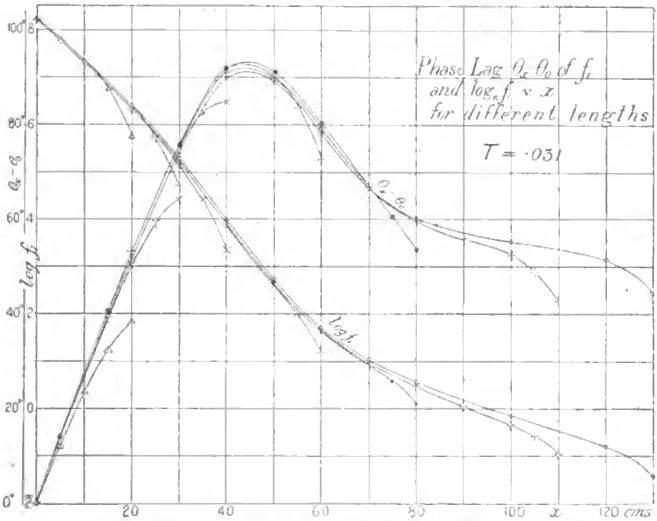
maximum leakage coefficient, and increases the distance from the origin at which these two maxima concur.

As a result of this series of experiments it is plain that what may happen at any one point of the rod, both as regards rate of retardation (or of advance as the case may be) of phase and leakage depends both on the position of the point and on the value of the resultant flux at the point, and if we followed previous investigators in computing a "velocity of magnetization" we should obtain different velocities for the same flux and frequency, provided the points for which the velocity is computed were at different distances from the magnetizing coil.

Fig. 6.



8. Specimen A was now reduced in length in successive steps by cutting each time equal portions off its two ends. For each length one series of observations was taken in which the initial flux and the frequency were approximately the same as in the series already given for the full length in Table III. The results obtained are given in detail in Tables VI. to XI., and the general effect of reduction in length on the lag in phase and on the logarithm of the amplitude of the first harmonic of the flux is exhibited in fig. 6.

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TABLE VI.—Specimen A reduced to $l=225$ cms. $T=\cdot0316$.

$C=1595 \sin \omega t+\cdot0054 \sin 3(\omega t-62\cdot4)+\cdot0007 \sin 5(\omega t-32\cdot4)$.
$F=f_1 \sin(\omega t-\theta)+f_3' \sin 3(\omega t-\theta-\beta_3)+f_5' \sin 5(\omega t-\theta-\beta_5)$.

x.	f_1.	f_3.	f_5'.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	3605	99·7	26·0	32·40	89·83	37·18	0	·0913
10.....	1446	60·0	9·0	59·98	86·18	28·68	27·58	·0961
20.....	553·3	15·8	1·5	85·28	91·70	35·00	52·88	·1078
30.....	188·3	3·0	...	106·80	100·78	...	74·40	·1355
40.....	48·56	0·42	...	123·28	118·29	...	90·88	·1206
50.....	14·53	0·13	...	121·93	137·48	...	89·53	·1027
60.....	5·201	·062	...	111·64	171·46	...	79·24	·0701
70.....	2·580	·034	...	99·18	173·02	...	66·78	·0474
80.....	1·607	·017	...	92·01	180·20	...	59·61	·0430
90.....	1·045	...	...	88·19	...	...	55·79	·0388
100.....	·709	...	...	84·81	...	...	52·41	·0451
105.....	·566	...	...	81·20	...	...	48·80	·0795
110	·380	...	...	74·94	...	...	42·54	

TABLE VII.—Specimen A reduced to $l=165$ cms. $T=\cdot0309$.

$C=\cdot1683 \sin \omega t+\cdot0063 \sin 3(\omega t-60)+\cdot0011 \sin 5(\omega t-33)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	3744	85·5	18·1	31·23	87·9	31·37	0	·0908
5.....	2378	95·9	17·7	44·70	88·08	28·94	13·47	·0922
15.....	945·7	37·6	4·84	71·78	89·15	33·77	40·55	·1087
30.....	185·0	1·9	...	106·68	102·12	...	75·45	·1332
40.....	48·84	0·57	...	123·06	115·74	...	91·83	·1224
50.....	14·37	0·15	...	122·20	141·90	...	90·97	·1025
60.....	5·153	·076	...	111·41	165·4	...	80·18	·0719
70.....	2·510	·028	...	97·95	173·9	...	66·72	·0676
75.....	1·790	·021	...	91·85	181·0	...	60·62	·1145
80.....	1·010	·011	...	84·91	183·0	...	53·68	

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TABLE VIII.—Specimen A reduced to $l=125$ cms. T=·0305.

$C=\cdot 1722 \sin \omega t+\cdot 0066 \sin 3(\omega t-57\cdot 5)+\cdot 0008 \sin 5(\omega t-27)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	3787	89·7	20·7	32·86	90·88	29·23	0	·0917
10.....	1514	61·4	11·6	60·17	87·42	23·57	27·31	·0969
20.....	574·2	17·3	...	85·23	91·36	...	52·37	·1156
30.....	180·8	2·9	...	108·09	103·31	...	75·23	·1360
40.....	46·42	0·61	...	124·62	110·18	...	91·76	·1238
50.....	13·46	0·13	...	122·71	140·8	...	89·85	·1212
55.....	7·34	0·07	...	116·67	152·6	...	83·81	·1596
60.....	3·31	0·04	...	105·61	163·5	...	72·75	

TABLE IX.—Specimen A reduced to $l=85$ cms. T=·0294.

$C=\cdot 1711 \sin \omega t+\cdot 0065 \sin 3(\omega t-59\cdot 8)+\cdot 0007 \sin 5(\omega t-31\cdot 7)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	3684	91·5	26·8	32·60	91·4	30·14	0	·0918
10.....	1471	66·6	13·6	59·91	87·31	25·79	27·31	·0974
20.....	555·5	15·9	...	85·32	92·21	...	52·72	·1212
30.....	165·3	2·76	...	107·53	99·57	...	74·93	·1436
35.....	80·62	1·00	...	115·06	100·70	...	82·46	·2114
40.....	28·01	0·26	...	117·12	107·58	...	84·52	

TABLE X.—Specimen A reduced to $l=65$ cms. T=·0311.

$C=\cdot 1718 \sin \omega t+\cdot 0067 \sin 3(\omega t-58\cdot 1)+\cdot 0008 \sin 5(\omega t-26)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	3794	87·6	26·0	32·66	89·0	31·67	0	·0903
10.....	1539	65·6	12·1	59·72	85·97	27·87	27·06	·0922
15.....	970·4	35·9	5·3	71·41	88·43	25·40	38·75	·1046
20.....	575·1	19·42	1·85	82·82	92·76	...	50·16	·1309
25.....	298·9	7·71	...	91·79	96·36	...	59·13	·2087
30.....	105·3	2·38	...	96·67	96·33	...	64·01	

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TABLE XI.—Specimen A reduced to $l=45$ cms.
$T=\cdot 0311$.

$C=\cdot 1734 \sin \omega t+\cdot 0077 \sin 3(\omega t-58\cdot 3)+\cdot 0003 \sin 5(\omega t-27)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x-\theta_0$.	λ_1.
0.....	382·0	94·9	29·1	32·01	88·37	27·95	0	·6862
5.....	248·0	106·5	21·9	43·93	86·38	28·88	11·92	·1032
10.....	150·2	75·3	15·4	55·44	86·16	28·13	23·43	·1139
15.....	85·0	41·1	7·2	64·26	89·41	31·94	32·25	·2048
20.....	305·3	13·9	2·5	70·11	91·65	30·36	38·10	

From the tables and from fig. 6 it will be seen that shortening the rod makes little perceptible difference either to the leakage of the flux or to its phase-lag until a point is reached at which the end effect begins to assert itself. Thus for the different lengths, flux-waves that are equal at the magnetizing coil are practically equal in all their characteristics after travelling equal distances along the rod, provided they have not reached to within 30 cms. or so from the ends.

9. The next specimen (B) tested was a rod of silver-steel of low permeability of the same section as specimen A. One frequency with one initial flux only was used, these being (q. p.) the same as in one of the series (Table III.) of experiments with specimen A, so that a direct comparison of the behaviour of the two materials could be readily made.

Details of Specimen B.

A straight cylindrical rod of silver-steel.
Length $=286\cdot 6$ cms. Diameter $=\cdot 653$ cm.
Section $=\cdot 3351$ cm^2.
Magnetizing solenoid and search-coils the same as were used with specimen A.
Specific resistance $=1\cdot 859\times 10^4$.

Statical Permeability for different inductions.

B	50	500	1000	2000	4000	6000	8000	10000	12500	15000
μ	73	96	122	170	248	298	316	290	222	140

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TABLE XII.—Specimen B. T = ·0305.

C = ·224 sin ωt; F = f_1 sin ($\omega t - \theta$).														
x	0	5	10	15	20	30	40	60	80	100	110	120	130	140	145
f_1	3735	1855	700	236	80·3	16·6	5·35	1·34	0·53	·265	·190	·142	·106	·071	·053
θ	23·4	35·7	47·5	53·7	54·7	47·7	42·8	39·0	38·6	38·5	38·5	38·2	39·5	37·4	37·6
$\theta_x - \theta_0$...	0	12·3	24·1	30·3	31·3	24·3	19·4	15·6	15·2	15·1	15·1	14·8	16·1	14·0	14·2
λ_1		·140	·195	·217	·216	·157	·114	·069	·047	·035	·033	·030	·029	·040	·058

TABLE XIII.—Static Leak. Specimen B. T = ∞.

Magnetizing Current C = ·2027 (abs.).													
x	0	5	10	15	20	25	30	35	40	45	50	55	60
F	3592	1777	695	235	81·5	37·5	17·6	9·35	5·63	3·72	2·57	1·91	1·34
λ		·141	·188	·217	·211	·155	·151	·127	·101	·083	·074	·060	·067

Tables XII. and XIII. give the results of the experiments with this specimen, the characteristics of the first harmonic of the flux only being given.

By comparing Tables III. and XII. one can see the difference in the behaviour of similar rods of iron and steel for the same initial flux and the same frequency. Fig. 7 illustrates the difference graphically. Thus, though near the origin the rate of retardation of phase ($d\theta/dx$) is nearly the same for both specimens (hence giving nearly equal fictitious velocities of magnetization), yet in the steel rod the retardation attains its maximum at less than half the distance from the origin at which it attains its maximum in the iron rod, and the maximum value in the latter rod is about three times what it is in the former. Again, for the steel rod the leakage coefficients are at first very much larger than for the iron one (see fig. 8), either at equal distances from the origin, or for equal fluxes; but when very low values of the flux have been arrived at, the leakage coefficients are quite as small in the steel as in the iron.

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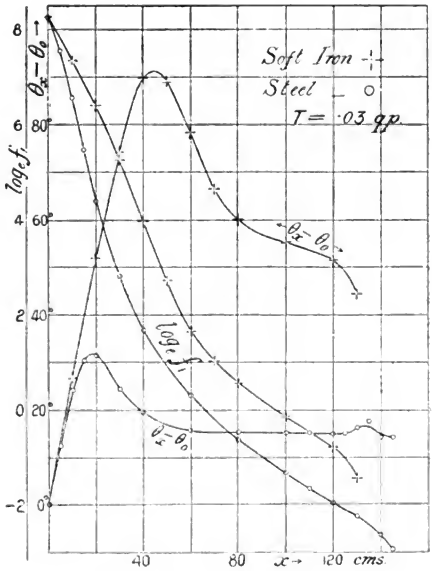
It will be noticed (see Table XII.) that in this specimen also the positions of max. retardation and max. leakage coefficient very nearly if not quite coincide.

10. The cause of the difference in the leakage-curves for the two rods is naturally to be looked for in their very different permeability-curves. Of the flux F which crosses any section

Fig. 7.



of the rod, $-d$F leaks out in the next elementary length dx, and F$+d$F continues in the rod, and the ratio of $-d$F to F$+d$F (*i.e.* $-d$F/F in the limit) will, when the magnetizing force is constant, be equal to the inverse ratio of the reluctances of the two corresponding paths. When the magnetizing force is periodic, the phenomenon is further complicated by the eddy currents set up in the rod, giving rise to back magnetomotive forces. If, for two different rods, the reluctances of two elementary leakage-paths (air) were equal, then

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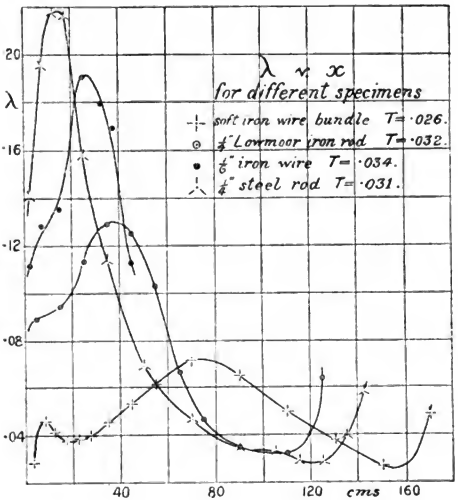
$-dF/F$ for the corresponding elements (and therefore λ, as $\lambda dx = -dF/F$) would be proportional to the reluctances of the second paths (partly in the materials), and thus would be greater for the less permeable substance. However, as λ depends on the ratio of two reluctances, it is quite conceivable that this ratio might be smaller for points on a rod of low permeability than for the corresponding point on a rod of higher permeability. (See fig. 8.)

Fig. 8.



Again, λ at a point near the end of a finite rod for a given initial flux and frequency must be greater than λ for a point equidistant from the magnetizing solenoid in an infinite rod of the same material and cross-section; for the reluctances of the elementary leakage paths through the air will be equal, while the second path for the infinite rod must have a lower reluctance than that for the finite rod, seeing that iron in the former takes the place of some of the circuit, which is air for the latter. Thus the "end effect" on the leakage coefficient is to make its value larger.

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11. C, the next specimen tested, was a long thick iron wire. Three series of experiments were made with it for three different frequencies and approximately the same initial flux.

Details of Specimen C.

A straight iron wire.
Length = 324 cms. Diameter = ·398 cm.
Section = 0·1244 cm².
Specific resistance = $1{\cdot}006 \times 10^4$.
Magnetizing solenoid and search-coils the same as were used with specimen A.

Statical Permeability for different inductions.

B.....	100	500	1000	2000	4000	5000	6000	8000	10000	12500	15000	17500	20000
μ.....	230	435	625	990	1385	1470	1455	1320	1100	800	540	330	170

In Tables XIV., XV., and XVI. are given the analysed results of the alternating flux experiments with this specimen, and in Table XVII. the results with the continuous flux.

TABLE XIV.—Specimen C. T = ·0545.

$C = {\cdot}1742 \sin \omega t - {\cdot}0091 \sin 3(\omega t + 1{\cdot}15) - {\cdot}0011 \sin 5(\omega t - 26{\cdot}1)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0	2360	189·0	41·8	22·65	3·28	45·08	0	·0898
5	1505	100·9	19·0	34·50	− 6·10	45·07	12·45	·1150
10	847·5	59·8	10·2	47·18	−11·81	40·74	25·13	·1245
20	244	16·5	1·6	69·82	−14·88	48·26	47·77	·1760
30	42·0	1·9	0·3	83·97	− 8·82	63·34	61·92	·1774
35	17·3	0·44	0·12	79·09	− 1·85	68·62	57·04	·1800
40	7·02	0·09	0·05	71·05	− 7·73	79·06	49·00	·1207
50	2·10	0·09	...	53·54	−11·45	...	31·49	·0759
60	0·98	0·06	...	46·23	− 5·57	...	24·17	·0510
70	0·59	0·05	...	39·72	+ 1·90	...	17·67	

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TABLE XV.—Specimen C. T=·034.

$C = \cdot 1737 \sin \omega t - \cdot 0096 \sin 3(\omega t - 0 \cdot 24)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0	2220	87	34	27·00	− 1·33	42·56	0	·1034
5	1326	54	15	40·55	−16·36	36·21	13·55	·1272
10	702	36	7	56·07	−20·51	36·24	29·07	·1351
20	181·7	8·9	...	81·94	−18·71	...	54·94	·1904
30	27·1	0·6	...	96·95	− 6·62	...	69·95	·1792
35	11·05	0·07	...	88·46	+16·98	...	61·46	·1687
40	4·75	0·04	...	77·14	66·04	...	50·14	·1123
50	1·55	0·04	...	56·41	130·2	...	29·41	

TABLE XVI.—Specimen C. T=·0234.

$C = \cdot 1803 \sin \omega t - \cdot 0072 \sin 3(\omega t + 3 \cdot 32) + \cdot 0016 \sin 5(\omega t - 6 \cdot 6)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3.	f_5.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0	2115	75	28	30·40	−15·46	42·70	0	·1148
5	1192	59	16	45·26	−26·67	35·00	14·86	·1388
10	595·4	29·4	6·4	62·07	−27·00	30·72	31·67	·1515
20	130·9	4·9	...	91·13	−22·21	...	60·73	·2053
30	16·80	0·18	...	104·61	+ 5·08	...	74·21	·1763
35	6·96	0·10	...	90·53	49·92	...	60·13	·1494
40	3·30	0·07	...	76·41	66·49	...	46·01	·1057
50	1·14	0·06	...	58·76	94·31	...	28·36	

TABLE XVII.—Static Leak. Specimen C. T=∞.

Magnetizing Current, C=·1844 (abs.).

x.....	0	5	10	15	20	25	30	35	40	45	50	55
F	2220	1476	919	559	318	162	77·5	31·6	12·6	5·83	3·01	1·98
λ		·082	·095	·099	·123	·135	·147	·179	·183	·154	·132	·084

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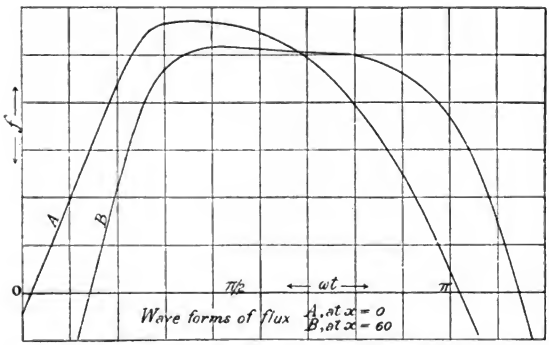
In the series of experiments with this specimen the flux density at the centre was much higher (18,000 instead of 10,000) than in the corresponding series with specimen A. The series with rod A, given in Table II., in which the induction at the centre was about 15,000, will, however, afford a comparison under more nearly similar conditions, and we can see that the leakage coefficients for the thinner rod are considerably higher than for the other (see fig. 8). On account of the smaller sectional area and greater leakage, observations were of necessity limited to a shorter range than for the other specimens, but within this range the phenomena observed and recorded in the preceding tables are similar in all respects to those observed with the previous specimens. In particular the leakage coefficient again attains a maximum value at or very near to the position for maximum phase-lag.

It is also worthy of notice that the value of $d\theta/dx$ near the origin is larger than for specimen A, the mean values of the fictitious "velocity of magnetization" for the first 20 cms. (a range within which in this specimen $d\theta/dx$ is fairly uniform) being

2750 cms./sec. when T = ·0545,
3850 cms./sec. „ T = ·034,
5060 cms./sec. „ T = ·0234,

values considerably smaller than those for specimen A. This

Fig. 9.



agrees with the results of some experiments that will be described further on, which show that reducing the diameter

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of the specimen, other things being equal, increases the space rate of retardation of the flux near the origin.

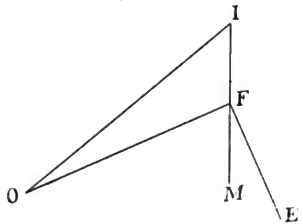
There might seem to be a discrepancy when the values of f_1 for $x=0$ in Tables XIV., XV., XVI. relative to the first harmonics of their magnetizing currents are compared with the value of F_0 in Table XVII. relative to the continuous current producing F_0. The iron seems to be quite as permeable under alternating magnetization as under continuous. This apparent discrepancy will disappear (and this remark will apply to the other specimens) when it is remembered (*a*) that the form of the current wave is peaked and its max. ordinate is greater than the amplitude of its first harmonic, (*b*) that the form of the flux-wave is flat and its max. ordinate is less than the amplitude of its first harmonic.

Thus the max. values of C and F_0 for the different values of T in the above are as follows :—

T.....	∞	·0545	·034	·0234
$C_{max.}$.....	·1844	·1854	·1850	·1866
$F_{max.}$.....	2220	2160	2100	2034

12. Eddy currents must play an important part in the phenomena under consideration, and their effects in this connexion may, in a general way, be explained as follows. If the vector OF (fig. 10) represent the resultant flux down

Fig. 10.



any cylindrical portion of a rod, the E.M.F. produced by its variation in a circuit in the rod round the outside of this cylinder, being in quadrature with OF and behind it in phase, may be represented by a vector FE where the angle OFE is a right angle. This E.M.F. generates an eddy

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current which generates a flux down the cylinder that lags behind FE in phase, and hence may be represented by some such vector as FM or IF, where IF=FM. Hence the vector OI will represent in amplitude and phase the flux that would have passed down the cylinder if the eddy current had not been present. Thus we see that the effects of eddy currents at any point of one of our specimens are (*a*) an increase in the phase retardation, *i. e.* in $\theta_x - \theta_0$; (*b*) a diminution in the amplitude of the flux there, which manifests itself as an increase in λ the leakage coefficient.

A further set of experiments was undertaken in which these eddy-current effects were much reduced by replacing the solid rod by a cylindrical bundle of soft iron wires, called specimen D, of which the details are as follows:—

Details of Specimen D.

A cylindrical bundle of soft iron wires.
Number of wires = 185.
Length of each wire = 368 cms.
Diameter " = ·0791 cm.
Area of iron section of specimen = ·909 cm.^{2}
Diameter of bundle = 1·5 cm.
Specific resistance of iron = $1{\cdot}366 \times 10^4$.
No. of turns on magnetizing coil = 366.
" " search coil central = 5.
" " " " *a* = 10.
" " " " *b* = 50.
" " " " *c* = 100.

Statical Permeability for different inductions.

B	100	1000	2000	4000	5000	6000	8000	10000	12000
μ	202	475	750	1140	1240	1260	1125	875	575

In Tables XVIII., XIX., and XX. are given the results of three series of experiments with this specimen at one frequency (T=·0255), but with different initial fluxes, and in Table XXI. those of one series at the frequency for which T=·0515, the initial flux being approximately the same as in one of the preceding series, while in Table XXII. are given the results for continuous magnetizing force also with approximately the same central flux.

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TABLE XVIII.—Specimen D. T = ·0255.

$C = \cdot 1697 \sin \omega t + \cdot 0296 \sin 3(\omega t - 58 \cdot 6) + \cdot 0091 \sin 5(\omega t - 65 \cdot 4)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0.....	16190	·086	·020	15·77	16·49	29·88	0	·0285
7·3.....	13150	·097	·021	19·52	12·33	26·98	3·75	·0460
10.....	11620	·107	·021	21·19	10·56	24·44	5·42	·0414
15.....	9440	·121	·021	24·27	8·34	20·73	8·50	·0278
20.....	7816	·132	·022	27·38	6·77	17·72	11·61	·0383
25.....	6456	·141	·025	29·97	5·68	15·62	14·20	·0396
30.....	5292	·151	·024	32·50	4·72	11·33	16·73	·0455
40.....	3360	·166	·033	36·39	4·17	9·86	20·62	·0530
50.....	1976	·179	·041	39·23	3·61	8·47	23·46	·0614
60.....	1070	·184	·045	40·97	4·35	8·58	25·29	·0716
80.....	255·4	·182	·046	41·20	5·52	9·46	25·43	·0651
100.....	69·5	·160	·041	37·90	6·98	12·83	22·13	·0501
120.....	25·47	·147	·037	33·38	9·14	15·56	17·61	·0381
140.....	11·91	·133	·031	30·43	9·04	18·42	14·66	·0270
160.....	6·94	·145	·027	28·40	11·32	19·95	12·63	·0487
180.....	2·62	·135	·036	26·83	12·52	19·43	11·06	

TABLE XIX.—Specimen D. T = ·0254.

$C = \cdot 0846 \sin \omega t + \cdot 0078 \sin 3(\omega t - 62 \cdot 17) + \cdot 0013 \sin 5(\omega t - 4 \cdot 49)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0.....	10030	·057	·005	18·98	26·27	39·69	0	·0304
7·3.....	8035	·065	·009	22·67	20·12	33·50	3·69	·0450
10.....	7121	·073	·010	25·07	16·81	30·69	6·09	·0433
15.....	5730	·085	·010	29·27	13·05	27·63	10·29	·0432
20.....	4618	·098	·012	32·16	10·86	24·44	13·18	·0437
25.....	3710	·106	·014	34·35	10·07	22·20	15·37	·0545
30.....	2826	·116	·014	37·23	8·71	20·85	18·25	·0545
40.....	1639	·136	·022	41·35	7·60	16·26	22·37	·0652
50.....	854	·142	·025	43·89	7·64	15·63	24·91	·0700
60.....	424	·149	·031	45·05	8·07	15·34	26·07	·0741
80.....	96·4	·136	·030	43·35	10·08	18·86	24·35	·0592
100.....	29·5	·118	·021	39·10	15·05	22·70	20·12	·0469
120.....	11·55	·101	·020	35·73	13·69	20·63	16·75	·0320
140.....	6·09	·092	·020	32·56	16·56	28·71	13·58	

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TABLE XX.—Specimen D. T=·0255.

$C = \cdot 0353 \sin \omega t + \cdot 0012 \sin 3(\omega t - 58 \cdot 29)$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0	295·8	·049	·006	13·72	27·77	35·36	0	·0469
7·3	210·0	·058	·006	17·55	21·02	32·80	3·83	·0761
10.....	171·0	·066	·008	19·42	19·47	28·70	5·70	·0751
15.....	117·5	·077	·011	22·77	17·29	28·91	9·05	·0745
20.....	80·9	·084	·014	24·68	15·51	26·75	10·96	·0771
25.....	55·0	·092	·013	26·03	14·98	25·48	12·31	·0814
30.....	36·64	·096	·016	26·98	14·98	24·07	13·26	·0805
40.....	16·38	·100	·019	27·35	15·60	25·08	13·63	·0824
50.....	7·18	·100	·019	27·39	16·49	25·41	13·67	·0756
60.....	3·37	·103	·018	26·40	17·95	28·16	12·68	·0677
70.....	1·713	·089	·015	25·77	18·58	28·79	12·05	·0591
80.....	0·948	·082	·016	24·73	18·51	26·98	11·01	·0458
90.....	0·600	·079	·010	24·35	17·68	30·36	10·63	·0459
100.....	0·379	·083	·013	23·54	16·81	30·66	9·82	·0345
120.....	0·190	·072	·005	20·19	18·16	30·96	6·47	·0287
140.....	0·107	·067	·015	18·85	19·50	31·50	5·13	

A glance at the columns for $\theta_x - \theta_0$ and λ_1 in Tables XVIII. to XXI. will show that so far as phase retardation and leakage coefficients are concerned, the general character of the results is the same as has been obtained from the other specimens; and, as was expected, not only are these quantities much reduced in magnitude, but also the effect of change of frequency on them is less marked. In addition the apparently coincident point of maximum phase retardation and maximum leakage coefficient is much further removed from the origin than in any of the other specimens.

13. A much more striking result, however, is the way the upper harmonics of the flux, and hence its wave form, vary as the flux-wave passes along the specimen. In these series the ratios f_3/f_1 and f_5/f_1 of the amplitudes of the third and fifth harmonics to that of the first are much larger than in the previous ones, where the flux wave was almost sinusoidal. We were therefore able to determine for this specimen these upper harmonics both in amplitude and phase with certainty for greater distances from the origin than for the other

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TABLE XXI.—Specimen D. T=·0515.

$$C = \cdot 0790 \sin \omega t + \cdot 0050 \sin 3(\omega t - 58\cdot 46) + \cdot 0006 \sin 5(\omega t - 64\cdot 86).$$
$$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$$

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0	9472	·065	·012	16·92	19·80	33·23	0	·0304
7·3	7592	·077	·013	19·90	14·58	28·61	2·98	·0454
10.....	6700	·083	·014	21·68	13·83	28·83	4·76	·0510
15.....	5193	·098	·017	25·15	10·83	23·91	8·23	·0441
20.....	4167	·115	·016	27·78	9·34	22·38	10·86	·0455
25.....	3319	·123	·019	30·09	8·81	17·68	13·17	·0519
30.....	2560	·136	·022	32·16	8·00	17·50	15·24	·0594
40.....	1414	·153	·033	36·00	7·75	15·05	19·08	·0653
50.....	736	·159	·032	38·16	6·59	13·65	21·24	·0721
60.....	358	·165	·039	39·00	7·42	14·31	22·08	·0789
70.....	162·9	·155	·036	38·38	8·22	14·35	21·46	·0727
80.....	78·7	·152	·038	36·91	9·33	14·39	19·99	·0600
100.....	23·7	·131	·028	33·41	10·42	18·50	16·49	·0443
120.....	9·76	·121	·026	30·06	11·16	22·35	13·14	·0333
140.....	5·01	·107	·014	28·73	12·78	19·78	11·81	·0269
160.....	2·92	·099	·023	28·26	10·25	22·50	11·34	·0354
170.....	2·20	·099	·023	26·27	14·24	28·24	9·35	·0596
180.....	1·13	·093	·020	25·65	14·62	29·65	8·73	

TABLE XXII.—Specimen D. Static Leak. T=∞.

Magnetizing Current. C=·0915 (obs.).

x...	0	7·3	10	15	20	25	30	40	50	60	75	90	100	110	120
F...	9775	7700	6517	5235	4154	3267	2542	1429	740	355	120	45·6	26·9	15·8	11·1
λ...		·0327	·0617	·0439	·0462	·0481	·0502	·0576	·0658	·0735	·0723	·0644	·0528	·0533	·0531

specimens; and the results given in the tables show that as we move away from the magnetizing coil we find (*a*) that the ratios f_3/f_1 and f_5/f_1 first increase, then both attain a maximum at approximately the same place, and after this diminish, and this maximum point is the same as that at

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which the phase retardation and the leakage coefficient of the first harmonic attain their maxima; (*b*) that the retardations in phase β_3, β_5 of these harmonics behind the primary first diminish, become minima, and then increase, and that the point of minimum values of β_3 and β_5 is the same apparently as that of maxima f_3/f_1 and f_5/f_1 as well as that of maxima $\theta_x-\theta_0$ and λ_1. The above may be differently stated as follows:—As the flux-wave moves away from the origin its form gradually changes, becoming flatter and flatter until it reaches the point of maximum lag and maximum leakage coefficient. After this as it continues to move on its flatness steadily diminishes. This change of form is exhibited in fig. 9, in which are plotted the two wave forms for $x=0$ and $x=60$ in the series given in Table XVIII. They are placed inaccurately as regards relative phase, but the ordinates of the one for $x=60$ are increased so that the amplitudes of the first harmonics of the two curves are the same.

Further interesting information as to the behaviour of the upper harmonics of the flux may be obtained by considering their actual amplitudes. Taking the third harmonics for any of the series in this paper, we find that the leakage coefficients are different from what we estimated they would be for a primary or first harmonic of the same amplitude and frequency at the same section of the rod. In some cases, indeed, as will be seen from the tables given, the amplitude of the third harmonic actually increases as the distance from the magnetizing coil increases.

Hence we cannot consider the upper harmonics as being independent of the first harmonic or of each other. Each seems to get continually reinforced by the first harmonic as well as by other harmonics lower in order than itself as it travels along. Such a transference of energy from the first harmonic to the higher ones must be accompanied either by a reduction of the amplitude of the first harmonic, which in these experiments would manifest itself as leakage in λ_1, or by a retardation of its phase helping to increase $\theta_x-\theta_0$, or by both; so it is probable that this reinforcement of the higher harmonics by the first has an important bearing on the theory of the experiments described in this paper.

14. In order to investigate the effect of change of diameter we made two smaller bundles with wires taken from specimen D. One of these called d_1 contained 46, and the other called d_2 12 wires, and as D originally consisted of 185 wires the sectional areas of D, d_1, and d_2 were very nearly in the proportions of 16 to 4 to 1 or their diameters as 4 to 2 to 1.

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One series of experiments was performed with each of d_1 and d_2 under approximately the same conditions of frequency and initial flux-density as the series already performed on D and recorded in Table XXI.

In Tables XXIII. and XXIV. are given the results of the new series, and it will be seen on comparing them with Table XXI. and with each other, that the effect of change of diameter is if anything more marked than that of permeability which was illustrated by the comparative series on specimens A and B.

Thus we find that the values of $d\theta/dx$ near the origin for the diameters 4, 2, and 1, are 4·75, 9·9, and 15·3 respectively; so that if we deduced a "velocity" from these figures we should get very different values for it, in the same material, with the same frequency and flux-density.

The change of diameter seems to have little or no effect on the max. value of the retardation, though it has a marked effect on the distance from the origin at which this maximum occurs. Thus the coincident points of max. retardation and max. leakage coefficient are at distances from the magnetizing solenoid approximately proportional to the diameters of the specimens in which they occur.

Again we find that reduction of diameter increases the leakage coefficients (for the same conditions of frequency and flux-density) approximately in the same ratio as the diameter is reduced.

TABLE XXIII.—46 wires from Specimen D.

T = ·050.

$C = \cdot 0742[\sin \omega t - \cdot 042 \sin 3(\omega t + 0\cdot 28) + \cdot 0035 \sin 5(\omega t - 06)]$.
$T = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0.....	2583	·0603	·0095	14·32	17·09	25·24	0	·0857
10.....	1096	·1113	·0158	24·23	7·07	14·75	9·91	·1181
20.....	336·6	·1530	·0302	31·41	5·59	9·76	17·09	·1508
25.....	158·3	·1630	·0345	33·45	4·83	8·93	19·13	·1596
30.....	71·3	·1583	·0374	32·86	5·69	9·35	18·54	·1559
40.....	15·0	·1400	·0321	31·18	8·44	13·64	16·86	·1147
50.....	4·76	·1214	·0225	28·14	10·37	18·80	13·82	·0742
60.....	2·32	·1033	·0205	25·92	10·45	19·94	11·60	

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TABLE XXIV.—12 wires from Specimen D.

T=·050.

$C = \cdot 0463[\sin \omega t - \cdot 05 \sin 3(\omega t + 1\cdot 63)]$.
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5)$.

x.	f_1.	f_3/f_1.	f_5/f_1.	θ.	β_3.	β_5.	$\theta_x - \theta_0$.	λ_1.
0.....	681	·080	·0164	15·60	11·35	16·24	0	·1690
10.....	125·6	·1534	·0312	30·88	4·88	6·61	15·28	·2820
15.....	30·7	·1637	·0396	34·72	4·89	6·92	19·12	·3142
20.....	6·37	·1485	·0312	32·10	7·61	11·04	16·50	·2230
25.	2·09	·1210	·0259	25·51	10·73	16·73	9·91	·1641
30.....	0·92	·0776	·0172	25·26	11·71	25·18	9·66	

In particular it is most striking that the max. values of λ_1, viz.:—

·3142, ·1596, and ·0789,

should be so nearly inversely proportional to the corresponding diameters, 1, 2, and 4 respectively, the corresponding products being

·3142, ·3192, and ·3156 respectively.

In addition we find with the smaller bundles d_1 and d_2 the same change of form of the flux-wave as it passes along them as in the case of specimen D, and also the same peculiarity in the behaviour of the upper harmonics both as regards their amplitudes and phases as has been drawn attention to in the preceding paragraph in relation to the thicker bundle.

15. The fact that so many of the characteristics of the alternating flux attain either their maximum or minimum values at practically the same point in the specimen (particularly in D, d_1, and d_2) is undoubtedly the most striking result of this investigation.

This point might be called the critical point for the specimen under the conditions of initial flux and frequency used, its distance from the magnetizing solenoid the critical distance, and the values of the different characteristics of the flux there the critical values. For the existence of such a critical point we are unable to offer any explanation.

A more fundamental knowledge than any we at present possess of the nature of magnetization and permeability is required. Our want of any exact knowledge of the amplitude and phase relations between induction and magnetizing force and of the relations between the different harmonics of an

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induction wave precludes the possibility of a satisfactory solution on the basis of our present knowledge.

16. It was thought, however, that a partial explanation of some of the phenomena observed might be obtained if the initial flux was made up of elementary magnetic lines very heterogeneous as regards their phases. As these passed along the specimen there would be a retardation of the phase of each line due to "velocity" as well as a change of the phase of the resultant flux at any section due to groups of lines in special phases leaking out at special distances from the origin. If this were so, the phases of the lines at different distances from the axis of the rod or bundle should be different, as at any section those lines about to leak out would be near its circumference. To test this the wires of specimen D were divided into three groups. The first group of 28 wires was taped to form a circular cylinder, and round it was wound a search-coil of 50 turns insulated. Over this central cylindrical bundle the second group of 45 wires was uniformly distributed and taped so that the whole formed a circular cylinder coaxial with the first. An insulated search-coil of 50 turns was wound round the middle of the compound bundle formed. The remaining group of 110 wires was now added and taped so as to be coaxial with the others, and a third search-coil of 50 turns was wound round the middle of the whole.

The bulging caused by the search-coils was not very great. The magnetizing coil was wound on a bobbin, and could be placed at any distance on the specimen from the search-coils. By means of a switch the search-coils could be connected in different ways, so that when continued to the wave-tracer the latter determined for us the flux-waves through the central bundle or through either of the surrounding tubular bundles of wires for different distances from the magnetizing coil.

The result of this investigation was to show that while at the centre of the magnetizing coil the flux-density and phase varied over the section—the phase of the first harmonic for the outer layer being 4·56 degrees ahead of that for the central portion and the amplitude varying from 67 lines per wire for the outside portion to 58·5 lines per wire for the central,—yet for distances greater than 15 cms. from the magnetizing coil there was no appreciable difference in the phases of the first harmonics, wave-forms, or densities of the fluxes through the three different portions of the compound bundle. Hence no such explanation as that imagined at the beginning of this paragraph can be tenable.

17. It was also thought that if we determined in amplitude and phase the magnetizing force required to produce the

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different resultant fluxes at different sections of the specimen in any one series of the transmission experiments, we might obtain some information that would help to explain the phenomena.

This was done for specimen A and for a cylindrical bundle made up of 46 of the iron wires from specimen D.

The specimen was inserted in a long solenoid and by means of the wave-tracer, using it as described in a former paper * by one of us, the amplitudes and phases of the magnetizing forces required to produce uniform oscillating fluxes of different amplitudes and phases were obtained and the results plotted. From these curves and the results given in this paper on specimen A, and similar results obtained from the narrow bundle, we were able to obtain the amplitude and phase of the *resultant* magnetizing force that acted at each section of the specimens in the transmission experiments. When these characteristics of the magnetizing force were plotted against corresponding distances along the specimen from the magnetizing solenoid, the curves obtained were less instructive than those for the characteristics of the flux at different distances which have been given in this paper.

18. To sum up:—When waves of magnetic flux that have been started by alternating currents in a short solenoid placed at the centre of a long iron rod, or bundle of iron wires, are transmitted along the rod or bundle, we find that—

(1) The retardation of phase of the first harmonic of the flux at any point distant x from the centre, behind the first harmonic of the initial flux, first increases with x, attains a maximum and then diminishes, and keeps diminishing until the flux is dissipated if the specimen is sufficiently long for this to be effected.

(2) The leakage coefficient λ_1 of the amp. f_1 of the first harmonic, which we define by the equation $\lambda_1 = -\frac{1}{f_1}\frac{df_1}{dx}$, first increases with x, attains a maximum, and then diminishes and keeps diminishing until the flux is dissipated.

(3) The distances from the magnetizing solenoid at which the phase retardation and the leakage coefficient become maxima are equal (or very nearly so), and the point at which these maxima occur we call the *critical point* of the specimen, and its distance from the magnetizing solenoid the *critical distance*, for the particular initial flux and frequency used.

(4) Previous investigators of this subject, using less sensitive methods, were only able to obtain observations within the

* T. R. Lyle, "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. vol. ix. p. 102 (1905).

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critical distance, and from their observations concluded (1) that the leakage coefficients were practically constant at all distances x, thus arriving at a logarithmic decrease of flux amplitude; (2) that the fairly regular space-rate of phase retardation $d\theta/dx$ observed would be completely accounted for by a velocity of magnetization v, deduced by the formula

$$v = \frac{2\pi}{T d\theta/dx}.$$

We find that neither of these conclusions is correct, as (*a*) λ, for the same rod and conditions of experiment, varies within wide limits along the rod, and (*b*) the retardation, after increasing as they observed within the critical distance, becomes stationary at the critical point, beyond which the phase of the flux advances: so that if their conclusion (2) were correct, we should have to admit an infinite velocity at the critical point and a negative velocity beyond it.

(5) As the flux passes along the specimens made of wires, the amplitudes f_3, f_5 of its upper harmonics get continually reinforced by a transference of energy from the first harmonic (see § 13, Tables XVIII.–XXI.). The ratios f_3/f_1, f_5/f_1 first increase with x, attain maxima, and then diminish; at the some time the differences β_3, β_5 between the phases of these harmonics and the phase of the first diminish first with x, attain minima, and then increase; and the positions of maxima of f_3/f_1, f_5/f_1 and minima of β_3, β_5 are at (or near) the critical point. From this it results that as the flux-wave moves away from the origin its form changes, becoming flatter as x increases (see fig. 9) until the critical point is reached, when its flatness is a maximum; beyond this as x increases its flatness diminishes.

(6) The effects of an end on a flux-wave approaching it are to increase the leakage coefficient and to cause an advance in the phase. (See fig. 6.)

(7) When a rod whose behaviour has been determined is shortened and subjected to the same magnetization as before, no difference is observed in the flux-waves until (for a $\frac{1}{4}$ in. rod) within 30 cms. from the new end. (See § 8 and fig. 6.)

(8) For the same frequency, initial flux, and cross-section, increase of permeability increases the critical distance, increases the critical value of the retardation (see fig. 7), and diminishes the critical value of the leakage coefficient. (See § 9 and fig. 8).

(9) Eddy currents diminish the critical distance, increase the critical value of the retardation, and increase the leakage coefficients. (See § 12.)

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(10) For the same frequency, initial flux-density, and material, increase of diameter of the specimen increases the critical distance, has little or no effect on the critical retardation (hence making $d\theta/dx$ near the origin less), and diminishes the leakage coefficients: in fact, we find that the critical distance is approximately proportional to the diameter, and the critical value of the leakage coefficient inversely proportional to the diameter. (See § 14 and Tables XXI., XXIII., XXIV.)

(11) For the same specimen and initial flux, increase of frequency increases the initial value of $d\theta/dx$, increases the critical values of the retardation and the leakage coefficient, and slightly diminishes the critical distance. (See figs. 1, 2, and the Tables.)

(12) For the same specimen and frequency, increase of initial flux increases the critical distance (see fig. 4), increases the critical value of the retardation (fig. 4), and reduces the critical value of the leakage coefficient (see fig 5): also for low values of the initial flux F_0, $d\theta/dx$ near the origin increases with F_1, but seems to approach an upper limit for high values of F_0. (See fig. 4.)

L. *On the Use of Chilled Cast Iron for Permanent Magnets.* *By* Albert Campbell, *B.A.* (From the National Physical Laboratory.)*

EARLY in the past year an interesting paper† was published by Mr. B. O. Peirce of Harvard University, drawing attention to the fact that chilled cast-iron is in many instances a suitable material for permanent magnets. As the subject is of interest to scientific experimenters and of considerable importance to instrument makers, I undertook some time ago a short research upon it, with a view, firstly of obtaining some measurements by standard methods, and secondly of finding, if possible, an easy method of chilling the material so as to give good results.

Form of Test Pieces.—The cast iron tested was of ordinary commercial quality and was obtained in the form of rods and rings. The rods, which were of rectangular section, were shaped to dimensions usual in such tests, viz., 10 cm. × 1 cm. × 1 cm.

* Communicated by the Physical Society: read January 26, 1906.

† Amer. Acad. Proc. xl. 22, pp. 701–715, April 1905. Dr. Watson has kindly drawn my attention to Mr. J. R. Ashworth's experiments on chilled cast-iron rods (Proc. Roy. Soc. vol. lxii. p. 210, Dec. 9, 1897); he found that the magnetic quality of these was comparable with that of tungsten steel.

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By using rods of these dimensions it is easy to compare the results with those for various kinds of steel which we have already tested or with those published by Madame Curie * and other observers.

Two rings were tested; they were of rectangular section, their mean diameters being nearly equal (12·5 and 13·0 cm.). Their cross-sections, however, were very different, being 1·00 sq. cm. and 6·0 sq. cm. respectively. The object of testing a thick and a thin ring was to find if our method of chilling was effective for the hardening of thick castings.

Heat Treatment.—All the test pieces were heated to 1000° C. in a gas muffle furnace, the temperature being measured by a thermo-junction in the usual way. Each piece was removed from the furnace and quickly chilled in water at the temperature of the room. As Mr. Peirce has pointed out, great care is necessary in handling the cast iron at this high temperature (so near its melting-point) for it becomes very brittle. For this reason the thick ring was placed in the furnace on a U-shaped piece of wrought iron, it was lifted out by means of this support, and the two were plunged together into cold water. By this method the brittle material could be handled without risk of breakage.

Tests on Rods.—After the chilling, the rods were magnetized to saturation by means of a very strong magnetic field. This was produced by a solenoid consisting of 70 turns with a length of 16 cm. and having a resistance of about 0·1 ohm. A large current was sent through this coil by connecting it for a very brief interval to a 50-volt circuit (by the process commonly known as "flashing"). Each rod was then tested for

(1) The maximum remanent flux density B (at the medial section of the bar).

(2) The coërcivity H_0, *i.e.* the value of the demagnetizing magnetic force required to annul this remanent magnetism.

(1) In order to measure the medial B, a small square search-coil of 20 turns of very fine wire was used; it was just large enough to slip along the rod. The search-coil was connected to a calibrated ballistic galvanometer, and, when the coil was slipped off the rod, the resulting deflexion gave the required B in the usual manner.

(2) The coërcivity was found by Madame Curie's method, as follows:—The magnetized bar was fixed at the centre of a long solenoid, a search-coil being so arranged that it could be

* *Bulletin de la Société d'Encouragement pour l'Industrie Nationale*, Jan. 1898.

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slipped off the bar from the mid position and replaced without removing the bar from the solenoid. By sending a measured current through the solenoid the bar was subjected to a known demagnetizing field. This field was gradually increased until the search-coil when slipped off gave no throw on the galvanometer, thus showing that B had been reduced to zero. The value (H_0) of the magnetic force when this took place is a good indication of the permanence of the remanent magnetism.

If H_0 is large, we should expect the bar to hold its magnetism very obstinately. The following table gives the results for four of the rods (Nos. 1 to 4). For the sake of comparison the corresponding numbers are given for exactly similar rods (M and A) of hardened magnet steel of well known makers, M being supplied by Marchal of Paris, and A coming from the Allevard Forge.

TABLE I.

Marks.	Material.	Maximum Remanence.	Coërcivity.
No. 1	Chilled Cast Iron.	1775	52·8
No. 2	" "	1670	48·9
No. 3	" "	1690	50·4
No. 4	" "	1850	52·1
M	Magnet Steel.	2550	55·5
A	"	2950	73·0

The above results show that the hardened cast iron rods are not very much inferior to the ordinary magnet steel (M) either in original strength or in power of resisting demagnetization.

Tests on Rings.—Primary and secondary coils were wound on each of the chilled rings, and the (H, B) curves shown in fig. 1 were determined by the ordinary ballistic method. In addition to these the coërcivities (H_0) were found for a number of different values of B, and the (H_0, B) curves are also shown.

Fig. 2 gives the (H, B) curves for the thin ring before and after chilling.

We see from fig. 1 that, after chilling, the thick ring is magnetically very similar to the thin one; and thus it is evident that, by the treatment already described, a quite heavy casting can be satisfactorily hardened throughout.

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Fig. 1.

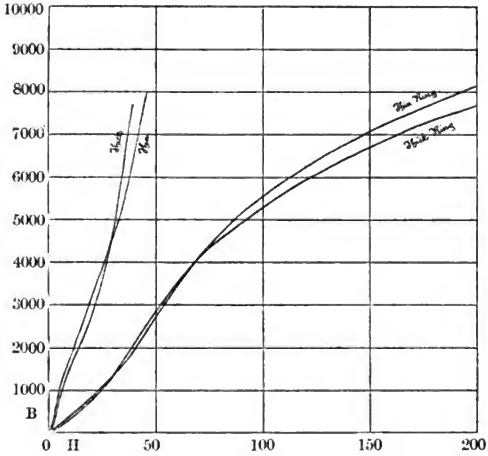
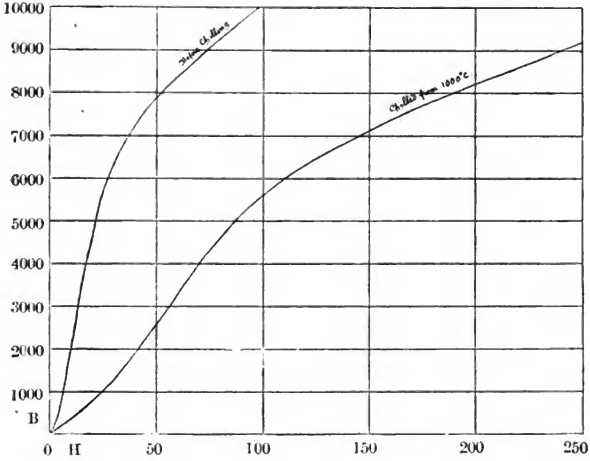


Fig. 2.



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Since the thin ring had the same cross-section as the rods, we may assume that its magnetic hardness was equal to theirs. The experiments here described amply confirm Mr. Peirce's results, and show that large cast iron magnets can be made cheaply and easily; we hope that this short paper will draw the attention of our instrument makers to the matter. In conclusion I would express my thanks to Dr. H. C. H. Carpenter for his kind help in the chilling of the specimens.

LI. *The Action of Electrolytes on Colloidal Solutions.* *By* E. F. Burton, *B.A.* (*Emmanuel College, Cambridge*), 1851 *Exhibition Science Scholar of the University of Toronto, Research Exhibitioner of Emmanuel College* *.

1. *Introduction.*

THE study of the coagulative action of comparatively small quantities of electrolytic solutions on irreversible colloidal solutions, besides being of intrinsic interest, has thrown most important light on the mechanism of such solutions; the suggested theories as to the forces which keep the minute particles in suspension in the liquid media rest to a great extent on the behaviour of these particles on the addition of electrolytes.

The importance of the phenomenon was first emphasized by the quantitative experiments of Linder and Picton †; they determined the coagulative power of different salt-solutions on a colloidal solution of arsenious sulphide, and discovered that this property depended directly upon the valency of the metal of the salt. They found that equivalent solutions containing monovalent, divalent, and trivalent metallic ions possess coagulative powers in the ratio of 1 : 35 : 1023. Additional work along the same line has shown that the valency of the metallic ion is important only in the case in which the colloidal particles are negatively charged. In fine, the coagulative power of any electrolytic solution on any colloidal solution depends on the valency of the ion which bears the charge opposite in sign to that on the colloidal particle.

The next fundamentally important step was made by the researches of Hardy ‡. Working with egg albumen, he found that the direction in which its particles move in an

* Communicated by Professor J. J. Thomson, F.R.S.

† Journ. of Chem. Soc. vol. lxvii. p. 63.

‡ Proc. Roy. Soc. vol. lxvi. p. 110; Jour. of Phys. vol. xxix. p. 26 (1903).

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electric field, and consequently the sign of the charge on the particles, depends on whether the liquid in which it is suspended is acidic or alkaline. If acidic, the particles are positively charged ; if alkaline, the particles are negatively charged. At the point (called the isoelectric point), where the sign of the charge changes, coagulation of the albumen took place. From analogy with this phenomenon, the generally accepted theory of the coagulation of all colloidal particles by electrolytes has been proposed: namely, that the particles have their charges neutralized by the absorption of the oppositely charged ions of an electrolytic solution and, at the isoelectric point, where the charge becomes zero, the colloid coagulates.

Following out this idea, Whetham * has given an explanation of the remarkable valency relations of the coagulative powers of electrolytic solutions. On the supposition "that in order to produce the aggregation of colloidal particles which constitutes coagulation, a certain minimum electrostatic charge has to be brought within reach of a colloidal group, and that such conjunctions must occur with a certain minimum frequency throughout the solution," he deduces that the coagulative powers of equivalent solutions containing monovalent, divalent, and trivalent ions respectively would be in the ratio of $1 : x : x^2$, where x is an arbitrary constant. If x is put equal to 32 this ratio reproduces, in a wonderfully exact way, the experimental numbers of Linder and Picton: the results, calculated and observed, are respectively—1 : 32 : 1024 and 1 : 35 : 1023. An apparent contradiction of this view is found in the recent work of Linder and Picton †, who observe that solutions containing tetravalent metallic ions (Pt and Zr) possess very feeble coagulative power towards colloidal arsenious sulphide; whereas, according to the above theory, solutions of these metals should have a coagulative power 30,000 times that of an equivalent solution of a univalent ion. An explanation of this fact is suggested later in the present paper.

Correlating Hardy's results with the Lippmann phenomenon regarding the connexion between surface tension and potential difference, Bredig ‡ suggested a very plausible theory as to the precise causes bringing about coagulation. The surface tension of mercury in contact with an electrolytic solution reaches a maximum when the potential difference between the two phases is zero. If we assume that the

* Theory of Solutions, p. 396.
† Journ. of Chem. Soc. vol. lxxxvii. p. 1906.
‡ *Anorganische Fermente*, Leipzig (1901).

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colloidal particles are kept in their state of fine subdivision in the liquid medium by the surface tension between the particle and liquid, then anything which tends to decrease the potential difference between the particle and the liquid will increase that surface tension. Such lessening of the potential difference is brought about by the absorption by the particles of ions bearing a charge opposite in sign to that on the particle; now, as the surface tension is increased, the particles tend to decrease the surface exposed to the liquid by uniting with one another and thus bring about coagulation.

2. *Purpose of Experiments.*

As is quite evident, the whole superstructure of the theory of the coagulation of colloids is built up on the results of Hardy's experiments on egg albumen. According to Noyes *, however, the behaviour of the albumen particles may be due to the fact that albumen is an amphoteric substance, capable of acting as a base towards acids and as an acid towards bases. The important question remains then—what effect have the ions of an added electrolyte on the charge of the colloidal particle in general?

In the experiments given below, the velocities of the particles of gold and silver solutions respectively are measured both before and after adding varying quantities of an electrolyte. Billitzer †, in making similar experiments on colloidal solutions of Pt, Hg, Ag, Au, Pd, to which he added gradually increasing amounts of various electrolytes, found that the velocity of the particle gradually decreased and eventually changed its direction, showing that even the sign of the charge was changed by the addition of the electrolyte. He added gelatine and urea to his solutions in order to prevent coagulation. Whitney and Blake ‡ disagree *in toto* with the conclusions of Billitzer, and fail to reproduce his results with colloidal solutions of gold and platinum, free from gelatine. They assign Billitzer's change in the direction of migration to the dissolved gelatine. In the following experiments, the addition of traces of aluminium sulphate to colloidal solutions of gold and silver (without the addition of either gelatine or urea) brought about in each case a change in the direction of migration of the particles in an electric field.

* Journ. Amer. Chem. Soc. vol. xxvii. No. 2, p. 85.
† *Ann. der Physik*, vol. xi. p. 903 (1903).
‡ Journ. Amer. Chem. Soc. vol. xxvi, No. 10. p. 1339 (1904).

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3. *Results.*

The solutions used were prepared in pure water by Bredig's * electrical method, and the velocities were measured as already described in a former paper by the writer †. Large quantities of very pure solution were quite easily prepared by the use of a motor to carry on the sparking between the metal electrodes underneath the surface of the pure water. The water used had a specific conductivity of 3×10^{-6} at 18° C.

In each case (silver and gold solutions) the velocity was measured for the pure solution; varying quantities of aluminium sulphate were then added to fresh samples of the stock colloidal solution and the velocity again measured. Aluminium sulphate was used because the metal ions being trivalent have a large coagulative power; if these ions in producing coagulation do diminish the charge on the particles, a very small addition of aluminium ions should have a perceptible effect on the velocity of the particles, while at the same time the specific conductivity, and consequently the current through the colloid, is small. $\frac{N}{1000}$ aluminium sulphate (pure, from Kahlbaum) was added drop by drop to some 40 c.cs. of the colloidal solution, the whole well mixed and the velocity measured; each experiment was completed in the course of two hours after adding the electrolyte. In Tables I. and II. the velocities corresponding to the various weights of aluminium per 100 c.cs. of colloidal solution are given: a gradual decrease in the velocity and final reversal of direction is shown in each case. The positive sign in the velocity column indicates motion of the particles toward the anode.

TABLE I.—Silver Solution.
Amount of silver per 100 c.cs. = 6·5 mgs.

No.	Grms. of Al. per 100 c.cs.	Spec. Conductivity of Solution at 18° C.	Velocity at 18° C.
1	0	31×10^{-6}	$+22{\cdot}4 \times 10^{-5}$
2	14×10^{-6}	$30{\cdot}3 \times 10^{-6}$	$+ 7{\cdot}2 \times 10^{-5}$
3	38×10^{-6}	$29{\cdot}7 \times 10^{-6}$	$- 5{\cdot}9 \times 10^{-5}$
4	77×10^{-6}	$28{\cdot}5 \times 10^{-6}$	$-13{\cdot}8 \times 10^{-}$

* *Loc. cit.*
† Phil. Mag. [6] vol. xi. April 1906.

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TABLE II.—Gold Solution.
Amount of gold per 100 c.cs. = 6·2 mgs.

No.	Grms. of Al. per 100 c.cs.	Spec. Conductivity of Solution at 18° C.	Velocity at 18° C.
1	0	$3{\cdot}6\times10^{-6}$	$+33\ \times10^{-5}$
2	19×10^{-6}	$5{\cdot}2\times10^{-6}$	$+17{\cdot}1\times10^{-5}$
3	38×10^{-6}	$6{\cdot}6\times10^{-6}$	$-\ 1{\cdot}7\times10^{-5}$
4	63×10^{-6}	$11{\cdot}6\times10^{-6}$	$-13{\cdot}5\times10^{-5}$

With solutions No. 2 and 3, both with silver and gold, the velocities given in the tables are those during the first 10 minutes after turning on the current, instead of that during the second 20 minutes as in former cases*; with these four solutions the motion was gradually interfered with by coagulation which was hastened by the current. But with Nos. 1 and 4 in both cases the motion was quite steady.

Samples of each of the solutions to which the electrolyte had been added were enclosed in test-tubes and the rapidity of coagulation observed. With the silver solutions, No. 2 coagulated within a few hours, No. 3 had settled slightly after standing all night, while No. 4 took longer to coagulate than No. 3. With the gold, solutions Nos. 2 and 3 both coagulated at the end of a few hours, while No. 4 had not completely coagulated after standing for 4 days. In each case the pure solution was stable for any length of time.

These results point quite clearly to the existence of an isoelectric point for such solutions, for it is quite apparent that the particle passes through a state of maximum instability at the time when its charge is changing from negative to positive.

4. *Calculation of the Charge on a Particle.*

In the curves given in fig. 1, the velocities of the particles in these solutions are plotted against the amount of aluminium per 100 c.cs. of colloidal solution. From them we may deduce the amount of aluminium which appears to be just necessary to neutralize the charge on the particles in those solutions; for silver it is about 26×10^{-6} grams per 100 c.cs., for gold, 37×10^{-6}. In such very dilute solutions we may take it that

* Phil. Mag. [6] vol. xi. April 1906.

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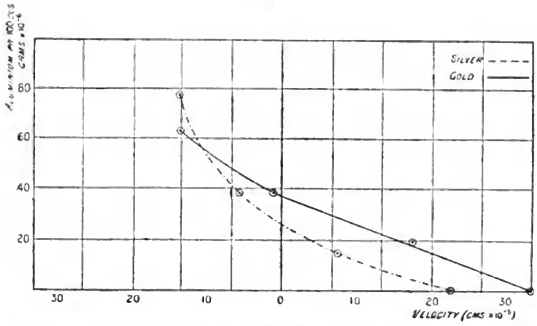
the aluminium salt is completely ionized, and therefore, assuming that all the metal ions go to neutralize the charges on the particles, we can determine the charge on each particle if we know the number in solution. From a large

Fig. 1.



number of determinations of the size of these silver particles, we may take their average volume to be 2×10^{-14} c.cs., so that since there was 6·5 mgs. of silver per 100 c.cs. of solution, there were 3×10^{10} particles per 100 c.cs. So that the charge carried by 26×10^{-6} grs. aluminium ions is just sufficient to neutralize the charge carried by 3×10^{10} particles. Therefore the charge on each particle is $2{\cdot}8 \times 10^{-2}$ electrostatic units.

From the amount of aluminium necessary to neutralize the charges on the silver and gold particles, we may calculate the total charge corresponding to a gram equivalent of silver and gold in the colloidal state in water:—

(*a*) Charge on gram equivalent of colloidal silver = ·04 of charge corresponding to a gram equivalent of a univalent ion.

(*b*) Charge on gram equivalent of colloidal gold = ·12 of charge corresponding to a gram equivalent of a univalent ion.

5. *The Reversal of the Charge on the Particles.*

A very striking result of these experiments is the fact that, after passing through the isoelectric point, an increase in the quantity of electrolyte added produces an increase in the stability of the solution. When the minutest traces of

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aluminium sulphate are added to the colloidal solution, it appears that all of the aluminium ions go to decrease the charge on the particle, and when aluminium is added in quantities just sufficient to neutralize that charge, coagulation of the particles is most rapid. When, however, the electrolyte is added at once in excess of this quantity, the particles act as absorbers of the metallic ions, and the charge on the particle is thus changed at once from a large negative one to a large positive; this positive charge on the particle induces the same surface tension effects as the negative charge, and so maintains the colloidal particles in the state of fine subdivision.

In this action may lie the explanation of Linder and Picton's results * on the coagulative power of zirconium and platinum mentioned above (p. 473). For the experiments detailed in the present paper the largest quantity of aluminium sulphate added to the colloidal solution was 10 drops $\frac{N}{1000}$ per 40 c.cs. of colloid. If the tetravalent metals are some 30 times as powerful as the trivalent (as the theory of Whetham demands), it is quite probable that Linder and Picton missed the isoelectric point altogether by adding much too large a quantity of the zirconium and platinum ions at first.

6. *Conclusion.*

The following are the points dealt with above :—

1. The addition of small quantities of aluminium sulphate to colloidal solutions of silver and gold has been shown to bring about a decrease in the charge of the particle and even a reversal of the sign of that charge.
2. The existence of an isoelectric point has been demonstrated, and thus the treatment of the case as analogous to Hardy's egg-albumen experiment has been justified.
3. A calculation of the size of the charge on each particle of silver solution has been made.

In conclusion I wish to thank Prof. J. J. Thomson for his kindly interest and helpful suggestions at all times.

Cavendish Laboratory,
June 1906.

* *Loc. cit.*

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LII. *Acceleration of Gravity at Johannesburg.* *By* Prof. R. A. Lehfeldt, *D.Sc.**

THE value of gravity has not, so far as I am aware, been measured anywhere on the South-African tableland. On taking charge of the physical laboratory at Johannesburg I thought it desirable to make a provisional determination with such means as were at hand. There was a pendulum, intended for student's use, of the usual (Borda's) pattern; the only standard of length available was the scale and vernier of a cathetometer (by Pye, of Cambridge), the scale being engraved on the steel upright of the instrument. For time measurements there was a chronometer, which, by the kindness of Mr. Innes, Director of the Government Meteorological Observatory, Johannesburg, could be rated telephonically by comparison with the observatory standard clock.

The dimensions of the pendulum were as follows:—

Ball: diameter 7·783 cms., mass 1730 gms.
Nut at top of ball: thickness 0·238 cm.
Wire (pianoforte steel): mass 0·011 gm. per cm.
Knife-edge attachment: mass 53 gms. (radius of gyration about knife-edge)^2 = 13, centre of inertia 2·1 cms. below knife-edge.

The laboratory possessed a steel beam, two metres long, forming part of an optical bench. In this a V groove was cut, and a set of eight steel bars, each about 248 mms. long with rounded ends, were made to slide in the groove. The beam was mounted vertically alongside the pendulum, and a set-square used to mark the position of (*a*) the top of the nut on the ball, (*b*) the plane (of glass) on which the knife-edge works. These levels were marked by scratches on paper pasted across the steel beam; the length of the suspending wire was so chosen that the distance between the scratches was a few millimetres longer than the length of a whole number of the steel bars. The beam was then laid horizontally, the bars placed in the groove, and the short lengths over at the ends measured by a travelling microscope. The lengths of the bars themselves were measured by the scale and vernier of the cathetometer, using the instrument as a pair of calipers.

Since the chief error of Borda's pendulum is usually taken to be the uncertainty as to whether the centre of inertia of

* Communicated by the Author. Read before the South African Association for the Advancement of Science at Kimberley, 1906.

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the ball coincides with its centre of figure, two lengths of wire were used—one about equal to seven rods, the other to four. The length from knife-edge to centre of ball was obtained as described, and the length of the equivalent simple pendulum calculated by means of the centre of inertia and moment of inertia of the whole system. The results were:—

179·19 cms.	104·07 cms.

To observe the time of vibration the chronometer, illuminated by an incandescent lamp, was placed on a table in front of the pendulum, a small mirror fixed to the wall behind it, and a telescope arranged to observe the seconds hand of the chronometer reflected in the mirror. The pendulum wire (slightly out of focus, of course) swung in front of the mirror and could therefore be seen along with the chronometer. A set of five thousand complete oscillations was observed, the time being noted for every hundredth up to 2000 and every five hundredth from there onwards. The period for the first two thousand and for the whole set was calculated in the usual manner (0 to 2500, 500 to 3000 &c.) and corrected for amplitude.

Semiamplitude at start ...	8 cms.	$5\frac{1}{2}$ cms.
,, finish ...	3 cms.	$2\frac{1}{2}$ cms.
Temperature	10° to 18°	7° to 13°
Chronometer loses 1 in 18,000.		
Corrected period	2·68845	2·0490

$$g = 4\pi^2 \frac{179{\cdot}19}{2{\cdot}68845^2} = 978{\cdot}74$$

$$= 4\pi^2 \frac{104{\cdot}07}{2{\cdot}0490^2} = 978{\cdot}59.$$

The difference between these numbers may be accidental or may be due to an error in the position of the centre of inertia of the ball. I think it is due to the former cause chiefly, and as the measurements cannot claim an accuracy beyond the first place of decimals I take the result to be

978·7.

The latitude of the physical laboratory is 26° 11′ south, and its altitude 1753 metres. Helmert's formula

$$g = 980{\cdot}62 - 2{\cdot}6 \cos 2\phi - \frac{h}{3300}$$

gives 978·50. For a tableland theory would give a correction for altitude only $\frac{5}{8}$ as great: it is usually stated

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that the tableland formula is inapplicable, owing to displacements in the density of the underlying strata; but this would seem not to be the case with the great South African tableland, as the formula

$$g = 980{\cdot}62 - 2{\cdot}6 \cos 2\phi - \frac{5}{8} \cdot \frac{h}{3300}$$

gives $g = 978{\cdot}70$ in agreement with the observations.

Since there are only two really large tablelands in the world, South Africa and Tibet, some geophysical interest attaches to the results, and it is hoped they may be repeated with better apparatus.

Transvaal University College, Johannesburg,
July 1906.

LIII. *Comparative Observations on the Evolution of Gas from the Cathode in Helium and Argon.* By Clarence A. Skinner, *Ph.D., Professor of Physics in the University of Nebraska* *.

In previous reports† of investigations on the subject of evolution and absorption of gases by the electrodes in vacuum-tubes, the writer has shown that during the first few minutes after starting a glow-current through helium, hydrogen is given off by a metallic cathode at a rate determined by Faraday's law for electrolytes. Further, if a carbon cathode be used in helium, nitrogen is evolved by it, within certain time limits, according to the same law. But if a current be passed between metallic electrodes in hydrogen, or between carbon electrodes in nitrogen, the increase in gas pressure in the discharge-tube (by which the evolution is detected) is relatively small; in fact, under certain conditions it decreases with duration of current. The difference between the observations with helium on the one hand, and with hydrogen or nitrogen on the other, was traced to anode absorption of the last two, that of helium being inappreciable. Under the *assumption* that the rate of evolution of gas from a given cathode is determined by the current alone, irrespective of the kind of gas in the discharge-tube, the experimental results indicate that the anode absorbs hydrogen and nitrogen, when these are used as filling, at a rate also determined by Faraday's law.

To verify the foregoing assumption is important. This of course can be done only in those gases which are not appreciably absorbed under the action of the current. In addition

* Communicated by the Author.

† Phys. Rev. xxi. pp. 1 & 169; *Physik. Zeitschr.* 6 Jahrg. p. 610.

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to helium, therefore, argon suggested itself as a gas which might fulfil this condition. This having proved true, the experiments described below were performed with the above-mentioned intent.

Experimental Arrangement.

The discharge-tube used was one which has served in the other investigations (*l. c.*). It was devised for making comparative tests of several different electrodes without in the meantime opening to the atmosphere. These electrodes could be removed for polishing or be replaced by new ones.

Helium and argon were obtained in sealed bulbs from Messrs. Thomas Tyrer & Co., London. Each gas was transferred from the bulb containing it to a storage-chamber connected by a stopcock to the system containing the discharge-tube, manometer, drying-chamber, and pump connexion.

The quantity of gas evolved being determined from the increase in pressure, it was necessary to measure the pressure very carefully. This was accomplished by means of a MacLeod gauge, which multiplied it one hundred times, and with which single observations were seldom more than one half per cent. off the mean of several.

The volume of the chamber occupied by the gas was obtained by attaching to the system a chamber of known volume, measuring the pressure when a given quantity of gas occupied the unknown volume, then the known in addition, and from these data calculating the desired value. This was found to be 455 c.c.

The electric current through the discharge-tube was furnished by a battery of small accumulators, measured by a Weston milliammeter, and regulated by a resistance of cadmium iodide in amylic alcohol.

Customary precautions were taken to avoid the presence of moisture and hydrocarbons.

The electrodes were given in all experiments a mirror polish, thereafter carefully cleaned with a dry cloth, then mounted in the discharge-tube, and this allowed to stand evacuated in connexion with the drying-chamber, at least twenty-four hours before making a test. To avoid the possibility of time changes in the metal, the plan was followed of testing one sample of it in one kind of gas, then following this with a test of another sample of the same metal (which had been prepared in a similar manner) in the second gas. This method proved to be wholly satisfactory.

Earlier experiments (*l. c.*) having shown that both

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aluminium and magnesium give off hydrogen much more freely than the denser and more electropositive metals, these were chosen for making the comparative study of the two gases.

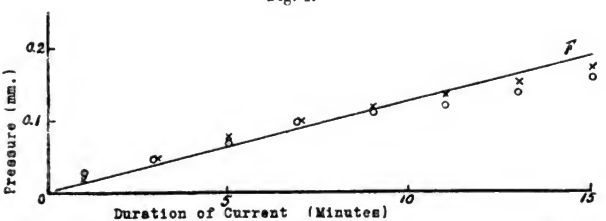
Observations with Aluminium Cathodes.

Six similar cathodes of aluminium (circular disks, discharge area, 1·8 sq.cm.) were prepared and mounted, as already described. These were then successively tested during an uninterrupted series of experiments. With a fresh sample each time as cathode—an aluminium wire serving throughout as anode—a definite current was passed through the discharge-tube and the gas pressure observed at given time intervals. The test of one being completed, the system was thereupon immediately evacuated, filled with fresh gas, and the next tested. In this manner three samples were tested in helium under currents of 1·0, 2·0, and 3·0 milliamperes respectively; and three others in argon.

The results of these tests are incorporated in Tables I., II., and III.; the observed values being recorded in the first, second, and fourth columns; while in the third and fifth is given the pressure of the evolved gas; and in the sixth the calculated value of the latter in case Faraday's law controls the rate of evolution from the start. At the top of each of the four medial columns is indicated the possible error of observation, though the probable error was not more than half as great.

For the sake of clearness these results are also represented graphically in figs. 1, 2, and 3; the pressure of the evolved

Fig. 1.



gas by the ordinates and the corresponding duration of current by the abscissas. The values obtained in helium are indicated by circles, those in argon by crosses. The line F in each figure represents the value required by Faraday's law.

Referring to fig. 1, we find for a current of one milliampere

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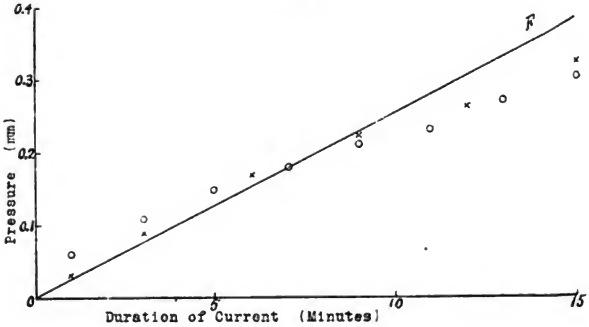
the values in helium and argon practically identical during the first seven minutes, and for the same period they coincide within the errors of observation with the line F. After that time the evoultion of gas falls gradually below the calculated rate, as indicated by the direction of the points dropping below that of the line. It may be noted that the values for argon are slightly nearer F throughout than those for helium. Fig. 2, representing the observations obtained under a current

Fig. 2.



of two milliamperes, shows a marked divergence in the results from the two gases during the first minute after starting the current, but agreement in rate during the next few minutes, after which helium falls below argon and both as before below F. Here again the results in argon are very near the calculated values until such a time is reached when we must conclude that the metal has been relieved largely of its available gas, namely, that in its surface-layer. After the first minute helium gives equally satisfactory results until a similar condition is reached. The results obtained with a current of three milliamperes (fig. 3) are practically identical in direction with F in the two gases between the first and third minutes, both, however, giving excessive values on starting the current. Contrary to the results in foregoing tests, the sample of aluminium used in this case in argon became depleted before that used in helium.

We may conclude from these experiments that Faraday's law is the controlling factor in the evolution of gas from the cathode, though as the current-density increases there is an

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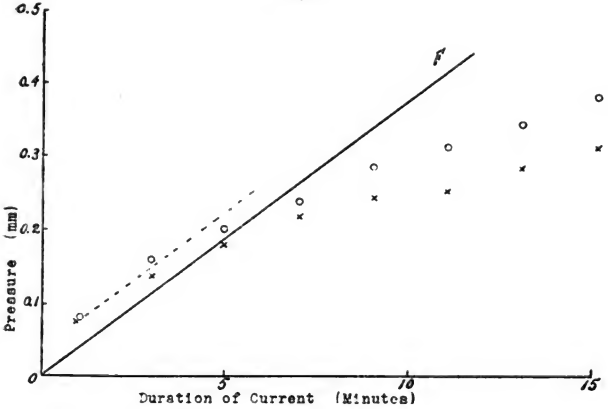
excess given off at the start. This excess was inappreciable in the case of argon until a current of three milliamperes was reached, but was very marked in helium for two milliamperes. This suggests the possible cause. For with argon

Fig. 3.



it was observed that the current-density was at no time more than twice its "normal" value, while in helium it varied from about twice its "normal" value at one milliampere to six times at three milliamperes. The excess may therefore arise from the abnormally high current-density. Another respect in which the two gases were remarkably different, namely in their cathode fall, strengthens the belief that the evolution of gas is not merely incidental. For each current used the cathode fall in helium was at least three times that in argon, hence the energy dissipated at the cathode was three times as great. If the evolution of gas be not controlled directly by the current, we should expect it to increase with this energy dissipated, which evidently is not the case.

Observations with Magnesium Cathodes.

Experiments similar to the preceding were performed with magnesium cathodes of the same form as the aluminium and subjected to a similar treatment. The results obtained from these are incorporated in Tables IV. and V., and represented graphically in figs. 4 and 5. The designations are the same

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as in the preceding figures. Fig. 4 exhibits the results with one milliampere in both helium and argon. Both sets of results follow the direction of the line closely, helium showing here again a slight excess, while argon holds closely to F until depletion makes itself evident. In fig. 5 the results

Fig. 4.

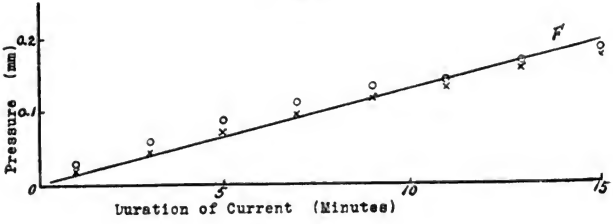
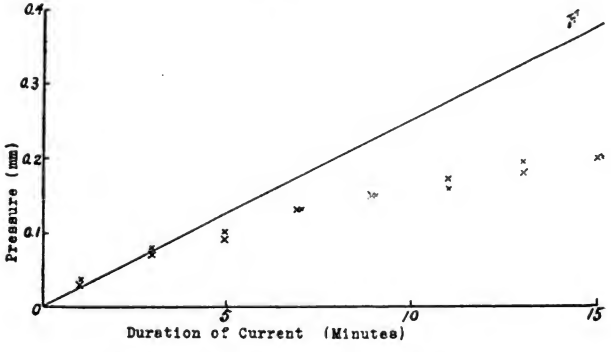


Fig. 5.



from the two samples of magnesium tested in argon, under a current of two milliamperes, serve to show the extent of agreement with Faraday's law, and also that within the errors of observation the values from two different samples are identical. They show further that magnesium does not possess as great a supply of available gas as aluminium.

The experiments with both kinds of metal support the assumption stated at the beginning, namely, *the rate of evolution of gas from the cathode is the same irrespective of the gas-filling*

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used; hence it also warrants the conclusion, derived from earlier experiments, that *the anode absorbs hydrogen and nitrogen, when these are used as gas-filling, at a rate determined by Faraday's law.*

The Brace Laboratory of Physics,
University of Nebraska.

TABLE I.

Aluminium Cathodes. Current, 1·0 milliampere.

Duration of current (minutes).	Sample No. 1 in Helium.		Sample No. 2 in Argon.		Calculated increment in pressure.
	Gas pressure (mm.). ±·02.	Pressure increment. ±·02.	Gas pressure (mm.). ±·01.	Pressure increment. ±·01.	
0	3·76	0	2·21	0	0
1	3·79	·03	2·23	·02	·013
3	3·81	·05	2·26	·05	·04
5	3·83	·07	2·29	·08	·06
7	3·86	·10	2·31	·10	·09
9	3·87	·11	2·33	·12	·11
11	3·88	·12	2·345	·135	·14
13	3·90	·14	2·365	·155	·16
15	3·92	·16	2·385	·175	·19

TABLE II.

Aluminium Cathodes. Current, 2·0 milliamperes.

Duration of current (minutes).	Sample No. 3 in Helium.		Sample No. 4 in Argon.		Calculated increment in pressure.
	Gas pressure (mm.). ±·02.	Pressure increment. ±·02.	Gas pressure (mm.). ±·01.	Pressure increment. ±·01.	
0	3·99	0	2·24	0	0
1	4·05	·06	2·27	·03	·025
3	4·10	·11	2·33	·09	·075
5	4·14	·15	...	...	·13
6	...	...	2·41	·17	·15
7	4·17	·18	...	...	·18
9	4·20	·21	2·46	·22	·23
11	4·22	·23	...	...	·25
12	...	...	2·50	·26	·30
13	4·26	·27	...	...	·33
15	4·29	·30	2·56	·32	·38

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TABLE III.—Aluminium Cathodes. Current, 3·0 milliamperes.

Duration of current (minutes).	Sample No. 5 in Helium.		Sample No. 6 in Argon.		Calculated increment in pressure.
	Gas pressure (mm.). ±·02.	Pressure increment. ±·02.	Gas pressure (mm.). ±·015.	Pressure increment. ±·015.	
0	3·76	0	3·19	0	0
1	3·84	·08	3·27	·08	·04
3	3·90	·14	3·35	·16	·11
5	3·96	·20	3·37	·18	·19
7	4·00	·24	3·41	·22	·27
9	4·05	·29	3·435	·25	·34
11	4·08	·32	3·445	·26	·42
13	4·11	·35	3·48	·29	·49
15	4·15	·39	3·51	·32	·57

TABLE IV.—Magnesium Cathodes. Current, 1·0 milliampere.

Duration of current (minutes).	Sample No. 1 in Helium.		Sample No. 2 in Argon.		Calculated increment in pressure.
	Gas pressure (mm.). +·015.	Pressure increment. +·015.	Gas pressure (mm.). +·01.	Pressure increment. +·01.	
0	3·25	0	2·28	0	0
1	3·28	·03	2·30	·02	·013
3	3·31	·06	2·325	·045	·04
5	3·34	·09	2·35	·07	·06
7	3·36	·11	2·375	·095	·09
9	3·38	·13	2·39	·11	·11
11	3·39	·14	2·41	·13	·14
13	3·41	·16	2·43	·15	·16
15	3·43	·18	2·45	·17	·19

TABLE V.—Magnesium Cathodes. Current, 2·0 milliamperes.

Duration of current (minutes).	Sample No. 3 in Argon.		Sample No. 4 in Argon.		Calculated increment in pressure.
	Gas pressure (mm.) +·01.	Pressure increment. +·01.	Gas pressure (mm.). +·01.	Pressure increment. +·01.	
0	2·27	0	2·22	0	0
1	2·30	·03	2·26	·04	·025
3	2·34	·07	2·30	·08	·075
5	2·36	·09	2·32	·10	·13
7	2·40	·13	2·35	·13	·18
9	2·42	·15	2·37	·15	·23
11	2·44	·17	2·38	·16	·28
13	2·45	·18	2·415	·195	·33
15	2·47	·20	2·42	·20	·38

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LIV. *On the Interference-Rings, described by Haidinger, observable by means of Plates whose Surfaces are absolutely Parallel.* By Lord RAYLEIGH, *O.M., Pres.R.S.* *

[Plate XVI.]

THE importance which these rings have acquired in recent years, owing to the researches of Michelson and of Fabry and Perot, lends interest to the circumstances of their discovery. It seems to be usually supposed that Haidinger merely observed the rings, without a full appreciation of the mode of formation. Thus Mascart† writes: "C'est par ce procédé, que Haidinger les a observées le premier avec une lame de mica, mais sans en donner la véritable explication." A reference to the original papers will, I think, show that Haidinger, in spite of one or two slips, understood the character of the rings very well, and especially the distinction between them and the rings usually named after Newton and dependent upon a variable thickness in the thin plate.

In the first memoir (Pogg. *Ann.* lxxvii. p. 219, 1849) the bands formed by *reflexion* are especially discussed. A spirit-flame with salted wick, seen by reflexion at considerable obliquity in a mica plate, is traversed by approximately straight bands running perpendicularly to the plane of incidence. Talbot had observed phenomena in many respects similar. ... But the yellow and black lines, observed by Talbot in thin blown glass, differ in character from the lines from mica, though both are dependent upon the interference of light. In the case of the glass the interference is due to the fact that the thickness of the glass is variable, and the lines are localised at the plate. The lines from the mica behave differently. However the plate may be turned round in its own plane, the yellow and dark lines remain perpendicular to the plane of incidence. The two surfaces of the mica are absolutely parallel to one another, and accordingly the phenomenon is the same in all azimuths. The lines appear sharper and more distinct, the nearer the mica be held to the eye, in contradistinction to the lines from glass which then become more and more indistinct and finally disappear.

The bands are due to interference of light reflected at the two surfaces. The difference of path for the rays reflected at the front and back surfaces amounts for the bright bands to a whole number of wave-lengths *plus* a half wave-length,

* Communicated by the Author.

† *Ann. de Chim.* t. xxiii. p. 128 (1871).

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for the dark bands to a whole number of wave-lengths simply.

The dark parallel lines are seen with the greatest distinctness in the reflected light. There is then a striking contrast between the reflected bright light and the black due to its absence, the plate being backed by a dark ground. If the plate is held in an oblique position between the eye and the flame, the parallel lines are seen directly, but there is a much less striking contrast with the bright parts.

At this time Haidinger had not succeeded in seeing the complete rings, but in a later memoir (Pogg. *Ann.* xcvi. p. 453, 1854) he returns to the subject and shows how the obstacle to the incident light caused by the head of the observer may be overcome with the aid of a glass plate inclined at 45°. The incident light on its way to the mica is reflected at the glass plate, while on its return it traverses the plate and so reaches the eye.

The observation of the transmitted rings is of the simplest possible character. It is sufficient to look through the plate of mica at a sheet of white paper illuminated either from in front or from behind by the homogeneous light of the spirit flame. The rings are complementary to those seen by reflexion. They are, however, much less intense, being due to the interference of the powerful directly transmitted light with the much feebler light twice reflected in the interior of the plate.

The distinction between "Berührungs-ringe" and "Plattenringe" is again emphasised, the former depending upon a variable thickness, the latter upon a variable obliquity. We may well agree with Haidinger when he concludes: "Die *Plattenringe* am Glimmer bilden also eine Classe von Interferenz-Erscheinungen für sich, die einfachste, die es geben kann, wie ich diess in der vorhergehenden Zeilen mit hinreichender Evidenz nachgewiesen zu haben glaube"*.

It is interesting to remark that Haidinger's rings, rather than Newton's, are those directly explained by the usual calculation due to Young, Poisson, and Airy, where plane waves of light are supposed to be incident upon a parallel plate. The application to a plate of variable thickness cannot be more than approximately correct. That the indirect rather than the direct application should have been (until lately)

* It should perhaps be noted that Haidinger omits the factor μ (refractive index) in the expression for the retardation on which the interference depends, viz. $2\mu e \cos\theta$, where e denotes the thickness of the plate and θ the angle of refraction. Also that, probably by a slip of the pen, he speaks of the retardation as increasing with the obliquity.

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the more familiar may be attributed to the great difficulty of preparing artificial surfaces of the necessary accuracy. The demand for equality of thickness is satisfied naturally in plates of mica obtained by cleavage, and again when a layer of water rests upon mercury *.

There is no difficulty in repeating Haidinger's observation. The transmitted rings are best seen by holding the mica close to the eye (focused for infinity) and immediately in front of a piece of finely ground glass behind which is placed a salted Bunsen flame †. If the mica be very thin, of the kind sold by photographic dealers—perhaps ·05 mm. thick, the rings are on too large a scale. But if the plate be inclined to the line of vision, the circular arcs are well seen and, owing to the enhanced reflexions, exhibit more contrast than is attainable at perpendicular incidence. When it is desired to examine the complete rings, the plate should be much thicker. I have experimented especially with two plates, ·185 mm. and ·213 mm. thick, and have observed some novel effects, evidently dependent upon the double refraction of the mica, hitherto it would seem not taken into account.

Very cursory observation on these plates, held squarely, showed that with the thinner ·185 mm. plate the inner rings were well seen, while with the thicker one they were not. Familiarity with Fabry and Perot's apparatus at once suggested that the complication might be due to a double system of rings, corresponding to the two D lines, accidentally coincident in the first case but interfering with one another in the second. It soon appeared, however, that the duplicity of sodium light was not the cause. The substitution of a *helium* vacuum-tube for the salted Bunsen made no material difference. And further, calculation showed that the two soda systems would be practically in coincidence in both cases. Thus, if we take as the mean thickness ·20 mm. and a refractive index of 1·5, the relative retardation is $2 \times 1{\cdot}5 \times {\cdot}20$ or ·60 mm. The wave-length for soda light is $5{\cdot}9 \times 10^{-4}$ mm., so that the order of the rings under observation is about $\cdot 60 \div 5{\cdot}9 \times 10^{-4}$, or very near 1000. Now the wave-lengths of the two soda lines differ by about one-thousandth part, and thus the two ring-systems are almost in coincidence. As the thickness increases from ·20 mm., the concordance would be lost, but

* 'Nature,' xlviii. p. 212 (1893); 'Scientific Papers,' iv. p. 54.

† According to Prof. Wood's recommendation the salting is best effected with the aid of a piece of asbestos, previously soaked in brine, wrapped round the tube of the Bunsen and forming a prolongation of it.

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complete discordance would not occur until a thickness of ·30 mm. was reached. Practically in both cases the ring-systems may be considered to be in coincidence. But although the duplicity of the soda line is not the cause, there is in fact a second ring-system, owing to the double refraction of the mica. That this is the case is easily proved with the aid of a nicol capable of rotation about its axis. When the ring-system of the thicker plate is examined with the nicol, there are four portions at right angles to one another at which the inner rings become distinct. But in adjacent positions, *i. e.* positions distant 90°, of the nicol the ring-systems seen are different. If one has a bright centre, the other has a dark centre. When no nicol is used, or when the nicol occupies positions at 45° to those above mentioned, the ring-systems interfere, and little or nothing is visible, at any rate near the centre. When the thinner plate is employed, the ring-system is really double, but does not appear to be so, since the components are approximately in coincidence. In this case the appearance is but little altered by the use of a nicol however held.

It is only near the centre of the system that the rings are obscured when the thicker plate is used without a nicol. Further out, the rings become distinct enough. A closer examination shows, however, that this statement needs qualification. Along four directions, apparently at right angles, radiating from the centre, there are regions of no definition. These regions are narrow, so narrow that they might at first escape observation, and they constitute, as it were, *spokes* of the ring-system. It was natural to suppose that these spokes represented places where the rings of each system bisected the intervals between the rings of the other system—a conjecture supported by the fact that the spokes disappeared when a nicol was introduced in the positions suitable for rendering the inner rings distinct. The effect was to make distinct the outer rings also all round the circumference. Further confirmation was afforded by the introduction behind the mica of cross-wires and a collimator-lens, serving to indicate a fixed direction. This was pointed at a spoke, so that without a nicol no bands were visible in the neighbourhood of the cross-wires. The nicol was then introduced in such a position as to give maximum distinctness, and the cross-wires adjusted to coincide with the centre of a bright band. A rotation of the nicol through 90° showed that in the band-system then visible the cross-wires marked the centre of a dark in place of a bright band.

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The disappearance of the rings at the places where the brightest places in one system bisect the intervals between the brightest places of the other system depends of course upon the width of the bright rings being not much less than half of the complete period. If, as in the Fabry-Perot apparatus, the bright rings are much narrower, both systems should become visible. I thought therefore that it would be of interest to silver lightly on both sides a portion of the thicker plate, the more as, apart altogether from the spokes, the whole effect would be improved owing to the enhanced reflexions. By the chemical method, as ordinarily used for silvering glass surfaces, I did not succeed; but there was no difficulty in getting the required deposits by the method of electrical discharge *in vacuo* using a silver cathode *. With the silvered plate the nature of the whole phenomenon, including the character of the spokes, was evident on simple inspection. At the spokes both ring-systems could now be seen, forming a compound system of half the original period. If we neglect the circular character of the rings, the effect may be imitated with straight bands, as shown in Pl. XVI. From a piece of striped stuff, in which the bright bands are of width equal to about a quarter of the complete period, a photographic negative was taken. The prints from this negative are made with *two* exposures, between which the paper is slightly rotated. In this way two systems of bands are impressed, crossing one another at a small angle.

The fact that the two concentric ring-systems cross one another proves of course that they cannot both be strictly circular. Complete circularity in all cases would require a wave-surface in the form of two concentric spheres, and such is not the character of optical double refraction. An exception would occur in the case of a uniaxal crystal cut perpendicularly to the axis, which would then be an axis of symmetry for the whole phenomenon. Another comparatively simple case would arise if the surfaces of the plate were parallel to the axis of a uniaxal crystal. In general one at least of the ring-systems would be elliptical as in the observations; and it would seem that the systems need not even be concentric. A more minute theoretical examination might be of some interest, especially in comparison with observations upon a plate whose optical constants were known.

Terling Place, Witham.
Oct. 10, 1906.

* Wright, Amer. Journ. Sci. xiv. p. 169 (1877).

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LV. *Unsymmetrical Diffraction-bands due to a Rectangular Aperture. By* C. V. RAMAN, *Demonstrator in Physics at The Presidency College, Madras**.

WHEN a pencil of monochromatic light coming from a slit in the focal plane of a collimating lens falls upon the object-glass of a telescope in front of which a narrow rectangular aperture is placed with its sides parallel to the luminous slit of the collimator, the diffraction-pattern seen in the focal plane of the telescope consists of a series of bright and dark bands symmetrically arranged on either side of the geometrical image of the slit, provided that the light falls normally upon the aperture. If, on the contrary, the aperture is held inclined to the incident pencil—its sides being still parallel to the slit—the diffraction-pattern is *not* necessarily symmetrical. The symmetry is not, however, *sensibly* departed from, unless the incidence be very oblique. The case in which this unsymmetrical pattern was first seen is this: place a prism on the table of a spectrometer and observe the image formed by the light reflected at very oblique incidence from one of the faces of the prism. With a prism of face-width 4·5 cms. and an incidence of 85°, the diffraction-pattern seen in the field is sensibly symmetrical, and the minima of illumination equidistant from one another. If the incidence is greater than 87°, this is no longer true. The bands are wider on one side of the pattern than on the other, those on the side towards the direct image of the slit being broader. This asymmetry increases greatly as the angle of incidence approaches 90°, and at the same time the number of bands on one side of the pattern—the side where they are broader—becomes smaller and smaller till at last they disappear altogether.

The facts can be explained quite easily. Let a be the width of the face of the prism and λ the wave-length of the light, and $\frac{\pi}{2}-\theta$ the angle of incidence. Then, in any direction making an angle $\frac{\pi}{2}-\phi$ with the normal to the face of the prism, there is no illumination provided

$$a(\cos\theta-\cos\phi)=\pm n\lambda, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where n is any whole number.

If $n=0$, $\theta=\phi$, and we have the position of the light

* Communicated by the Author.

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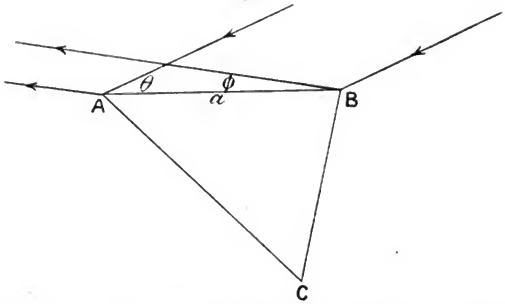
reflected according to the usual law. If θ is not small,

$$2a \sin \frac{\theta+\phi}{2} \sin \frac{\phi-\theta}{2} = \pm n\lambda$$

$$\phi-\theta = \pm \frac{n\lambda}{a \sin \theta}; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Fig. 1.



and the diffraction-pattern is identical with that produced by the effective aperture of the prism-face, and is symmetrical. If θ is small, (2) is no longer true, and a reference to the Tables shows that if the angle is small, for equal increments of its cosine, the increments of the angle are large and by no means equal. This shows that the bands are fairly broad, and that the minima are not at equal angular distances from one another.

I give this example worked out from the following data :—

$$a=3 \text{ cms.} \quad \lambda=7000 \text{ A.U.} \quad \theta=1^\circ\ 9'.$$

Angular Distance from	
The 4th minimum to the 3rd	202″
3rd „ to the 2nd	212″
2nd „ to the 1st	223″
1st „ to the central band ...	233″
Central band to the 1st minimum	247″
1st minimum to the 2nd	266″
2nd „ to the 3rd	287″
3rd „ to the 4th	311″

Further, the smallest value of ϕ admissible is zero. There is therefore a limit to the number of bands possible on one

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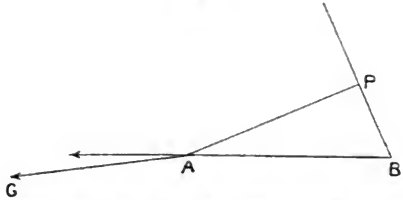
side of the pattern. There can be one or two or more, the number being the greatest integer in

$$\frac{a(1-\cos\theta)}{\lambda}.$$

This fact can be put in another way. If AB be the face of the prism and BP the incident wave-front, the limit to the

Fig. 2.



diffraction-pattern is set by the direction BA, for points on the surface AB obviously cannot send out wavelets in the direction AG.

Measurements were made on the diffraction-pattern by means of a micrometer, in order to test the theory. The measurements given below were made at an incidence where the asymmetry was not very marked yet sufficient to be easily seen.

$a=4{\cdot}57$ cms. $\lambda=6500$ A.U. $\theta=1^\circ\ 24'\ 55''$.

Angular Distance from			Observed.	Calculated.
The 5th minimum		to the 4th	110″	108″
4th	„	to the 3rd	105″	110″
3rd	„	to the 2nd	111″	113″
2nd	„	to the 1st	118″	115″
1st minimum on one side to the 1st on the other			234″	237″
1st	„	to the 2nd	125″	123″
2nd	„	to the 3rd	127″	126″
3rd	„	to the 4th	132″	130″
4th	„	to the 5th	132″	134″

For the same angle of incidence and with approximately homogeneous light of mean wave-length 7100, the observed width of the central band was $261''\pm2''$, the calculated value being 260″. Theory and observation agree as to the number of bands on one side of the pattern, if it is not more than six or seven.

The facts described above suggest that by holding a fairly wide rectangular aperture very obliquely in the pencil of light, we should get an identical system of diffraction-fringes.

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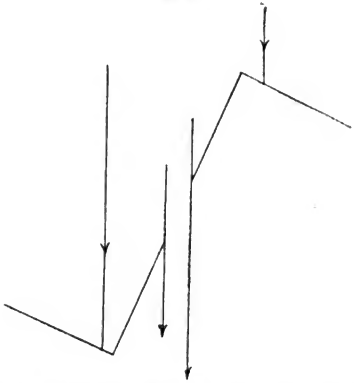
This was verified by experiment. An aperture 2 cms. wide was cut in a thin sheet of zinc, which was then bent into the shape

Fig. 3.



shown (fig. 3). The side-vanes served to cut off the light from the other portions of the collimator and to support the sheet on the table of the spectrometer. It was found that the diffraction-bands were sensibly symmetrical when the incidence was moderate, and asymmetrical when it approached 90°. The minima in this case are of course given by the usual equation

$$a(\cos\theta - \cos\phi) = \pm n\lambda.$$

The angle θ could not be measured; only the relative positions of the bands could be determined. The table gives some measurements.

$a=2$ cms. $\lambda=6500$ A.U. θ unknown.

From	
The 2nd minimum to the 1st	306″
1st ,, to the central band ...	322″
Central band to the 1st	364″
1st minimum to the 2nd	422″

The intensity of illumination at any point of the diffraction-pattern is given by the expression

$$I\frac{\sin^2\dfrac{\pi a}{\lambda}(\cos\theta-\cos\phi)}{\left\{\dfrac{\pi a}{\lambda}(\cos\theta-\cos\phi)\right\}^2}.$$

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It is perhaps worth noting that Bridge's Theorems do not hold for this unsymmetrical system; *i. e.*, the scale as well as the relative distribution of the maxima and minima in the unsymmetrical pattern is altered by altering the width of the face of the prism or the wave-length of the light, the incidence being constant. If, however, the ratio $\frac{a}{\lambda}$ is kept unchanged, the diffraction-pattern is not altered.

The experiments and observations recorded in this note were made at the Presidency College Physical Laboratory.

LVI. *Remarks on Professor Jeans' Article "On the Thermodynamical Theory of Radiation."* *By* L. B. Tuckerman, Jr.

In the July number of this Magazine*, Professor Jeans has a short article on the "Thermodynamical Theory of Radiation," in which he draws conclusions which do not seem to me to be justified. He says: "Since, however, σT^4 is to be the amount of energy per unit volume, the physical dimensions of σ are known.

"The thermodynamical argument by which, in this first theory, the formula σT^4 is reached, is concerned only with phenomena taking place in the æther. Thus we should naturally expect that it would be possible to evaluate σ in terms of quantities which measure the properties of the æther."

That, however, is not the case, as the dimensions of σ are dependent also on the dimensions of T, a quantity which is and can be defined only in terms of the properties of matter, and in fact in terms of properties which are common to all matter.

If, then, the second method mentioned by Professor Jeans gives us a valid relation between σ and e (the charge of an electron) which enables us to evaluate e in terms of σ and *vice versa*, this relation does not in the least invalidate the derivation by means of the "Thermodynamical Theory of Radiation," but merely adds another universal constant to those already thermodynamically deduced.

The apparent paradox obtained by assuming an "ideal" matter, with electrons bearing the charge $\frac{1}{2}e$, is therefore no more and no less fatal an objection to the thermodynamics of radiation, than the paradox obtained by assuming an "ideal" fluid for which Clapeyron's formula does not hold, is to the thermodynamics of ordinary matter.

Berlin, Physikalisches Institut,
July 30, 1906.

* Phil. Mag. 1906, vol. xii. no. 67, pp. 57–60.

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LVII. *Fluorescence and Magnetic Rotation Spectra of Sodium Vapour, and their Analysis.* By R. W. Wood *.

[Plates XI.–XV.]

PREVIOUS work, which has been recorded in the Philosophical Magazine†, convinced me that a careful study of the remarkable optical properties of the vapour of metallic sodium would, in time, furnish the key to the problem of molecular vibration and radiation. This opinion has been strengthened by the work of the past year, and though much remains to be done, it seems best to place the results already obtained on record. In no other case that I know of is the molecular mechanism so completely under the control of the operator. Its periodicities can be studied in a variety of ways: by absorption, by cathode-ray stimulation, by excitation with light, either white or monochromatic, and lastly by its remarkable selective magnetic rotation of the plane of polarization.

The vapour is, in every case, that obtained by heating metallic sodium in steel or porcelain tubes, usually highly exhausted. From a study of the dispersion of the vapour, it seems probable that we may be dealing with clusters of molecules with which a certain amount of hydrogen may be associated.

As I have shown in a previous paper ‡, if a pool of sodium is heated in a highly exhausted horizontal tube, the top of which is cooler than the bottom, the vapour has an enormous optical density close to the surface of the pool, and a very small density along the roof, the non-homogeneous layer acting as a prism. The only way in which I can reconcile this state of things with the kinetic theory, is to assume that the vapour leaves the metal in the state of molecular clusters, which gradually break up into smaller clusters and eventually into molecules. This is of course only an hypothesis, and I mention it in the present paper merely to indicate that our vibrating mechanism may be an aggregate and not a single molecule. It is also possible that hydrogen atoms are associated with the sodium, for the work on the dispersion indicated that there was present always a small trace of some gas other than sodium, which no amount of pumping would remove; that

* Communicated by the Physical Society: read October 26, 1906.

† "Magneto-Optics of Sodium Vapour," Phil. Mag. Oct. 1905. The "Fluorescence of Sodium Vapour," Phil. Mag. Nov. 1905.

‡ "A Quantitative Determination of the Dispersion of Sodium Vapour," Phil. Mag. vol. viii. p. 293 (Sept. 1904).

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is, it appeared to be tangled up in the sodium vapour, condensing with it in the cooler parts of the tube. All of this is, however, irrelevant, for we are for the present merely engaged with the study of a certain remarkable vibratory mechanism, and for the present need not concern ourselves whether it is a molecule, a cluster of molecules, or a compound molecule.

We will begin by a description of the various spectra which we shall study and compare in the present paper. The spectrum region with which we are concerned lies between wave-lengths 4600 and 5700, *i.e.* the region of the green-blue channelled absorption spectrum.

The Absorption Spectrum.

Photographed with the twelve-foot concave grating, the absorption spectrum is found to consist of a multitude of fine lines, to the number of about 1500 in the region specified. Its appearance has been found to be profoundly affected by the presence of hydrogen or any inert gas. It is shown on Pl. XI. (fig. 1, *g*), photographed in the second order with a twelve-foot concave grating. In hydrogen at atmospheric pressure its appearance is shown by spectra *f* and *h*, the fluted appearance being very marked. In a high vacuum its appearance is shown by spectrum *b*, the flutings having entirely disappeared. The chief change appears to lie in the increased absorbing power of certain lines or groups of lines. Spectrum *d* is taken under nearly the same conditions, the sodium vapour being less dense, however. A careful study of the effect of the inert gas has not yet been made, and it is mentioned here only on account of its relation with the subject of the paper. Comparatively few of the absorption-lines have any relation with the fluorescent and magnetic rotation spectra, *the ones concerned, however, being those which are affected by the presence of the inert gas.*

Moreover, both the fluorescence and magnetic rotation disappear, *i. e.* cannot be excited, when the sodium vapour is formed in an atmosphere of hydrogen or other inert gas. Of this matter more will be said later on.

The Magnetic Rotation Spectrum.

It was found last year that a number of vapours, showing fine and sharp absorption-lines, when placed in a powerful magnetic field, rotate the plane of polarization for wave-lengths agreeing with that of the absorption-lines, not all of the absorption-lines showing this rotatory power, however.

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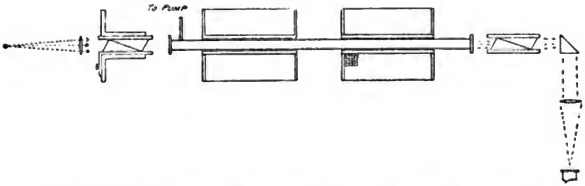
The arrangement of the apparatus for showing the bright-line magnetic spectrum of sodium is shown in fig. 1.

Fig. 1.



A piece of thin seamless steel tubing of such a diameter as to slip easily through the hollow cores of the electromagnet, from which the conical pole-pieces have been removed, is procured. A short piece of small brass tubing is brazed into one end, through which the tube is exhausted.

A lump of sodium the size of a walnut is melted in an iron crucible, and poured out into a V-shaped trough made of thin sheet iron. As soon as the bar is solid it is placed in the iron tube, one end of which has been previously closed with a small piece of plate glass cemented on with sealing-wax. The tube is introduced into the magnet, the sodium bar pushed to a position midway between the helices, and the other end closed with a piece of glass in a similar manner. The ends of the tube should be coated while hot with sealing-wax before the introduction of the sodium. One has then only to wave a Bunsen flame over them and press on the piece of glass, previously heated; the sealing-wax should come into optical contact with the glass to insure an air-tight joint. The tube is now connected with an air-pump which will produce a vacuum of a millimetre or two. If the air-pump leaks, it is a good plan to place a glass stopcock between the pump and tube to prevent the entrance of traces of air after exhaustion. For purposes of demonstration it is sufficient to heat the tube gradually with a Bunsen burner turned down low. In the present work, however, where constancy of temperature was essential, electrical heating was invariably used.

The light from an arc-lamp, made parallel by a lens, is passed through a Nicol prism, the steel tube, and a second nicol, after which it is brought to a focus by means of a second lens upon the slit of a spectroscope. With the steel tube cooled below the point at which sodium vapour forms, the nicols are set for complete extinction, and the field of the spectroscope becomes dark. The tube is now heated and

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the magnet turned on, the air-pump being worked occasionally to remove the hydrogen which is given off from the sodium. A bright yellow spot will appear on the slit of the spectroscope, which is seen to be made up of radiations chiefly in the immediate vicinity of the D lines. The phenomena at the D lines have been fully described in the paper already alluded to (Magneto-Optics).

When the vapour acquires a considerable density, a most magnificent bright-line spectrum appears in the red and green-blue region. Each bright line corresponds to a dark line in the absorption spectrum, but only a small percentage of the dark lines appear to exercise a rotatory power. Some of the strongest absorption-lines are absolutely unrepresented in the magnetic-rotation spectrum, which indicates that there is some radical difference in the absorbing mechanism.

It is with the bright-line spectrum in the green-blue region that we are now concerned. This spectrum has been photographed with the large, three-prism long-focus spectrograph, and also with the twelve-foot concave grating. Reproductions of the prism spectrograms are given on Pl. XII., *f* and *m*. The magnetic spectrum made with the large grating and the absorption spectrum recorded on the same plate are reproduced on Pl. XI., *c* and *d*.

Only about sixty lines appear in this spectrum, in contrast to the 1500 in the absorption spectrum. The intensities are very variable and apparently bear no relation to the intensities of the corresponding absorption-lines. The rotatory lines in many cases coincide with the heads of the groups of absorption-lines, though the centre of the line appears to be slightly displaced beyond the head of the group of absorption-lines. The displacement is, however, very slight, not more than half the width of the line. A list of the wave-lengths of all the lines visible on the negative follows (p. 503). The approximate intensities are represented by numerals, 10 indicating the maximum intensity and 1 the minimum.

At first sight there appears to be no regularity whatever in the distribution of the lines, except perhaps above wave-length 502, where they appear to be about equally spaced in small groups of three or four lines each. Without the aid of the fluorescence spectra of the vapour excited by monochromatic light, it is doubtful whether any regular series of lines could be found in the magnetic spectrum, for, as has been subsequently found, more than half of the lines in the series are absent, and there are six or more series present. The fluorescence spectrum with white-light excitation is shown on Pl. XI., *e*, which is from a negative made with the

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Green Rotation Spectrum.

1	5225·34	7	5040·65	2	4839·56
1	5218·49	2	5033·54	9	4837·49
1	5212·02	2	5025·66	2	4819·43
1	5186·70	3	5003·12 broad	1	4814·60
1	5179·71	10	5001·57	1	4812·68
1	5172·98	5	4979·34	3	4810·16
1	5171·98	2	4970·85	3	4802·62
1	5169·04	1	4967·10	5	4792·67
1	5165·85	1	4964·39	3	4782·89
1	5147·50	9	4962·85	1	4777·00
1	5140·71	1	4958·62	1	4766·94
3	5133·73	4	4933·93	6	4756·69
3	5126·54	5	4932·64	1	4752·04
2	5119·34	3	4924·32	2	4738·51
1	5095·70	4	4912·10	4	4727·52
2	5094·78	1	4904·67	1	4716·90
7	5087·31	1	4903·38	1	4715·63
7	5079·78	1	4896·65	1	4703·78
2	5071·58	1	4894·58	2	4692·54
1	5052·83	1	4892·77	2	4670·30
1	5049·56	2	4883·81		
5	5048·49	3	4865·59		

twelve-foot grating. As will be seen, the bright lines coincide with the bright lines of the magnetic spectrum, though much broader. It will be easier to explain how the series were picked out after we have commenced the study of the fluorescence. I have added to Pl. XI. spectrum *i*, the magnetic spectrum with the series indicated. There appear to be five distinct series and a number of lines which thus far have not been brought into any definite relation with one another. These series we will number 1, 2, 3, 4, and 5. All the lines belonging to the first series have one dot under them, those belonging to the second have two dots, &c. These series are shown separate on the chart (Pl. XV.) at the top. Absent lines are indicated thus : ◊.

The fifth series is at the top, the fourth next, and so on down, the extra lines being indicated in the lower row. This arrangement is considered provisional : it is the best that I can do at the present time, and I believe that it is correct in the main. We shall see presently, however, that photographs of the fluorescence stimulated by monochromatic radiation will have to be made with the large concave grating before we can be absolutely sure of all the lines. We will drop the magnetic spectrum for the present, and consider

The Fluorescence Spectrum.

In the previous paper I have described some of the remarkable changes which take place in the distribution of

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energy in the fluorescence spectrum of sodium vapour when the wave-length of the exciting light is changed. With white-light stimulation the general appearance of the spectrum is shown in Pl. XIV. fig. 1, A. There is, in addition, a broad double band at the position of the D lines, and a red-orange spectrum which, when the vapour is dense, is distinctly banded. In the present paper we shall be concerned chiefly with the portion figured, for it is in this region that most of the remarkable changes occur. As will be seen, it is comprised between wave-lengths 460 and 570, and is devoid of any apparent regularity in the distribution of its lines, except in the region above $\lambda=505$ where we have lines spaced with considerable regularity, the spacing becoming less as the wave-length increases. The distribution of intensity in this portion of the spectrum is such as to give it a fluted appearance, the flutings being most conspicuous in the region between $\lambda=505$ and $\lambda=535$. With white-light stimulation the flutings cannot be made out above 540, as can be seen from Pl. XIV. fig. 1, B, in which the upper limit of this part of the spectrum is shown. The fine lines are present in this region, becoming, however, less and less distinct as the upper limit of the spectrum is approached. If, now, instead of stimulating the vapour with white light, we employ blue light in the region 460–465 obtained from a spectroscope for the excitation the fluorescent spectrum presents a totally different appearance (fig. 1, C). The blue region, corresponding in its range to that of the exciting light (indicated by a double arrow), appears as before, and the upper limit of the spectrum between wave-lengths 540 and 565, *the intermediate portion being entirely absent*, as shown in the lower spectrum of fig. 1, C. Furthermore, at the upper or yellow end, there now appear the flutings which were absent when the fluorescence was stimulated with white light.

If, now, we gradually increase the mean wave-length of the exciting light, the region of maximum intensity in the fluorescence spectrum moves down from the yellow into the green, as is shown by the remaining photographs in fig. 1, C (Pl. XIV.). Moreover, as I pointed out in the previous paper, the positions of the fluted bands change slightly, the positions of the individual lines which make up the bands remaining fixed however, the shift resulting from a change in the distribution of intensity among the lines. The reason of this curious phenomenon will appear when we come to the study of the fluorescence spectrum excited by strictly monochromatic radiations.

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The spectrum stimulated by white light I have named the "complex fluorescent spectrum," for it has been found that it is a superposition of a number of simpler spectra, any one of which can be independently excited by suitably controlling the wave-length of the stimulating light. Indications of something of this sort were found last year, and were described in the preliminary paper. An insufficient number of photographs were obtained, however, at the close of the university year, to make anything like a complete analysis of the complex spectrum possible.

During the past winter and spring a careful study has been made of the relations existing between the complex fluorescent spectrum, the absorption spectrum, and the bright-line rotation spectrum described in the earlier paper. The fluorescent spectrum has at last been photographed with the twelve-foot concave grating, enabling a study to be made of its more minute structure.

Some very remarkable effects have been observed with monochromatic stimulations obtained by the isolation of certain lines from metallic arcs, which yield comparatively simple fluorescent spectra made up of widely separated sharp lines, placed in many instances at nearly equal intervals along a normal spectrum. A given series of lines can be brought out by stimulating with light of any wave-length corresponding to that of some line in the series, but when the stimulations occur at certain points, some of the lines may be absent, gaps appearing in the series. The most conspicuous example is the case of stimulation with the cadmium line 480, which will be considered in detail presently. It will be remembered that certain lines are absent in each series, in the magnetic spectrum.

The apparatus employed in the experiments was essentially the same as that used in the earlier work. It consisted of a seamless tube of thin steel three inches in diameter and thirty inches long, with a steel retort at its centre in which a large amount of sodium could be stored. The retort was made by fitting two circular disks of steel to a short piece of tubing, just large enough to slip snugly into the larger tube. The circular ends of the retort were provided with oval apertures, as shown in Pl. XIII., fig. 1. The retort was half filled with sodium, the molten metal being poured in through one of the apertures. It was then introduced into the tube and pushed down to the centre, after which the plate-glass ends were cemented on, as shown in the figure. This arrangement prevented the rapid diffusion of the vapour, and enabled a

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large supply of metal to be kept at the centre of the tube. The tubes used in the earlier work required re-charging after two hours' continuous operation, while the retort-tube could be operated for several hundred hours on a single charge.

The tube was exhausted with a Fleuss pump and heated at the centre with a large burner, the ends being kept cool by jackets of absorbent cotton which dipped into pails of water.

The illuminating beam of either white or monochromatic light was focussed just within one of the oval apertures of the retort, falling upon the opposite wall a little to one side of the other aperture. By covering the further end of the tube with a black cloth, the fluorescent spot showed against the dead black background of the second oval aperture, and its spectrum was therefore uncontaminated with the exciting radiations.

A large three-prism spectrograph was constructed for photographing the spectra. The prisms were of clear dense flint four inches in height, and the focal length of the lenses thirty-six inches.

Since only lenses such as are used for telescopes were available, the spectrum lines are not so sharp as one would wish, except near the axis of collimation. By adjusting things so that the centre of the fluorescent spectrum fell at this point, the definition was pretty fair throughout its extent, and wave-lengths could be determined with an error not greater than one or two Ångström units.

The photographs of the complex spectrum of the fluorescence excited by sunlight, obtained with this instrument Pl. XIV. fig. 1, showed peculiarities which made it appear of the utmost importance to study the spectrum under higher dispersion. The green fluorescent spot had, after repeated improvements in the apparatus, attained such brilliancy that I felt sure that records could be obtained with the twelve-foot concave grating. An all-day exposure was found to be sufficient, the resulting spectrogram, with the iron comparison lines and the wave-length scale, being reproduced on Pl. XI. fig. 2. The scale was printed separately, and slight errors occur, due to stretching and shrinking of the prints. They are not greater in any case than 1·5 A.E. This plate shows the minute structure of the complex spectrum, and enables us to measure the wave-lengths of the bright lines with far greater accuracy than could be done with the plates made with the prism-spectrograph. As much of the detail is lost during the process of reproduction, I have prepared a very careful

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drawing of one of the groups of bands at wave-length 5200 (fig. 2). The drawing was made from a print with the aid

Fig. 2.

of a hand-magnifier, and the peculiarities shown are found throughout the entire spectrum.

The bright lines are sharp, and quite as narrow as the iron lines of the comparison spectrum. We must remember, however, that the slit was not very narrow, 0·2 mm. perhaps, and it is quite possible that a further contraction would not decrease the width of the fluorescent lines. Each bright line is in general accompanied by two lateral wings, which terminate quite sharply the narrow spaces between, every two adjacent wings appearing as narrow dark lines. These wings do not in general appear with strictly monochromatic stimulation. In the work of last year, when studying the remarkable changes which occur in the spectrum stimulated with the fairly homogeneous light furnished by the monochromatic illuminator with very narrow slits (see previous paper), I observed that as the wave-length of the light was very gradually altered, the fluorescent lines appeared with wings first on one side and then on the other, the change in the appearance of the line reminding one of a flag flying first on one side of the mast and then on the other. With the strictly monochromatic illumination obtained with the isolated metallic arc lines, the fluorescent lines are usually devoid of wings, though in some instances the wings are found, and sometimes the wings appear without the lines. These circumstances appear to indicate that the wings are due to the stimulation of the electron by frequencies slightly greater and slightly less than its own natural frequency.

I have not yet had time to repeat last year's experiments with the monochromatic illuminator, and plan to make a further study of the changes which accompany very gradual changes in the wave-length of the exciting light. The observations are very difficult and uncertain, as the light furnished by the monochromatic illuminator is not very bright when its slits are made as narrow as possible, and the fluorescence spectrum can only be observed by carefully

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rested eyes. Probably by using a *large* prism-spectroscope as an illuminator better results can be obtained. What is most desired is a light siren!

Analysis of the Complex Spectrum. Stimulation with Monochromatic Light.

It was found impracticable to use the monochromatic illuminator for the study of the simple spectra which made up the complex spectrum. Even with its slits very narrow, the wave-length range of the emitted light was wide enough to cover several of the absorption-lines of the vapour. The earlier work had shown that the light of the cadmium spark was capable of exciting fluorescence, and experiments were accordingly started with metallic arcs. Just at this time came the very opportune invention of the fused quartz metallic arc-lamps by Stark, working in the Heraeus laboratory at Hanau. Two of these lamps were immediately ordered, one filled with cadmium, the other with zinc. Their form is shown on Plate XIII. fig. 1. The lamp is kept in communication with a mercury pump during its operation and stands in a dish of water. The cadmium lamp worked well on a circuit of 110 volts direct, but the zinc lamp gave better results on the 220. They are started by a small induction-coil, one terminal of which is connected to the negative pole of the lamp, the other twisted around the quartz U-tube. A blast lamp is directed against the tube until the portion above the metal electrodes is red-hot, the coil is then started, and the arc usually forms at once. As exposures of eight or ten hours were often necessary, and the lamps have a trick of going out every half-hour or so, an automatic starter was devised, which turned on the coil the moment the lamp went out. As soon as the arc struck again, the coil was stopped. This arrangement is figured on Plate XIII. fig. 2, and consisted of a small electromagnet in circuit with the lamp, which pulled a steel spring away from a brass screw as long as the lamp burnt. The spring and screw were inserted in the primary circuit of the coil.

The cadmium lamp burns with a greenish-blue light of dazzling brilliancy, the zinc lamp with a curious purple light, which causes all the woodwork in the room to appear blood-red, while most other objects appear bluish white or purple. Both lamps excite a brilliant fluorescence of the sodium vapour when their images are thrown upon the oval aperture of the retort. In this case the fluorescence is excited by several different radiations. Various devices were used for picking out one line at a time. The cadmium radiations

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which are capable of exciting fluorescence have wave-lengths 5086, 4800, and 4678. Colour screens and the Fuess monochromatic illuminator, as well as the thin crystals of chlorate of potash (described in the Phil. Mag. for June), were tried; also a block of quartz, cut perpendicular to the axis, placed between two nicols. The arrangement which gave the best results, and appeared to be accompanied with the least loss of light, is the one figured on Pl. XIII., fig. 2. One vertical tube of the lamp is used as the source, the light from which, after collimation, passes through a large bisulphide-of-carbon prism, and is focussed upon the retort by an achromatic telescope objective with an aperture of 12 cms. and a focal length of 2 metres. The dispersion of the prism was sufficient to separate completely the monochromatic images of the lamp, any one of which could be thrown into the aperture, the light passing by the edge of the 90° prism by means of which the fluorescent light was reflected through a lens and thence upon the slit of the spectroscope.

The sources of light which have thus far been successfully employed in stimulating the fluorescence of the vapour are the following: quartz arc-lamps containing cadmium, zinc, and thallium; ordinary arcs between lead, silver, bismuth, and copper electrodes; lithium, sodium, and barium arcs, and vacuum-tubes containing helium and hydrogen.

Unfortunately the quartz lamps are very expensive, and become almost useless after a run of about thirty hours, the surface becoming granular and an opaque black deposit forming on the inner walls. As exposures of eight hours are sometimes necessary, it will be seen that lamps at thirty dollars apiece, with an average life of thirty hours, make the investigation an expensive one.

The photographs of the fluorescent spectra obtained with monochromatic stimulation are reproduced on Pl. XII.

After each exposure the D lines were recorded on the plate, so that the different spectra could be brought into coincidence for purposes of comparison: the D lines will be found at the extreme right in each spectrum. The photographs have been reproduced as negatives, and the point or points coinciding with the wave-lengths of the stimulating light are indicated by arrows. A large-scale drawing or chart of the most interesting of these spectra, together with others too faint for reproduction, has been made on cross-section paper, the points of excitation being indicated by arrows, as in the photographs. (Folding Plate XV.)

Drawings of the complex spectrum and the magnetic-rotation spectrum made from the photographs obtained with

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the large concave grating will be found at the bottom of the chart: the former is a positive. At the top I have given a composite drawing which represents a superposition of all the simple fluorescent spectra thus far obtained. Immediately below it will be found the spectra excited by the complete radiation of the cadmium and zinc tubes. In each case there are three different exciting radiations simultaneously applied, yet it is almost impossible to find two fluorescent lines which coincide. The other spectra are excited for the most part by a single monochromatic radiation, the wave-length of which is indicated by the arrow. I have not yet obtained photographs of the fluorescent spectra excited by the separated radiations of the copper arc or by the separated zinc lines 468 and 472; consequently these have been drawn together. It is possible, however, by comparing the spectrum excited by copper with the one excited by zinc 4811, to make a guess as to which lines belong together.

We will begin by a study of the spectra excited by the cadmium radiations.

Cadmium Stimulation.

Photographs of the fluorescent spectrum obtained with the cadmium-arc excitation are shown on Pl. XII., *h*, *i*, *j*, *k*.

Of these, *i* and *k* are excited by all three cadmium lines; the former taken with a much narrower slit than the latter. Spectrum *h* was taken when the sodium vapour was excited by the line 480; observed visually, it consists of twelve bright lines, in groups of two and four, as shown on the chart immediately above the complex spectrum at the bottom. Midway between the groups very faint lines can be perceived if the eye is carefully rested. The strong lines are arranged thus: two, absent line, four, absent line, two, absent line, four. The absent lines, or, more correctly speaking, the faint lines, evidently belong to the same series, and taken collectively the lines will be found to be very nearly equidistant, measured along a *normal spectrum*.

The wave-lengths of the lines in this series, as determined from measurements of the plates obtained with the prism-spectrograph, are as follows:—

λ.	λ differences.		λ.	λ differences.	
4760			..	$\frac{76}{2}$	38
4800	.....	34	5095	.....	38
4838	.....	38	5133		
..	$\frac{70}{2}$	35	..	$\frac{74}{2}$	37
4908			5207		
4946	.....	38	5245	.....	38
4983	.....	37	5283	.....	38
5019	.....	39	5321	.....	38

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These wave-lengths I consider to be accurate to within about 2 A.E. or $\frac{1}{3}$ of the distance between the D lines.

It is, of course, of the utmost importance to determine the law which governs the spacing of the lines in the simple spectra. A criterion may perhaps be obtained by referring to the magnetic rotation spectrum of the vapour, the lines of which correspond in general to the lines of the fluorescence spectrum. This spectrum has been photographed with a large concave grating and the wave-lengths determined certainly to within a tenth of an Ångström unit. The strong lines at the following points of the spectrum form a series analogous to the series obtained with the cadmium 480 series.

λ.		λ differences.
5119·34		
	.....	39·56 A.E.
5079·78		
	....	39·13
5040·65		
	.....	39·08
5001·57		
	.....	38·67
4962·85		
	.....	38·52
4924·32		

This we have called the first series.

The wave-length differences are, in this case, much more nearly constant, and decrease progressively.

The lines of this series are especially conspicuous in the magnetic rotation spectrum (Pl. XI., fig. 1, *c*), hence I have mentioned it first; they appear in the fluorescence excited by the lead line 5001, as will be seen by reference to the chart.

If now we try to fit one of the magnetic series to the cadmium 480 fluorescence series, we find that the third magnetic series coincides with it between 5019 and 5134, while in the violet region it coincides with the fourth magnetic set. I do not feel sure whether this peculiarity is due to slight errors in the determination of wave-lengths or not. I think not, however, for I have very carefully superposed the two negatives (cadmium fluorescence and magnetic rotation spectra), both taken with the same instrument, and find the same disagreement. We cannot be sure of anything, however, until the cadminm series has been photographed with the grating.

During the coming year I expect to photograph the fluorescence spectrum excited by cadmium and zinc radiations with the large concave grating. It will then be possible to determine the wave-lengths of the lines to within a tenth of a unit. One of the most remarkable facts connected with the appearance of the lines of a series is that the distribution

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of energy among the individual lines depends upon the point of excitation.

Unfortunately there are very few arc lights bright enough to excite fluorescence. It was found, however, that the silver line 5207, which coincides with one of the fluorescent lines of the cadmium 480 series, was bright enough for the purpose. The silver was carefully freed from copper, as the neighbouring copper lines are powerful exciters; in their absence, no prismatic separation was necessary, as the rest of the silver lines were inoperative. A photograph of the fluorescence spectrum obtained with silver stimulation is shown on Pl. XII., *g*. The series in this case *has no gaps* in it, the line at 5170, which is absent with cadmium excitement, coming out strong (see chart as well). The monochromatic illuminator, with its slits reduced to the width of a hair, was arranged to furnish light of wave-lengths corresponding to other lines in the series, and photographs obtained which are recorded on the chart. It will be seen that faint or missing lines occur in each case, but that their position varies with the point of excitation. If we consider each line caused by a single electron or vibrator, the phenomena suggest that the vibrators are united in some way, perhaps in a closed ring, and that when the system is set in vibration there are nodal points, the position of which depends upon the point in the chain where the periodic force is applied. Moreover, as has already been pointed out, if the force is applied at the "high frequency" portion of the chain, the regions excited are those of highest and lowest frequency, the intermediate portion appearing to be at rest. This is especially noticeable in the case of the bismuth excitation (Pl. XII., *e*; and chart, Pl. XV.).

In addition to the lines enumerated above, there are a number of others at the upper end. These do not appear to be distributed with the same regularity, though some of them may form an extension of the series, or more probably may be the beginnings of other series. In general it has been found that in the simple spectra the lines are regularly spaced between the extreme violet end and a point at about $\lambda = 5350$. Above this point the spacing is generally very irregular, and it is difficult to unravel the spectrum. Of this more will be said later.

We will next take the fluorescent spectrum excited by the green cadmium line 5086.

This spectrum is remarkable in that it is made up of eleven pairs of lines regularly spaced (Pl. XII., *j*, and chart). The other two cadmium lines appear on the plate, as the spectrograph was not shielded from the diffused light from the lamp.

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A series in the magnetic spectrum coincides with the series formed by the shorter λ member of each pair. The wave-lengths and differences are given in the following table:—

λ.	λ differences.	λ.	λ differences.
5165·85	..... 39·31	.....	$\frac{77·64}{2}$ 38·82
5126·54	..... 39·23	4970·85	
5087·31	..... 38·82	4932·64	..... 38·21
5048·49		4894·58	..... 38·06

I am quite at a loss as to how the series formed by the other members of the doublets is originated. It appears to coincide with the series excited by helium 5014, as will be seen by reference to the chart. It appeared at first as if the exciting line might lie between two adjacent fluorescent lines, and in that way excite a double series; but cadmium 5086 is slightly on the short λ side of the magnetic line 5087·3, while the wave-length of the other line of this pair is 5092, *i. e.* on the long λ side.

This is the only case recorded where a spectrum of doublets is excited by monochromatic stimulation, though I am of the opinion that the copper line 5152 behaves in the same way.

What is still more remarkable is the fact that if the excitation is at a different point we no longer get doublets. The lithium line at 4971 takes hold of one of the more refrangible components of one of the doublets, but only a single series of lines appears in the fluorescence spectrum (see chart, Pl. XV.). The other series, *i. e.* the less refrangible components, can be separately excited by stimulation with the helium line 5014 (see chart). If we are dealing with anything in the nature of electron doublets, we should expect both the lithium and helium radiations to excite a fluorescence showing double lines.

If we try to explain the phenomena by assuming two chains of electrons fastened together at the point 5086, we must account for the fact that the 5086 vibrator excites the other chain when it is acted upon by light of its own frequency, but not when it is vibrated by the lithium radiation acting at a different point on the chain. I have adopted this hypothesis of electron chains merely to aid in describing the physical phenomena, and not with much hope that it will explain anything.

It seems much more likely that the different lines represent vibrations of different frequencies of the same system. We must not try to make the molecule too much like a piano.

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The vibrations may be ripples running over its surface, or they may be unlike anything with which we are familiar. If we had never seen a bell, it would be difficult to work out the theory of its very complicated vibrations from a study of a set of simple pendulums. Possibly stimulation at some other points might give rise to the double lines.

I attempted to do this with the monochromatic illuminator, but without success. The band of exciting light cannot well be made much narrower than the distance between the components of the doublets. Even with the instrument set at 5086, I could detect no evidence of the doublets. I am planning to investigate this matter further with a larger monochromatic illuminator designed to furnish more nearly monochromatic light.

In addition to the eleven pairs of lines in the fluorescence spectrum excited by cadmium 5086, there are two strong lines at wave-lengths 5305 and 5341. These seem to belong to the same series, and the former has a faint companion, the two forming a doublet. The line 5341 is also accompanied by a companion, which, however, is so faint as to be barely distinguishable.

The spectrum excited by the more refrangible of the two blue cadmium lines 4678 is reproduced only on the chart. It consists of a regular series of five lines in the blue region, and a large number of irregularly spaced lines of widely different intensities in the yellow-green region. None of these lines appear to be represented in the magnetic spectrum.

This spectrum illustrates well the characteristic peculiarity of the sodium fluorescence spectrum, that stimulation at the more refrangible end excites powerful fluorescence at the opposite end. The lines which form the regular series we may call directly excited, the others in the yellow region indirectly excited. The latter in all cases seem to be irregularly spaced. The great problem to solve is to determine the nature of the mechanism and find out how the low-frequency vibrators are set agoing by the stimulation of the high-frequency ones, while they remain quiescent when the stimulation is at the middle of the spectrum. Speculations on these points must be deferred for the present.

Zinc-arc Excitation.

The complete fluorescent spectrum excited by all three of the zinc lines (*i. e.* the total radiation of the lamp) is shown near the top of the folding chart, just below the cadmium

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fluorescence. It is scarcely possible to find a coincidence of two lines. The two spectra placed side by side make a striking picture of the variation produced by different excitations of the same fluorescing medium. A photograph of the spectrum is reproduced on Plate XII. *c*.

Exciting with the zinc line 4811 alone gives us the spectrum shown on Plate XII. *b*. The other two zinc lines are of course present on account of diffused light. Referring now to the chart (Pl. XV.), we find that the violet end of the spectrum agrees pretty well with the fifth magnetic series, though other lines are present. The two strong lines at 5188 and 5225 also appear to belong to the same series. The lines on the whole are much less regularly distributed than in the case of the cadmium 480 excitation. The three wide pairs between 523 and 535 are peculiar to this excitation.

The fluorescence excited by the other two zinc lines, 4680 and 4722, is also recorded on the chart. These lines are so close together that it was found impossible to illuminate the vapour with light from but one of them, and have at the same time sufficient illumination to excite much fluorescence. I have not yet found much evidence of regularity in the distribution of the lines in this case, though there is undoubted evidence of two series in the immediate vicinity of the exciting lines. I have indicated with the letters A and B the lines which appear to belong together (compare with helium excitation). We have an enormous number of lines in the yellow-green region, since we have a double stimulation at the opposite end of the spectrum. There seems to be some regularity here, but it is difficult to say which lines belong together.

Bismuth Excitation.

The light of the bismuth arc makes a beautiful stimulus for the fluorescence, since it contains but a single operative line, the strong one at 4724. It gives rise to a very regular series in the blue-violet region, the lines appearing to fall midway between the lines of the third and fourth magnetic series (Plate XII. *e*, and Chart). Though the wave-length of the exciting line is only two Ångström units longer than that of the zinc line 4722, the spacing of the series in the two cases is quite different. The same thing has been noticed in the case of the shortest cadmium and zinc lines, which makes it seem possible that interesting results may be obtained by altering the wave-length of the exciting line, either by

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pressure or a powerful magnetic field. Experiments in this direction will be made next winter.

In both of these cases, in each of which we have excitation by lines of nearly the same wave length, the wider-spaced series is produced when the stimulation is by the longer wave-length. It remains to be determined whether we take hold of different absorption-bands and excite entirely different series, or whether we stimulate the same vibrator in each case, the spacing of the resulting lines depending upon how nearly we approach its natural period in our exciting vibrations.

In addition to the regularly spaced lines in the violet, we have a complex assortment of lines in the yellow-green region, the intervening portion being totally devoid of lines. One of these lines, $\lambda=5300$, has a broad diffused wing, and it is perhaps worthy of remark that in the spectrum excited by the two zinc radiations we have a hazy doublet at this point, in the spectrum excited by zinc 4811, a single line, and in the spectrum excited by cadmium 480, two faint lines. Some of the other lines have wings, as will be seen from the chart, and at wave-length 546 we find a broad hazy band. All of these peculiarities complicate things; and I have drawn attention to them merely to show that we must not expect to explain matters by too simple a mechanism.

A word or two about the bismuth arc may not be out of place. Various plans were tried, such as immersion of the electrodes in water, burning in the carbon arc, &c. The best arrangement was found to be a shallow iron dish about 4 cms. in diameter (pounded from a piece of thin sheet iron), filled nearly full of molten bismuth, and kept hot over a small burner. The dish of metal formed the positive electrode, the negative being a bar of iron which could be raised or lowered by a rack and pinion. The arc required constant attention, fresh metal being put into the dish every ten or fifteen minutes, and as exposures of eight hours were necessary, it will be seen that an enormous amount of very fatiguing work was necessary in all cases where open-air arcs were used.

Copper Excitation.

I have been unable thus far to obtain photographs of the fluorescence excited by the separated copper radiations. The lines are close together, and the arc climbs about over the electrodes. I hope next year to improve matters in this respect. On Plate XII. *a*, is seen the fluorescence excited by

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the total copper radiation. Only the three green lines are operative in stimulating the vapour. The lines in the fluorescence spectrum appear to bear no very definite relation to the lines of the magnetic spectrum, as will be seen by the chart. There are many coincidences, however, with lines in the spectrum excited by zinc 4811, and by zinc 468 and 472. By comparing the three spectra I have made a provisional determination of the lines which belong together in the spectrum excited by the copper radiation. These lines are indicated by crosses and vertical dashes placed above them; other lines, which do not appear in the zinc spectra, have not been marked. I suspect that excitation with copper 5152 will produce a doublet at this point, and probably at other points, just as does cadmium 5086. An attempt will be made to verify this surmise. I have on one or two occasions, when trying to stimulate the vapour with this isolated line, been of the impression that I saw doublets distinctly, but at the time I attributed it to incomplete separation of the exciting lines.

Lead Excitation.

A lead arc, operated in a manner similar to the one described for bismuth, was used for exciting the vapour. The only line operative was the one at 5001, and it gave rise to a well-marked series of fluorescence lines, which coincided exactly with the first series of the magnetic rotation spectrum. It is worthy of remark that one of the extra lines of the magnetic spectrum lies very close to the exciting line, yet none of these lines appeared in the fluorescence spectrum. The fluorescence excited by lead was very feeble; and even with an eight-hour exposure the lines were very faint.

Helium Excitation.

A large "end on" helium tube with a 3 mm. bore was constructed for the investigation. This tube could be run continuously with an induction-coil yielding a heavy 10-inch spark. An exposure of about twelve hours was given. Two of the helium lines are operative: line 5015 gives a well-marked series, the lines of which fall exactly midway between the lines of the second and third magnetic series; line 4713 gives a good series in the blue and at least six distinct lines in the yellow-green, the wave-lengths of which can be seen from the chart. The exciting line in this case coincides with one of the lines in the spectrum excited by zinc 468; and there is

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perfect agreement in position between the fluorescent lines in both cases, not only in the blue, but also in the yellow-green region. The lines in the spectrum excited by line 468, or at least as many as could be identified, have been marked. The identification was of course made by comparison with the spectrum excited by helium.

Lithium Excitation.

An arc was caused to play between a carbon rod and a large carbon block on which the lithium salt was placed. The image of the red flame was projected upon the window of the retort and excited a bright fluorescence. Two of the lithium lines were operative,—one at 4601, the shortest monochromatic stimulation thus far found, which gives the series in the violet (see chart, Pl. XV.), and a large number of lines in the yellow-green ; and another at 4971, which gives a beautiful series in the green, coinciding exactly with the second magnetic series. The 4971 stimulation should be especially interesting, since there are several lines in the magnetic spectrum very close to it. The line is unfortunately not very bright, and the fluorescence lines were so feeble that they could only be measured with difficulty.

The lines in the yellow-green region are also of considerable interest, since they result from a single monochromatic stimulation applied practically at the extreme lower end of the fluorescence spectrum. In fact, this line is considerably below the limit of the fluorescence spectrum as usually seen with white-light stimulation : this limit is not far from wave-length 4670, which is the shortest thus far detected in the magnetic spectrum photographed with the grating, though faint lines are visible even below 4600 on negatives made with the prism spectrograph. There seems to be evidence of a number of series in the yellow-green region, the spacing, however, decreasing with increasing wave-length, just the opposite of the state of things which holds in the blue-green region. It is more likely, however, that the apparent decreasing of the spacing as the yellow end of the spectrum is approached is due to other series similar to those which are found in the green and blue, the nearness of the lines resulting from the large number of superposed series. With white-light stimulation no trace of the lines can be seen above wave-length 555, and they are so faint as to be almost indistinguishable for a considerable distance below this point. The broad flutings seen in the spectrum stimulated with white light are doubtless to be referred in some way to the

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circumstance that the lines of the different series get into and out of step periodically: they may thus be considered analogous to the bands seen when two diffraction-gratings of slightly different spacing are superposed.

Barium Excitation.

The fluorescence excited by the barium arc appears to be due chiefly to the line 4934, which coincides with one of the extra lines in the magnetic spectrum. Line 4932 of the second magnetic spectrum is also very near it; and we find that the fluorescence spectrum contains lines which coincide with the magnetic lines of the second series, as well as lines which coincide with some of the extra magnetic lines. The barium arc contains a good many other fainter lines which may give rise to some of the fluorescent lines. It will be necessary to repeat the experiment with the 4934 line isolated.

Sodium Excitation.

As I showed in the earlier paper referred to, if we stimulate the vapour with intense sodium light, we obtain a yellow fluorescence which the spectroscope shows to be made up of two lines in the position of the D lines. We have here a re-emission of light of the same wave-length as the exciting light, and nothing else. This I have called resonance radiation, as we may find that it is different from fluorescence, though the two are doubtless intimately related. As there are a number of pairs of lines in the ultra-violet which belong to the same series as the D lines, it seemed of great importance to determine whether these appeared in the spectrum of the fluorescence excited by the sodium flame. The sodium tube was provided with a quartz window, and the light of the oxyhydrogen flame, heavily charged with sodium, focussed upon the aperture of the retort with a glass lens. White light from the arc was also used, as this excites the D-line vibrations in the fluorescence. The spectrum was photographed with a small quartz spectrograph, and though the D lines were greatly over-exposed, no traces of any of the ultra-violet doublets were found on the plate. Conversely, illumination with ultra-violet light in the region of the first ultra-violet pair of lines failed to produce any visible fluorescence. It was hoped that a faint yellow fluorescence might be produced in this way, due to emission in the region of the D lines. I have not yet tried stimulation with D_1 and D_2, alone, to see whether both D lines appear in the fluorescence. This will be a very difficult experiment, and I

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am saving it for the last. It will settle the question as to whether the principal series of sodium is a series of doublets or two series of single lines.

Cathode-ray Excitation.

The cathode rays, I find, excite a fluorescence similar to white light. The lines of the principal and subordinate series appear as well, some of them of overpowering intensity. The apparatus for the electrical excitation is shown on Plate XIII. fig. 3.

It consisted of a steel tube 3 cms. in diameter and 35 cms. in length, one end closed with a glass plate, the other cemented with sealing-wax to a glass tube carrying the cathode. The mercury-pump was kept in continuous operation to remove the hydrogen liberated from the sodium. On looking into the tube through the glass window a blazing spot of yellow light 2 cms. in diameter was seen at the point where the cathode rays entered the vapour. Its spectrum was photographed with the prism spectrograph, and is reproduced on Plate XII., *n*. In addition to the fluorescent spectrum, and the sodium lines of all three series, the hydrogen lines come out strong. I have never been able to eliminate them entirely. Very few experiments have been made on the electrical excitation, but some very curious phenomena have been observed. In some cases, by looking into the tube in an oblique direction, it was seen that at the point where the cathode rays entered the mass of vapour there was a bright green spot of fluorescent light, while at the point of exit there was an orange-yellow spot, *the intervening space being non-luminous.* Seen in a direction oblique to the direction of the rays, the two spots were seen completely separated. This I consider a very remarkable circumstance, and a spectroscopic study of the two spots of light will undoubtedly yield very fruitful results. Unfortunately the condition is a difficult one to keep fixed, for the phenomenon only appears when the density of the sodium vapour is just right and the surrounding vacuum high. As I have shown in the paper on the dispersion of sodium vapour, we can have a dense mass of the metal vapour, bounded on each side by a very high vacuum, a very anomalous condition from the point of view of the kinetic theory of gases. My impression is that the green spot will show the fluorescent spectrum, and the yellow spot the lines of the principal and subordinate series, as found in the sodium arc, but as yet I have not found time to make even a visual examination. Several attempts have been made, but by the

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time the image of the spot was thrown upon the slit in the proper direction to pass the light through the prisms, and the eye brought to the instrument, the conditions in the tube changed.

It is difficult to account for the absence of luminosity of the centre of the mass and the two bright spots. Perhaps the condition under which the rays excite fluorescence exists only where the vapour mass is in contact with the vacuum, *i. e.* in the region where the hypothetical clusters of molecules are breaking up and flying to the cooler walls of the tube. Even assuming this to be the fact, the difference in the colour of the two spots is still to be accounted for. Possibly the cathode rays excite the green spectrum, while the canal rays travelling towards the cathode excite the orange-yellow luminescence. I have made one experiment with a similar tube arranged so as to deliver a stream of canal rays against the vapour. The luminescence was bright yellow, but the tube cracked before a spectroscopic examination was made.

On the other hand, it may be that whatever causes the green luminescence is removed from the ray-bundle by absorption, the residue exciting the yellow luminescence at the point of exit. If this is the case, we should expect the same amount of yellow light in each spot; and I am of opinion that the green light is much too pure for this to be the case. Further experimenting will be necessary before it is possible to draw any very definite conclusions.

In the spectrum excited by the cathode rays the D lines are of immense brilliancy, running together into a single band of light. On each side of this are seen three or four symmetrically placed bands, decreasing in brilliancy as they recede from the D lines in each direction. No trace of these bands appears in the magnetic spectrum, which in this region shows only fine lines arranged in narrow groups, which do not coincide with the bright bands of the cathode luminescence.

A photograph of these bands is reproduced in Pl. XIV. fig. 2. They have some connexion with the D lines, I feel sure, for they are symmetrically arranged on each side of them. If the photograph had been made with a grating, we should of course call them ghosts. It may be that they are analogous to satellite lines; but if they are, we are certainly dealing with the phenomenon on a grand scale, for the fourth one is not far from the sodium doublet at 5688! All of these points will be more fully investigated during the coming year.

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Other Possible Excitations.

It has occurred to me during the preparation of this paper that very interesting results would be obtained by exciting the fluorescence with the light selectively rotated by the vapour in a magnetic field, *i. e.* by the magnetic bright-line spectrum. This light is fairly intense; and it would be interesting to see whether the intensity distribution among the excited fluorescence lines was the same as in the magnetic spectrum.

What I most need, however, is a set of screens which will enable me to separate lines such as those of copper without resorting to the systems of prisms and lenses. A good collection of solutions of the rare earths would probably be very useful in the work. Erbium, praseodymium, and neodymium I have, but I should feel very grateful for the loan of any others which might prove serviceable, or for any suggestions regarding other possible sources of monochromatic light. As I have said before, the instrument most needed is a light siren!

Composite Excitation.

At the top of the chart just below the magnetic series will be found a spectrum containing about two hundred lines. This is a composite drawing made by superposing all of the drawings made of the simple spectra excited by monochromatic stimulation. It contains many lines not found in the complex spectrum excited by white light. In the latter, between wave-lengths 5000 and 5100, we find but ten or a dozen lines, while in the composite spectrum there are at least twenty. This circumstance is of interest in connexion with the periodic dark regions of the complex spectrum, which give it a fluted appearance. The formation of these flutings requires further study, as their position shifts as we alter the wave-length of the exciting light, which in this case is a rather broad band isolated from the continuous spectrum with the monochromatic illuminator. The phenomenon was more fully described in the earlier paper, but requires further study.

The Series in the Magnetic Spectrum.

As we have seen, the complex fluorescent spectrum is made up of six or more series of lines, the individual lines of each series being about 38 Ångström units apart, the spacing becoming less as we pass from yellow towards violet. The

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fact that the lines in the magnetic spectrum coincide with the lines of the complex spectrum makes it seem certain that the same series will be found there. By comparing the various fluorescent spectra with the magnetic spectrum, and by measuring carefully the distances between the lines of the latter, I have made a provisional assignment of the magnetic lines thus far observed into five series.

The wave-lengths and wave-length differences are given in the following tables :—

First Series.

λ.	λ differences.
5119·34	
	39·56
5079·78	
	39,13
5040·65	
	39·08
5001·57	
	38·67
4962·85	
	38·52
4924·32	

Second Series.

λ.	λ differences.
5165·85	
	39·31
5126·54	
	39·23
5087·31	
	38·82
5048·49	
	$\frac{77·64}{2}$ = 38·83
4970·85	
	38·21
4932·64	
	38·06
4894·58	
	$\frac{75·15}{2}$ = 37·57
4819·43	
	36·54
4782·89	
	$\frac{112·89}{3}$ = 37·63
4670	

Third Series.

λ.	λ differences.
5211·71	
	38·89
5172·82	
	39·09
5133·73	
	38·95
5094·78	
	$\frac{115·34}{3}$ = 38·45
4979·34	
	$\frac{75·96}{2}$ = 37·98
4903·38	
	37·79
4865·59	
	$\frac{113·55}{3}$ = 37·85
4752·04	
	36·41
4715·63	

Fourth Series.

λ.	λ differences.
5219·60	
	39·29
5179·71	
	39·00
5140·71	
	$\frac{115·05}{3}$ = 38·35
5025·66	
	$\frac{113·56}{3}$ = 37·85
4912·10	
	$\frac{74·61}{2}$ = 37·30
4837·49	
	$\frac{109·97}{3}$ = 36·68
4727·52	
	34·98
? 4692·54	

Fifth Series.

λ.	λ differences.
5225·34	
	38·64
5186·70	
	39·20
5147·50	
	$\frac{76}{2}$ = 38·00
5071·50	
	38·00
5033·54	
	$\frac{74·92}{2}$ = 37·46
4958·62	
	$\frac{74·81}{2}$ = 37·40
4883·81	
	$\frac{73·65}{2}$ = 36·82
4810·16	
	$\frac{71·66}{2}$ = 35·83
4738·5	

Extra Lines.

5096·00
5052·83
5049·56
5003·12
4964·39
4967·10

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As will be seen by reference to the chart, the first series has the largest *average* spacing, and the fifth the smallest, the "scale," if the term is allowed, decreasing gradually from the first to the fifth, the two coming into coincidence at about wave-length 4860.

Doubtless these series could be extended to wave-length 5500 or thereabouts by making use of the grating-photograph of the complex fluorescence obtained by white-light stimulation. The lines are, however, so diffuse in their nature, with overlapping wings and other peculiarities, that I have not yet attempted any further extension. I think that by employing a much denser vapour the magnetic spectrum can be considerably extended; and as the lines are much sharper in this case, an extension of the series will be an easy matter. The other series necessary to give the close spacing found in the yellow may be discovered in this manner.

A theoretical discussion of the results will have to be deferred to a subsequent paper. In fact, I prefer to have this side of the subject attacked by those who have given especial attention to the theory of molecular radiation. The absence of many lines in each series in the magnetic spectrum, and the absence of certain lines in the fluorescent spectrum, are especially suggestive. We have similar conditions in other series of lines, as is well known; but the present case is, so far as I know, the only one in which we can, by varying the exciting conditions, bring about a change in the position of the absent lines. It appears to me that the data furnished us by sodium vapour ought in the end to enable us to choose between the various theories proposed to account for spectrum series.

The investigations recorded in the present paper have been made possible through generous aid given from the Rumford Fund by the American Academy of Arts and Sciences.

I feel also under great obligation to my assistant Mr. F. W. Cooper for the many hours of very fatiguing work which he devoted to the research.

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LVIII. *Notices respecting New Books.*

The Principles of Electric Wave Telegraphy. By J. A. FLEMING, *M.A., D.Sc., F.R.S.*, Professor of Electrical Engineering in University College, London. Longmans, Green & Co.: London, 1906. Pp. xx + 671.

A BOOK by Dr. Fleming is always looked forward to with eager anticipation by all serious students of electrotechnology, for the author has established for himself a unique reputation as a writer of clear and eminently readable text-books, and the circle of admirers who are indebted to him for much of their progress is a very wide one. The present volume is, therefore, in spite of its bulk and price, sure to meet with a warm welcome. Wireless telegraphy is one of those subjects which have taken a strong hold on the popular imagination: and in order to meet the demand for a popular account of the subject numerous books and booklets, to say nothing of the flood of articles in various periodicals, have been issued from time to time. Serious works dealing with the subject in a thorough manner have not been so numerous, and the rapid developments taking place in it have rendered it extremely difficult, if not impossible, to write a well-balanced and up-to-date account of it. From his close association with the practical aspects of wireless telegraphy, and his important contributions to it both as a theorist and an inventor, Dr. Fleming is preeminently qualified to deal with the very difficult task of coordinating and presenting in proper perspective the various branches of the subject on which he writes.

The book is divided into three parts, dealing respectively with Electric Oscillations, Electric Waves, and Electric Wave Telegraphy. The first two parts occupy over 400 pages; so that it will be seen that about two-thirds of the book are devoted to an account of the scientific foundations on which the practice of wireless telegraphy is based. These first two parts will therefore be found to be of very great interest to the student of pure science as well as to the technical man. Part I. consists of three chapters, dealing with the production of high-frequency currents and electric oscillations, high-frequency measurements (resistances, inductances, dielectric strengths, capacities, dielectric constants, currents, frequencies), damping and resonance. The three chapters in Part II. deal with stationary electric waves on wires, electromagnetic waves, their detection and measurement. This part of the book will appeal strongly to the student of electromagnetic theory, as it contains an admirable account of numerous difficult problems treated in an exceptionally clear and interesting manner. Part III., which also consists of three chapters, contains a historical account of the early development of wireless telegraph systems, of the recent work of the numerous inventors who have busied themselves with this

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subject, and of the principles of electric-wave telegraphy. This concluding part will more particularly appeal to those interested in the purely technical aspects of the subject. There are three appendices, the first of which contains the Wireless Telegraphy Act of 1904, the second bibliographical references, and the third a list of British patents relating to wireless telegraphy.

Throughout the entire work, there are numerous references to original sources of information, and the accounts of experimental researches having a direct or indirect bearing on the subject under treatment are so complete that the work may well be regarded as an encyclopædic one. Its preparation must have involved an immense amount of labour, and it will undoubtedly rank as the standard treatise on the subject for some time to come. From another point of view, perhaps, the very wealth of the information contained in the book is liable to prove somewhat perplexing to the reader who approaches the subject with only a slight knowledge of its scientific principles. There are a number of minor blemishes, but most of those which have come under our notice are either obvious printer's errors or the result of hasty writing. The book is beautifully printed and well and lavishly illustrated.

Anleitung zu Wissenschaftlichen Beobachtungen auf Reisen. Herausgegeben von Dr. G. von NEUMAYER, Wirklicher Geheimer Rat. Dritte völlig umgearbeitete und vermehrte Auflage in zwei Bänden, mit zahlreichen Holzschnitten, photographischen Abdrücken und zwei lithographierten Tafeln. Hannover: Dr. Max Jänecke. 1906. Vol. I., pp. xxiv + 844: Vol. II. pp. xxvi + 880.

A BRIEF notice of the Introductory portions of this important standard work has already appeared in these columns, and now the work has reached completion. It is by more than 30 authors, each of whom is an acknowledged specialist in the particular subject on which he writes. The names of the various authors are in themselves a sufficient guarantee that the information contained in the book is thoroughly trustworthy and up-to-date, and there is no other publication which for completeness can compare with this "explorer's encyclopædia."

Vol. I. deals with the determination of latitude and longitude, route surveying, geology, earthquakes, the earth's magnetic field, meteorology, oceanography, and astronomical observations, and contains general hints regarding the fitting out and conduct of a scientific expedition. Vol. II. deals with anthropological and ethnographical investigations, the study of diseases, agriculture, the collection of cultivated plants, plant geography, the geographical distribution of animals, botanical and zoological collections.

The directions contained in the book are clear and explicit, and the lists of questions given in many sections should go a long way

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towards enabling the leader of the expedition to collect information of real importance. All that the resources of modern science and technical skill can do for the explorer in enhancing the value of his observations and researches appears to have received due notice in this remarkable work, and no truly scientific explorer can afford to ignore its existence.

The general editor is to be congratulated on the successful manner in which he has accomplished an arduous task, and a word of praise is due to the publishers for the excellent way in which their part of the work has been done.

Die Physik Roger Bacos. Inaugural-Dissertation von SEBASTIAN VOGL. Erlangen: Junge & Sohn. 1905. Pp. xii + 106.

To those interested in the history of physics, the present pamphlet, which deals with Roger Bacon and the science of his time, will be very welcome. The result of the author's researches appears to be that Bacon's knowledge of physical science was derived from Greek and Arabic sources, and that although no original contributions can be traced to Bacon himself, yet the truly philosophical and critical spirit in which he examines the science of his time entitle him to be regarded as a great man. When it is remembered that during the greater part of his life Bacon was practically deprived of means of experimental research, and that he was thwarted in his aspirations at every turn by his superiors, it is hardly to be wondered at that he failed to enrich the science of his time by any striking original discoveries.

Cours de Physique de l'Ecole Polytechnique. Par M. J. JAMIN. Troisième Supplément, par M. BOUTY, Professeur à la Faculté des Sciences de Paris. Paris: Gauthier-Villars. 1906. Pp. vi + 420.

THE rapid developments which have recently been taking place in many branches of physics have rendered it desirable to bring many of the old-established standard treatises on the subject up to date. In the case of M. Jamin's well-known work, this is being done by the issue of special supplementary volumes in which an account is given of recent advances in physics. The book before us forms the third supplement, and deals with Radiation and Electricity. Under the head of Radiation, the author deals with the emission of black bodies, the pressure of radiation, the emission of gases, the infra-red spectrum, dispersion, Hertzian waves and wireless telegraphy. The next section of the work is headed "Electricity," and is devoted to the electromagnetic effect of electric convection, the experimental study of magnetism, alternating currents, electrolysis, the ionic theory, and Nernst's theory of electrolytic solution pressure. The concluding section, on "Ionization," deals with

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the condensation of aqueous vapour on electrified nuclei, the general properties of conducting or ionized gases, ionic movements, different cases of ionization, radio-activity, the dielectric constant and dielectric strength of gases, the spark-discharge, the theory of discharges through rarefied gases, various electrical measuring instruments, and recent forms of high-efficiency incandescent lamps. As will be seen, the range of subjects dealt with is very wide, and it is hardly to be wondered at that in some instances the treatment is rather sketchy. We can warmly recommend the book to all students desirous of having a clear and interesting account of some of the most remarkable advances which have taken place in physical science within recent years.

A First Course in Physics. By ROBERT ANDREWS MILLIKAN, *Ph.D.*, Assistant Professor of Physics in the University of Chicago, *and* HENRY GORDON GALE, *Ph.D.*, Instructor in Physics in the University of Chicago. Ginn & Co. Pp. viii + 488.

THERE is no lack of effort in these days to make the study of physics as interesting as possible to the beginner, and the latest book having this aim in view we owe to two American writers. The manner in which the subject is presented cannot fail to prove attractive to the youthful mind, and one is tempted to envy the good fortune of the present young generation in having its tastes ministered unto in so pleasant a fashion. Any very striking display of originality in an elementary text-book on physics is, of course, hardly possible at the present day; but the authors display commendable good sense in the arrangement of the subject-matter, and their method of handling it clearly reveals the skilful touch of the trained teacher. An important feature of the work is the method of illustrating scientific principles by examples drawn from the region of applied science—an arrangement well calculated to impress the pupil with the importance of science in modern life. The book is embellished with sixteen portraits of the great makers of physics.

La Double Réfraction Accidentelle dans les Liquides. Par G. DE METZ. Paris: Gauthier-Villars. 1906. Pp. 100.

IN this monograph, which forms No. 26 of the "Scientia" series, the author gives a very complete account of the present state of our knowledge regarding the phenomenon of double refraction in liquids, and describes the various experimental arrangements for the purpose of studying it. The effects of dynamical stress, eddying motion, electric and magnetic stress are considered in succession, while the remainder of the work is devoted to theoretical considerations, many of which are highly original and uggestive.

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LIX. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 176.]

June 27th, 1906.—Sir Archibald Geikie, D.C.L., Sc.D., Sec.R.S., President, in the Chair.

THE following communications were read:—

1. 'Interference-Phenomena in the Alps.' By Mrs. Maria M. Ogilvie Gordon, D.Sc., Ph.D., F.L.S.

In former papers * the authoress pointed out that, while the leading axis of the Alpine mountain-system extended in an east-and-west direction, and it was generally accepted by Alpine geologists that Alpine areas had endured horizontal compressive stresses in a north-and-south direction, her own studies of Alpine geology had led her to the conclusion that there had also been cross-deformation throughout the whole region, including both plicational and overthrust-effects; and that this cross-deformation had been induced in relation to horizontal pressures which had acted from the Hungarian Basin westward over Alpine areas. In a quite similar way, radially-directed horizontal pressures had acted round the northern, eastern, and southern periphery of the Hungarian Basin, and had originated the deformational systems of the Carpathian Mountains.

On the Alpine side of the Hungarian Basin, owing to the resisting character of the previously-plicated rocks composing the Palæozoic Alpine chain, the ancient chain had been broken up by the east-and-west compression into a series of cross-segments or fault-blocks, and there had been a general westward crush of the series. The leading cross-segments named by the authoress were—(1) the Western Alps, (2) the Engadine, and (3) the Styrian Alps.[1(2)] Overthrust-effects had been produced not only at the western margins of the segments, but also in some cases at the eastern margins, *e.g.* notably in the Western Alps, and at the Judicarian Fault in the Eastern Alps. Further, these same cross-segments in the Central Alps had been from their first initiation, interrupted on the north and south by ancient leading east-and-west faults, in relation to which sagging or downthrow-movements took place towards the Central Alpine band, and overthrust-movements occurred in opposite directions.

The present paper, so far as it deals with the general structure of

* M. M. Ogilvie Gordon, (1) 'Torsion-Structure in the Alps' Nature, Sept. 7th, 1899, vol. lx, pp. 443–46; (2) 'The Crust-Basins of Southern Europe' Seventh International Geographical Congress, Berlin, 1899 (Proceedings of the Congress, Part II, pp. 167–80 & pl. vii); and (3) 'The Origin of Land-Forms through Crust-Torsion' Geogr. Journ. vol. xvi (1900) pp. 457–69.

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the Alps, was completed in April 1905; but, acting on the advice of Sir Archibald Geikie, the authoress has since endeavoured to strengthen her line of argument by taking as a type the series of structural changes undergone in the largely-igneous mountain-massive of Bufaure in the Dolomites, and elucidating the successive phases from the point of view of historical geology.

After describing in detail the geology of the Bufaure Massive, the authoress discusses the structural relation of the Western Alps and the Engadine to one another and to the whole mountain-system. From the particular arrangement of overthrusts, as well as from the distribution of the igneous intrusions in the Western Alps and in the Engadine, the authoress concludes that these were areas where leading cross-faults intersected the east-and-west Central Alpine band, and shows how the coalescence of these cross-faults with E.N.E.–W.S.W. faults on the north side and W.N.W–E.S.E. faults on the south side defined two leading fault-curves, the one passing through the Engadine and continuing in the Dalmatian Alps, the other passing through the Western Alps and continuing in the Apennines. These strike-curves are essentially peripheral to the western side of the Hungarian Basin.

The cross-segment comprising the Rhine-Ticino district between the Western Alps and the Engadine is, according to the authoress's interpretation, anticlinal in character, segments having been down-thrown from it both towards the west and towards the east, and overthrust masses having crept eastward and south-eastward from the Western Alps, and westward from the Engadine.

The authoress then discusses the relation of the French Jura Mountains to the Alpine System, pointing out that the Swiss-French Plain flanking the Western Alps presents the same essential features of structure in relation to the Western Alps on its east side and the French Jura Mountains on its west, as those that she has elucidated for the Rhine-Ticino cross-segment. She consequently interprets the strike-curve round the west formed by the Jura Mountains and the ranges of Dauphiné as the outermost peripheral plicational system in the Alps, showing that the whole region between the Hungarian Basin and the ancient mountain-groups of Central France has been under the influence of the westward thrust.

The general principle of structure treated of above—namely, the sagging of crust-blocks by means of normal faults towards bands or localities of crust-weakness or subsidence, and the reverse or overthrust-movements which may take place from within these bands or localities—is that which the authoress demonstrated in the Dolomite-Massives in 1893, and has ever since advocated as a leading principle in the interpretation of Alpine structures, the important consideration being that where, as in the Alps, the principle is applicable in relation to two intersecting axes of deformation, the phenomena produced must necessarily be of the nature of 'interference-phenomena.'

A leading feature of the paper is the evidence that it affords of

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differential rates of movement in different parts of a thrust-mass, or fault-block undergoing horizontal displacement, both in respect of the laterally-adjacent parts of a thrust-mass and also of the subjacent layers. The writer's maps and sections show that the actual deformations which characterize a thrust-mass have a different direction of strike on either side of an axial band of maximum horizontal displacement; for example, if the horizontal movement of a thrust-mass is westward, the deformational phenomena (faults, folds, etc.) in the western front of the thrust-mass curve on the north side towards some S.W.–N.E. or W.S.W.–E.N.E. direction, and on the south side towards some N.W.–S.E. or W.N.W.–E.S.E. direction. The authoress interprets these observations on the basis of the deflection of the general movement of the thrust-mass by the strains set up between the axial region of maximum horizontal displacement and the lateral regions where from any cause the horizontal displacement is less.

Another feature which may be mentioned is her description of several examples in the Dolomites where there had apparently been a local reversal of the regional westward movement, and her reference to the familiar examples of eastward overthrusts in the Judicarian district, and on the eastern and south-eastern front of the grander mountain-massives of the Swiss and French-Italian Alps. While each individual case demands special examination, she indicates an explanation that satisfies certain cases which she has examined. At localities where the base of the thrust-mass is open to inflows of igneous rock, the igneous material may ascend and be carried onward with the gliding mass, undergoing consolidation during the movement, and inducing contact-changes in neighbouring rock-material. After consolidation of such igneous inflows, they present resisting bodies within the thrust-mass, which, in the same way as any massive developments of hard sedimentary material, impede the advance of rock-material in the same direction as before, and thus cause local deflections. The tendency is for the material of the thrust-mass to be strongly plicated and faulted as it is driven against any such resisting body, widening out in a direction roughly parallel with the resisting mass, and piling up the material in front to such an extent that local reversal of the direction of overlapping is produced.

2. 'The Influence of Pressure and Porosity on the Motion of Sub-Surface Water.' By William Ralph Baldwin-Wiseman, M.Sc., Assoc.M.Inst.C.E., F.G.S.

The author commences the paper with a brief historical summary of the researches which have been conducted since 1830 on the motion and behaviour of underground water, more especially dealing with the question of sub-surface flow and the delimitation of cones of depletion.

In the second part of the paper, in discussing the influence of the porosity of a rock on the rate of flow of water through it, he describes in detail the variations in porosity which may occur in

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restricted areas of the same rock, due to superincumbent pressure, faulting, and the intrusion of dykes, illustrating the various points with data collected in the field. He also discusses and describes experiments on the rate of desiccation and soakage of various rocks.

He then describes a lengthy series of laboratory-experiments, which he conducted with specially-devised apparatus of his own design to afford a constant pressure and to eliminate all possible errors due to lateral flow; and in which he demonstrates that there is not a uniform relation between flow and pressure in various rocks over a considerable range of pressure, and discusses the various phenomena which were manifested.

In the third portion of the paper he describes the various attempts at determining the range of the cone of depletion in various strata, and then proceeds to outline a method based upon an experimental determination of the variation of internal pressure in a rock-mass when charged with water and subjected to a considerable difference of pressure on the two faces; and in further elaboration of the theory, he outlines a method of estimating the percentage-interference of two contiguous wells in the same strata.

In the concluding portion of the paper he discusses data collected during various hydrological surveys, and points out the influence of surface-configuration and stratigraphical sequence on the sub-surface water-contours.

LX. *Intelligence and Miscellaneous Articles.*

ATOMIC CONDUCTIVITIES OF THE IONS.
BY PHILIP BLACKMAN.

A CORRECTION:—Phil. Mag. 1906, xii. p. 151, lines 7 and 8, instead of

"By adding ..... $\mu_{II}=340$,"

read:—

"By performing all the possible subtractions and adding together the equations thus produced, then, on solving the resulting equation $4{\cdot}749\mu_{II}=1618$, it is found that $\mu_{II}=340$."

The tables for K, K_1, K_2, &c., and Rv_{IIX}, Rv_{IIX_1}, Rv_{IIX_2}, &c., used in the construction of the table of "atomic conductivities," will be found in full in the 'Chemical News,' 1906, vol. xciv. p. 164, "Molecular Conductivities: Quantitative Relation."

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THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

DECEMBER 1906.

LXI. *The Strength and Behaviour of Ductile Materials under Combined Stress.* By WALTER A. SCOBLE, *A.R.C.Sc., B.Sc., Whitworth Scholar* *.

1. *Previous Tests and their Differences from those given.*

THIS branch of the testing of materials, although of considerable theoretical and practical importance, has been seriously neglected until quite a recent date. Experiments on wires under combined tension and torsion are mentioned in Lord Kelvin's article on Elasticity in the *Encyclopædia Britannica*, but numerical results are not given. Excellent work was done by Mr. J. Guest † using thin tubes under tension, torsion, and internal pressure. A preliminary report has also recently appeared of tests on the effect of combined stresses on the elastic properties of steel, by Mr. E. L. Hancock ‡. In addition to the above there are the experiments by Tresca upon the ultimate strength under combined stress.

The ultimate practical value of experiments of the kind under consideration will greatly depend on the limiting condition of the material which is selected as the basis for comparison. Reasons are given below for selecting the yield-point. This point was taken by Mr. Guest, the choice being justified in his paper. This being so, it may appear that the present results will only be a confirmation of previous tests,

* Communicated by the Physical Society: read October 26, 1906.

† Proc. Phys. Soc. vol. xvii. p. 202, and Phil. Mag. [5] vol. 1. p. 69.

‡ Phil. Mag. Feb. 1906.

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but the following points should be noted :—Mr. Guest used thin tubes, and although this course appears to be justified, experiments on solid bars are desirable, the distribution of stress being different. The tensions were applied either directly or by internal pressure. Perhaps the most important practical instance of combined stresses is that due to torsion and bending. Due to bending the stress varies gradually from a maximum tension to a maximum compression, the shear stress due to torsion being zero at the centre and a maximum at the surface. This is a further difference in the distribution of the stress. Several tests with different loadings were made on each of the tubes mentioned, and the results show that in certain cases at least the properties of the material changed appreciably, making accurate comparison impossible, or at least difficult. It was impossible to allow the specimen to yield very much, so that the value found for the critical stress was open to error. The writer preferred to use separate specimens, with possible slight differences in properties, and to allow the yield to be quite definite and of considerable magnitude.

2. *Separation of Metals into Ductile and Brittle.*

Before discussing the theories of elastic strength, it will be well to separate the materials into two classes—ductile and brittle. It is unnecessary to discuss these at length as the difference in properties is sharply defined. Ductile materials yield considerably before fracture, drawing out at and near a particular section when under tension, whereas those of the brittle class yield very little, or are very "short." The planes of fracture vary with the two classes when tested under the same conditions, and it is significant that with ductile specimens the planes of fracture approximate to those of greatest shear. This indicates that after yield a ductile material behaves like a viscous fluid, the large yield supporting this view.

3. *The Behaviour of Brittle Materials should not be judged by Tests on Ductile Specimens.*

Perhaps the most common examples of combined stresses in practice occur in vessels under internal pressure and shafts subjected to bending and twisting. The material used for the first may be ductile or brittle, but with ductile the thickness will be comparatively small and the maximum stress or maximum shear theories will lead to little difference when calculating the dimensions. From the distinction drawn

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above, it will be noticed that it is not justifiable to apply the results of tests on ductile materials when dealing with brittle specimens. The critical case in which internal pressure is applied is that of a thick cylinder of cast iron, and it would not be safe to treat this in the same way as a comparatively thin boiler-shell. In the latter case the two theories held in England give very similar results, but in the former the material has properties different from those of the specimens tested. This distinction should be remembered when reading all that follows.

4. *The Theories of Elastic Strength.*

The real need is a determination of the true complete theory of elastic strength. The theory of elasticity and all engineering applications are based on Hooke's law, that strain is proportional to stress. When this law fails the assumptions are no longer true, consequently the formulæ deduced are incorrect and it is impossible to reason further with certainty. In certain cases there will be a redistribution of stress, usually varying from that satisfying Hooke's law to a state of uniform stress, reached after considerable yield, through all the intermediate conditions. By the theory of elastic strength is meant the conditions which determine when the law fails and the uncertain stage is reached, the body considered being also seriously deformed. Three theories have claimed much support. That adopted by English engineers assumes that the material yields when the maximum principal stress reaches a certain value. The view generally held by Continental elasticians is that a definite maximum strain determines the failure of the specimen. A third theory states that yielding occurs when the maximum shearing stress becomes a specific amount. Mr. Guest has shown that neither the maximum stress nor maximum strain is even approximately constant. The maximum shear is constant within close limits; and it has been suggested that the variation is probably due to something analogous to friction, related to the force perpendicular to the plane of maximum shear. This point will be considered later when the results are discussed.

5. *The Yield-Point selected as the Criterion of Strength.*

Remembering that our accepted formulæ are based on Hooke's law, which holds to the elastic limit, it would at first appear that this point should be selected as a basis for comparison. The yield-point has here been used throughout. If the

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material is satisfactory, the stresses at the elastic limit and yield-point are nearly proportional and it makes little difference which is taken. Faulty specimens will usually have a low elastic limit whereas the yield-point is little affected, and the same applies to changes in the metal due to any special treatment to which it may have been subjected. Taking these facts together, it is evident that the yield-point is much more nearly constant than the elastic limit, and in making a simple test it is correct to consider both points in relation to each other. The yield-point, unfortunately, is not quite definite, but that taken for the purpose of these tests will be precisely stated. Having decided exactly how the yield-point is to be found, it is more easily obtained accurately than the elastic limit. The error in proportionality between the stress and strain is small to the yield-point. The view is now generally held that the lack of proportionality immediately after the elastic limit is due to local yielding, parts of the material being either weak or under internal stresses. The considerable lowering of the elastic limit with poor specimens supports this view very strongly; and if it is accepted, the yield-point is more important than the elastic limit, being less dependent on unique conditions. It will be seen later that when a specimen is tested to yield under combined stresses, there is yield in both ways. We therefore have definite loads of both kinds. Mr. Hancock has shown that when the elastic limit is reached under one loading, a large stress is needed to reach it with the other. A bar tested to the elastic limit in tension needs a considerable torque before this point is reached in torsion. For comparative work a difficulty arises. The first load reaches the elastic limit and the second can do no more except reach it with the other yield, therefore the first load can be taken with any fraction of the second to give the combined loads. No doubt the first kind of stress affects the elastic limit under the other kind, but the conditions are not so definite as when the yield-point is taken. Unfortunately more than one position can be ascribed to the yield-point, the point at which the curvature of the stress-strain curve becomes large, or the stress causing considerable yield. As the distribution of stress due to bending varies from a maximum tension to a maximum compression, and due to the torque from zero to a maximum shear, elements will yield in succession, this being especially the case under combined stresses with irregular distribution, the maximum shear stress only occurring at certain places. On this account Mr. Guest used thin tubes to obtain nearly uniform stresses. Since the elastic strain is so small a little yield at certain

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points causes a complete redistribution of stress across the section, and the yield-point stress extends to other areas. For this reason the stress at the first sign of yield should be taken. When a simple tension test is considered, there is still the rounded portion, due to local yielding and eccentricity of loading, although the stress is finally uniformly distributed over the section. When the stress-strain curve becomes approximately horizontal there is general yielding, and the corresponding stress should be taken as the yield-point stress. In the present case there are both causes for the rounded portion of the curve, local yielding and redistribution of stress: one operates to cause the first sign of yield, and the other complete yield, to be taken as the limiting condition. As the object of the tests is comparative, it matters very little which is taken, the results being in a sense proportional within the limits of experimental error and those arising from taking curve-readings. It is quite impossible to accurately locate the point at which yield first occurs, therefore the selection has been made as described below. This seems quite satisfactory when considering a suitable formula for bending and twisting. The same moments have been tabulated and the corresponding stresses calculated, making the usual assumptions. These have been found to bear a definite relation to the stresses corresponding to the first sign of yield, so nearly as could be read from the curves, when the usual assumptions hold good, and stresses can easily be changed to those calculated on the assumption of a uniform distribution of stress.

6. *Determination of the Yield-Point.*

The yield-point was determined as indicated in fig. 4, test 11. As the intermediate state between perfect elasticity and great flow, or the approximately vertical and horizontal portions on a stress-strain curve, is due to successive yield at various planes of weakness, most elasticians neglect this part of the curve altogether, and there is considerable justification for this course. This has practically been done here, the lines representing the two states being produced to meet in a point. Supposing this course was not justified, at least this is a definite, easily determined point to deal with, and any probable error would not be greater than that which is likely to arise when taking a point less closely defined.

7. *The Nature of the Loading.*

It has already been stated that the most important case of combined stresses with ductile materials is that of bending

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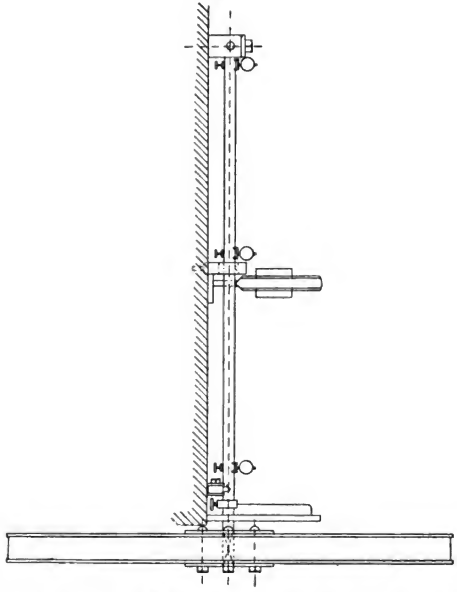
and twisting. Fortunately, tests of this kind could be made with the limited apparatus at the disposal of the writer, and they have the advantage that the distribution of tensile and compressive stress is similar to that in the practical case, whereas it is usual in tests under combined stresses to apply the tension directly.

8. *The Apparatus and Specimens.*

A general view of the apparatus is given in fig. 1. The

Fig. 1.



bars tested were 36 inches long and turned to $\frac{3}{4}$-inch diameter, with $\frac{3}{4}$-inch squares at each end. They were of steel, all taken from the same batch, and two were prepared from each length supplied. One end was held so that it could not turn but was not constrained against bending. The grip is given in fig. 2. The squared part of the rod fits into a suitable slot and the cover-plate grips it. There are

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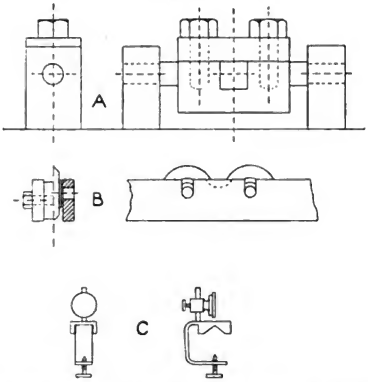
two pins branching from the central portion which fit into bearings, these steel pins being carefully ground in place. Drawings of the support at the other end are also given in fig. 2. The bar rests on two rollers of large diameter which are in turn supported on their side pins by the metal strips.

Fig. 2.



This arrangement reduces the resistance to twisting at this end to a minimum. The torque was applied through a light wooden pulley having metal plates bolted to it, these having $\frac{3}{4}$-inch square holes cut in them in line. Suitable wire ropes were attached to this pulley, and each passing over another pulley, weights could then be attached and considerable motion was possible. The bending load was applied directly by placing weights on a platform attached to a knife-edge resting on the bar. Deflexions were measured by means of a scaled strip which rested on the specimen and fitted easily in a slide with a vernier cut on the side. This arrangement was quite satisfactory, although it does not appear very sensitive, but it should be remembered that the deflexion is large even before the yield-point is reached, and it was possible to read accurately to 0·005 of an inch. The twist was measured at four points. At three of these, mirrors (see fig. 2) were attached to the bar. Telescopes were fixed at approximately a metre from the mirrors, and there were vertical scales; the arrangement being similar to that in common use in connexion with galvanometers. Outside the support, on the pulley side, a pointer was fixed to the bar and moved over a fixed scale as the rod twisted. This served to give fairly accurate

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readings of the twist directly. The length of the beam, or the distance from the centre line of the pins to the rollers, was 30 inches. The load was applied at the centre and the deflexion measured at 14 inches from the roller support. The end mirror was 1½ inches from the roller support. The pin joints at the one bearing, and the combined roller and pulley arrangement, were tested to determine the amount of constraint due to friction. This was in each case small, but has been allowed for in working out the results.

9. *The Tests Grouped.*

The tests may conveniently be divided into three classes. The first includes the tests with one kind of loading only, these giving the loads necessary to cause yield either by bending or twisting. For the second class a definite fraction of the critical bending load was put on and the specimen then twisted to yield. In the third set a given part of the yield twisting moment was applied and then sufficient bending load put on to cause the bar to give. Each kind of load was applied directly by weights, so that it increased by stages and not gradually. When the elastic limit was reached the load added was considerably smaller than those put on at first, it being of the order of one pound. The pulley was lightly tapped after the addition of each twisting load to minimize the effect of friction or sticking. If a result appeared doubtful, or if the increments of load had been too large at the yield-point, preventing its accurate measurement, the test was repeated on a new bar.

10. *Table of Results.*

The results of the tests are tabulated below :—

Group of Test.	Number of Test.	Bending Moment.	Twisting Moment.	Tensile Stress due to Bending.	Shear Stress due to Twisting.	Max. Principal Stress.	Min. Principal Stress.	Max. Shear Stress.
1.	I. ...	2660	0	64600	0	64600	0	32300
	III. ...	0	2400	0	29170	29170	−29170	29170
2.	IV. ...	667·5	2280	16220	28250	37500	−21300	29400
	V. ...	1331	2120	32350	25750	48200	−15800	32000
	VI. ...	2000	1899	48600	23050	57800	− 9200	33500
	VII. ...	2420	1171	58750	14240	61980	− 3220	32600
	XII. ...	2000	1720	48600	20900	56740	− 8140	32440
3.	VIII. ...	2558	645	62100	7840	63080	− 980	32030
	IX. ...	2310	1335	56100	16220	60450	− 4350	32400
	XI. ...	1454	2033	35330	24700	48060	−12740	30400

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The principal stresses are $T/2 \pm \sqrt{S^2+T^2/4}$, and the shear stress, equal to half the difference between the greatest and least principal stresses, is $\sqrt{S^2+T^2/4}$.

11. *Facts not included in the Table.*

The following interesting facts may be noticed in connexion with the tests. Considering Test IV., a quarter of the bending load was put on and torque increased to yield. There was simultaneous yielding by bending and twisting (fig. 3), but a large addition to the bending load was needed to cause further appreciable yield. When half, or more, of the bending load had been put on originally, the bar gave in both ways simultaneously, and a small addition to the bending load caused further yield (fig. 3). When the rod could stand a further load without much deflexion, the bending yield was almost sudden at the critical load, there being none preceding it that could be measured (Test IV.). With larger fractions of the bending load, the yield could be detected before the critical load was reached and deflexion was plotted against twisting moment, in addition to the twist; in fact the specimens gave first by bending in Tests V., VI., and VII. The yield by twisting, due to the bending load, was small in Test VIII., fig. 4, but the sensitive method of measuring readily detected it. The bar could stand an increase of torque with little more than normal twist. In Tests IX. and XI. the yield was large, and a small addition to the twisting load produced a large twist, the increase of load being unnecessary as the bar was badly deformed in this way without it. The critical loads causing yield in the two ways are given below:—

Test.	Original Load.	Yielded by Bending at	Yielded by Twisting at	Nature of Load being added and stated.
IV. ...	$\frac{1}{4}$ bending load.	77 lbs.	77 lbs.	Torque.
V. ...	$\frac{1}{2}$,, ,,	71·5 ,,	72·25 ,,	,,
VI. ...	$\frac{3}{4}$,, ,,	64 ,,	65·25 ,,	,,
VII. ...	$\frac{3}{5}$,, ,,	39·5 ,,	41·5 ,,	,,
XII. ...	$\frac{4}{5}$,, ,,	58 ,,	58·5 ,,	,,
VIII. ...	$\frac{1}{4}$ torque.	356 ,,	341 ,,	Bending.
IX. ...	$\frac{1}{2}$,,	308 ,,	310 ,,	,,
X. ...	$\frac{3}{4}$,,	223 ,,	194 ,,	,,

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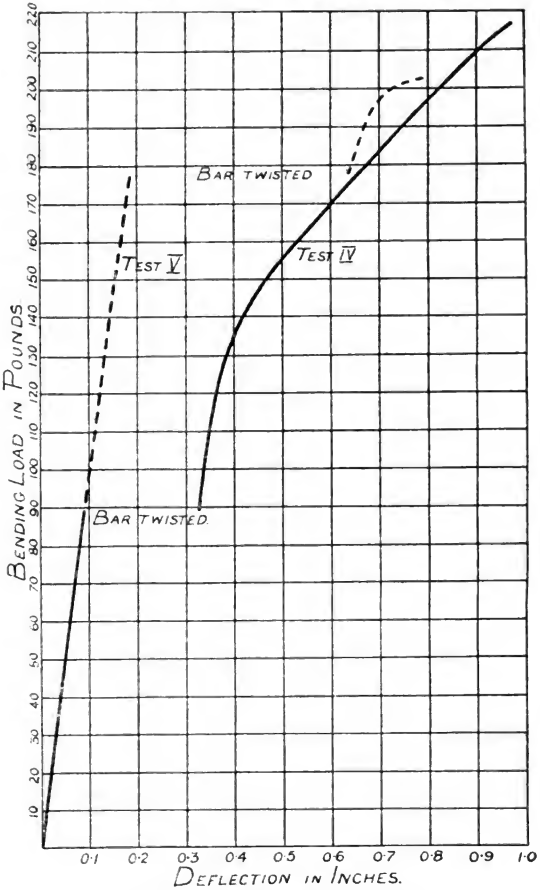
Fig 3.—Tests IV. & V. Bending Load and Deflexion.



Test V. ½ bending load on originally.
Test IV. ¾ bending load on originally.

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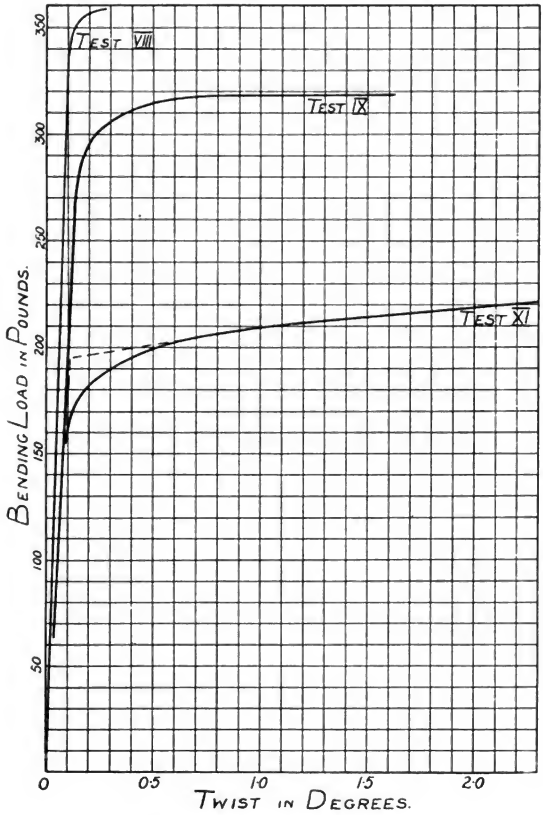
Fig. 4.—Tests VIII., IX., & XI. Bending Load and Twist.



Test VIII. $\frac{1}{4}$ Twisting load on originally.
Test IX. $\frac{3}{16}$ Twisting load on originally.
Test XI. $\frac{3}{4}$ Twisting load on originally.

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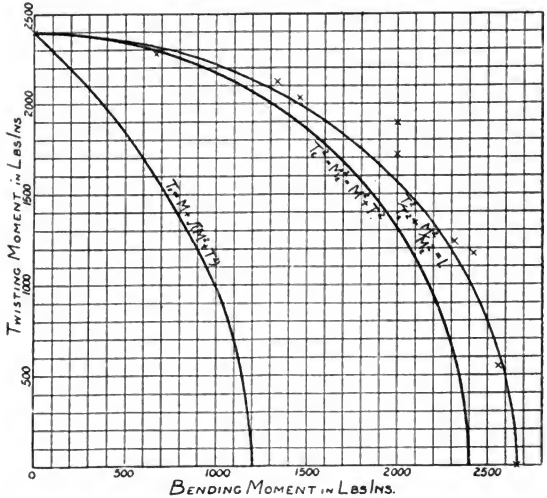
12. *A Formula for Combined Bending and Twisting.*

The corresponding bending and twisting moments required to cause yield are plotted in fig. 5. The curves are an ellipse

Fig. 5.—Twisting and Bending Moments at Yield.



through the points representing the limiting cases with load of one kind only, a circle with the smaller moment as radius, and a curve representing the common formula $T_e = M + \sqrt{M^2 + T^2}$, T_e being the equivalent twisting moment, M the actual bending moment, T the actual twisting moment. The last is ridiculously wrong, and it is difficult to understand why it has found so much favour. Putting $T=0$, $T_e = 2M_e$. This is only true if the tensile and torsional shear strengths are equal, whereas all experimental results indicate that this is far from correct. The ellipse fits the results best, but the circle is recommended on account of an explanation of the difference between the limiting bending and twisting moments which is given later. Thus the formula for combined bending and twisting becomes $T_e = M_e = \sqrt{M^2 + T^2}$.

T_e = the equivalent twisting moment.
M_e = ,, ,, bending ,,
T = the actual torque to which the shaft is subjected.
M = ,, bending moment to which the shaft is subjected.

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When determining the size of a shaft under both loadings it should be calculated from the working tensile strength, by using the equivalent bending moment if this is less than twice the working torsional shear strength, or from the shear strength if this is less than one half the tensile strength. In other words, the tensile strength should be taken as twice the shear strength, and the smaller value taken when obtaining these from the values given by tests, or used as working stresses. A simple case will make this quite clear. Unwin gives 13200 lbs/in.^{2} and 5400 lbs/in.^{2} as the tensile and torsional strengths of mild steel under the same conditions of loading. These should be taken as 10800 and 5400 lbs/in.^{2} when working with the formula given, thus taking the smaller values. If the twisting moment was small, there would be no danger in using 13200 and 6600 lbs/in.^{2}, thus approximating to the ellipse given in fig. 5. The formula based on the ellipse is slightly more complicated and may be given as

$$\frac{T^2}{4f^2_{\text{torsion}}} + \frac{M^2}{f^2_{\text{tension}}} = \frac{T_e^2}{4f^2_{\text{torsion}}}$$

or

$$T^2 + \left(\frac{2f_{\text{torsion}}}{f_{\text{tension}}}\right)^2 M^2 = T_e^2;$$

the letters having the same meanings as before.

13. *An Explanation of the Variation of the Maximum Shear Stress.*

The complete theory of elastic strength has yet to be determined. The table, section 10, gives the principal stresses, the maximum in one case being almost twice that in another. The maximum shear stress is nearly constant and may be taken as the determinant to a first approximation. After allowing for experimental errors, it is difficult to explain the differences in the values. Mr. Guest suggested a force corresponding to friction due to the force normal to the plane of greatest shear, the same having been mentioned by other writers. It is now necessary to examine the results more closely to determine whether they justify this explanation. When the principal stresses are $T/2 \pm \sqrt{S^2 + T^2/4}$, the distribution is that indicated in fig. 6. Assuming a force similar to friction this would assist the shearing force, so that the yield in tension would take place with a lower stress. With a force the reverse of friction, opposing the shear stress if there is tension across the plane, the tensile strength should be more than twice the shear strength. In the case of bending, there

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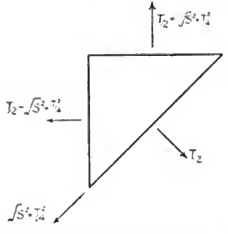
being both tension and compression, either kind of additional force would reduce the strength, causing the bending moment to be less than the torque. This is contrary to the results of the experiments, so that the idea must be abandoned.

Fig. 6.



Unwin gives the working tensile stresses as more than twice the torsional shear stresses for nearly all materials, in certain cases the proportion being exact, thus opposing the idea of friction. Referring to Mr. Guest's results, friction would cause the maximum shear stress at yield to diminish with increase of tension, and anti-friction would have the reverse effect. In the case of tube 1, the tension was greater than twice the shear stress. With tubes 4, 6, 7, 8 and 9, the tension was less than twice the shear stress. All these tubes were steel. With the copper tubes the shear strength was more than half the tensile stress. The brass tubes gave no definite evidence either way. The bulk of these results are contrary to the working stress proportion, and there is an exception amongst them. To trace the matter more completely, the results were analysed to trace the connexion between the maximum shear and the tensile stress across the plane. It was found that there was absolutely no connexion between them. As an example, taking the twenty tests on tube 8, the maximum shear stress and the tensile stress across the plane have been tabulated :—

Max. Shear.	Tension.	Max. Shear.	Tension.	Max. Shear.	Tension.
23735	22320	21760	20290	20975	19220
23175	22325	21600	19600	20800	19000
22810	21590	21460	20040	20600	20600
22500	0	21370	19630	20535	18765
22260	20340	21250	0	20480	18720
22100	0	21125	17000	20100	20100
22000	13400	...	...	20050	18450

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There is no sign of a regular increase or decrease of the maximum shear corresponding to an increase of the tensile stress across the plane of shear. The present tests have a rough indication of increase of maximum shear stress with increase of bending moment, and therefore of both tension and compression, but it is by no means conclusive. It is opposed to the idea of either positive or negative friction. The true reason of the variation in the maximum shear stress, after allowing for errors in measurement, appears to be that the shearing resistance varies in different directions. Bauschinger tested the shear strength of various specimens in six different directions. For annealed puddled plate it varied from 8·89 to 19·68 tons/in.^{2}, for rolled iron bar from 10·15 to 22·55 tons/in.^{2} With steel the variation was less, for Bessemer plate from 21·45 to 27·35 tons/in.^{2}, and for another specimen of the same from 25·05 to 29·2 tons/in.^{2} Remembering these results, assuming the maximum shear stress to be the criterion determining yield, and also that with tension and torsion the planes of shear are different, it cannot be expected that the tensile strength of steel will be exactly twice the shear strength. The fact that sometimes it appears to be more, and at others less, points to this being the true explanation of the variation of the maximum shear stress at the yield-point under combined loading. The relation noticed above between the stresses in the present tests is just the kind of connexion that would be expected if the assumption is correct, remembering that different specimens were used, no doubt differing slightly in properties and strength.

14. *Final Conclusions.*

It must be concluded that the maximum shear stress determines when yield takes place, but this will vary slightly on account of the difference in the shearing resistance in various directions, and any idea of a force analogous to friction must be abandoned. For the common case of combined bending and twisting either the circular or elliptic formula may be safely used, or to approximate to the elliptic formula $\sqrt{M^2+T^2}$ may be equated to the equivalent moment, taken as being of the same kind as the larger which is actually operating, and the corresponding true working stress used. It is not admissible to extend the conclusions to brittle materials until further evidence is available, and the author hopes to make experiments in this direction shortly. The writer wishes to acknowledge his debt to Mr. Guest for the results borrowed from his paper, and to thank Lord Blythswood for affording the opportunity of making the tests.

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LXII. *Double Refraction in Colloids produced by Electric Endosmose.* By H. W. Malcolm, *M.A., B.Sc., Ph.D., Carnegie Fellow of Aberdeen University**.

Introduction.

DOUBLE refraction in colloids has been the subject of numerous investigations; a summary is given by G. Quincke, *Ann. d. Phys.* x. p. 482 (1903), and *Ann. d. Phys.* xiv. p. 849 (1904). The latter paper deals with double refraction in gelatine produced by soaking and shrinking. It is shown that on cooling warm gelatine solution an optically isotropic jelly is obtained, built up of cells with walls of an oily nature, *i. e.*, showing surface-tension with the surrounding liquid. Prisms of this jelly when bent become doubly refracting, positively at places of dilatation, negatively at places of contraction, with optical axis parallel to the stress direction, as in the case of bent glass strips.

When an electric current flows through a jelly consisting of oily, water-poor liquid A with water-rich content B, one would expect, through the contact electricity at the places of contact of the heterogeneous substances, a transference of liquid along the cell-walls to take place, and simultaneously a displacement of the still-liquid, viscous cell-walls in the opposite direction, producing a state of strain and consequent double refraction.

Gelatine in Capillary Tubes.

A capillary tube 30 cms. long, outer diameter 1·2 mm., was filled with a 10 per cent. solution of gelatine and the lower end sealed off. Several such tubes were kept for a quarter of an hour in a long test-tube full of water at 100°, and slowly cooled, so that the gelatine solidified without strain and without double refraction; finally the upper end was sealed off too. As required, pieces 4·5 cms. in length were cut off and cemented to a glass slide. Over the ends of the tube were placed cover-glasses, supported by small empty pieces of the same tube. A drop of water was brought under each cover-glass, into which dipped platinum wires 0·5 mm. thick as electrodes; in this way disturbance due to gas-bubbles was avoided.

The slide was laid on the table of a microscope and could be examined between crossed nicols with different magnifying powers; between the prisms a wave-length selenite plate could be inserted, showing the red of the first order. The

* Communicated by the Author.

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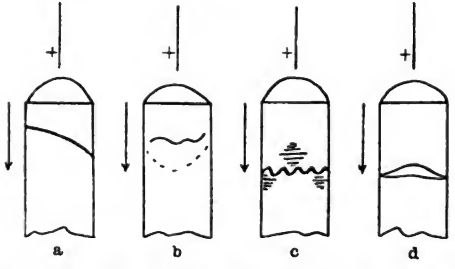
tube was so placed that a yellow colour corresponded to an expansion or positive dilatation, a blue colour to a contraction or negative dilatation, parallel to the axis of the tube. An eyepiece fitted with a Babinet's compensator could also be used, the upper nicol being slid to the side and a second, analysing prism put on top; the indications of the compensator were tested as to sign by means of a bent glass strip*. Current was obtained from the town mains or from a battery of cells.

When the circuit was closed there appeared, one or two millimetres from the positive end of the tube, a fine line, yellow with selenite plate, indicating a positive dilatation parallel to the axis of the tube. This line was to be seen the sooner the higher the voltage; its appearance varied—it was sharp or blurred, quiet or in rapid movement, according to the nature of the cell-walls that had been formed in the jelly. In general the line had a screw or wave-motion, in the direction of its length, and therefore perpendicular to its motion into the tube (fig. 1 *c*). This wave-motion was the more pronounced the higher the voltage and the more dried-up the gelatine.

Fig. 1.



Before the line, at the sides, were two blue spots indicating a negative dilatation parallel to the tube axis; these blue places came only after the yellow line had gone some distance into the tube, and not while its course was through soaked jelly; there the line was quiet without wave motion (fig. 1 *a*). This motion reappeared when the line had penetrated further into the tube. The line seemed sometimes the boundary of a parabolic surface (fig. 1 *d*†). Before the line a yellow colour spread slowly out; behind

* G. Quincke, *loc. cit.*

† G. Quincke, *Ann. d. Phys.* ix. p. 794 (1902).

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came a blue spot and a varied and irregular mass of yellow and blue, simplest in the case of soaked tubes. The displacement of the line parallel to the tube axis and the transversal wave-motion both grew slower as the line got further into the tube; it spread out, lost its sharpness and grew blurred; the wave-motion gave way to an intermittent one. One could see the displacement of the central line in the compensator slowly rise to a maximum and suddenly fall to zero. Table I. gives the displacements over the yellow line

TABLE I.

Voltage applied.	Age of tube: days	Time: min.	sec.	Displacement λ.	Current amp. $\times 10^{-6}$.
10·9	21	9	0	0·18	14
		11	45	0·16	12
		16	0	0·15	9
		20	15	0·16	8
		26	0	0·15	7
		43	0	0·14	4
12·0	16	18	0	0·20	14
		44	0	0·20	11
		78	0	0·21	10
24·5	15	10	0	0·22	28
		41	15	0·26	20
		103	0	0·27	12
50	10	18	0	0·10	30
		44	0	0·11	20
50	10	15	30	0·09	19
50	27	16	0	0·31	98
		44	0	0·30	64
		73	30	0·29	48
		100	0	0·29	30
100	23	5	45	0·18	60
		16	0	0·20	34
		34	0	0·20	22
		59	0	Max. 0·29	18
		81	0	„ 0·22	16
200	22	1	0	0·14	240
		2	0	0·18	125
		3	30	0·20	98
		7	0	0·18	64
		14	0	0·12	45
		28	0	0·24	32
220	37	17	0	Max. 0·30	108
		38	0	„ 0·27	16
220	14	12	30	0·18	126
		35	0	0·18	40

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at different times after completing the circuit: the displacement is fairly constant in a given case, and is greater the higher the voltage and the longer the time that has elapsed before using the tube.

At the cathode a yellow colour (positive dilatation parallel to the tube axis) spread slowly out to meet the yellow line from the anode; for small voltages the action was stationary before meeting had taken place. At meeting the line was often destroyed by a wave or mass of liquid running along the side of the tube. Afterwards movement and colour-change became irregular. The tubes kept their colours; the cell-walls had become solid through loss of water. An old used tube tried with higher voltage showed only small and slow changes. After the current had been passing some time, there appeared at the cathode shreds of gelatine and the water grew turbid; with the microscope numerous hexagonal crystals were to be seen at this end of the tube. The speed with which the yellow line moved into the tube was measured by reading off its distance from the positive end of the tube on a glass scale placed beneath the microscope slide; the figures in two cases are given in Table II. The

TABLE II.

228 volt; $2r=1{\cdot}4$ mm.			438 volt; $2r=1{\cdot}1$ mm.		
T.	s.	$s^2/T.$	T.	s.	$s^2/T.$
	mm.			mm.	
0′ 0″	0		0′ 0″	0	
25	1	2·4	22	4	43·6
40	2	6·0	42	6	51·6
57	3	9·0	1 0	8	64·0
1 21	4	12·0	27	10	66·6
47	5	14·3	2 0	12	72·0
2 20	6	15·4	43	14	71·3
57	7	16·3	36	16	71·4
3 37	8	17·4	4 4	17	70·8
4 22	9	18·7	33	18	73·3
5 11	10	19·3	5 5	19	72·0
6 10	11	19·6	41	20	70·6
7 12	12	20·0	6 18	21	69·6
8 17	13	20·3	7 0	22	69·1
9 24	14	20·8	40	23	69·0
10 39	15	21·1	8 22	24	69·1
13 21	17	21·7	9 47	26	71·8
15 47	18	20·6	10 42	27	68·4
16 15	19	22·2	11 37	28	67·2
			12 32	29	67·3
$(s^2/T)=21{\cdot}0$			$(s^2/T)=70{\cdot}2$		
$s^2r^2/TV=0{\cdot}51$			$s^2r^2/TV=0{\cdot}49$		

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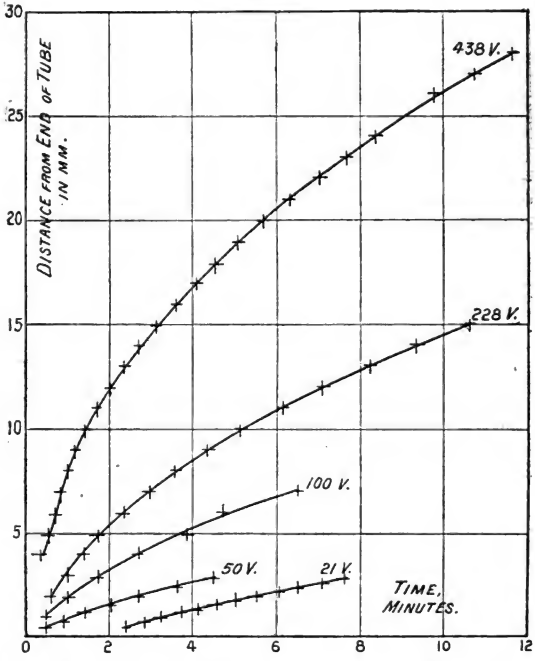
further the water has penetrated into the tube, the more it is dammed and the slower grows the motion; this may be caused by particles dragged along by the water blocking up the holes. Or the water may pass through holes in a series of transverse lamellæ; by hydrostatic pressure these lamellæ are pressed together and shut like a kind of valve. If the velocity is proportional to the voltage divided by the distance travelled, or $ds/dt = Au/s$, then $s^2 = Aut +$ a constant, which is

Fig. 2.



to be put 0. In Table II. are given the values of s^2/T; these increase till about one-fifth of the tube has been traversed; this increase is partly to be explained by the fact that the curves in fig. 2 do not begin from the origin. The values of s^2/T multiplied by the square of the radius and divided by the voltage are approximately equal.

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If the cells are open and communicate with one another, water can be carried along the cell or tube walls; if they are shut, however, the water can only enlarge or diminish the cells by dissolving in the substance of the wall or in the contents of the cells, and thus produce a temporary dilatation positive or negative, with corresponding positive or negative double refraction. By the pressure of liquid into such stretched-out open cells, the observed temporary double refraction, positive perpendicular to the direction of transference of the water, would be expected; the later-appearing spot showing a negative dilatation is probably produced by reaction. At the sides of the glass tube the water is less moved than in the middle, on account of the transverse walls there; at the sides appears as a consequence a temporary positive dilatation perpendicular to the tube. Behind the first line appears a series of places with positive and negative double refraction, as in the case of shrinking of gelatine in short capillary tubes in air, alcohol, or glycerine*.

Gelatine in Shallow Dishes.

Shallow glass dishes, either square (5 cms. × 5 cms. × 1 cm.) or round ($2r=8$ cms.) were filled to a depth of 1 cm. with a 10 per cent. gelatine solution and examined at the end of 24 hours. The electrodes only touched the surface, otherwise the bubbles rising up had a disturbing influence.

At the anode came a furrow, at the cathode a swelling. The anode was surrounded by two hemispheres, the inner more distinct, and both sharp. The cathode was surrounded by a white hemisphere and by a second, outer one, difficult to see. The white colour came from countless, very small (0·003 to 0·01 mm.) crystals. Round the electrode the crystals were packed and stuck together. The crystals were of two kinds: (*a*) spherical, showing a black cross parallel to the long diagonals of crossed nicols, and (*b*) rhombohedral ($CaCO_3$?), growing dark and reappearing as the turn-table was rotated. This crystal formation may come (1) from loss of water, (2) from loss of gelatine, the presence of which may increase the solubility of the salt. If the cell-walls are covered with a layer containing salt, layer and cell-contents will be displaced in opposite directions, eddies will be formed and the salt layer loosened to crystallize out later. The furrow round the anode was about 1 mm. deep, and had smooth sides with sharp edges; the swelling round the

* G. Quincke, *Ann. d. Phys.* xiv. p. 849 (1904).

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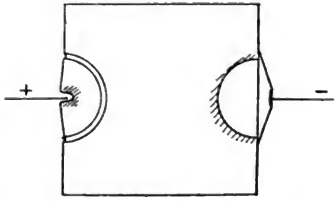
cathode was about 2 mms. high (see fig. 3 for the case of a cube of gelatine with electrodes touching the centres of

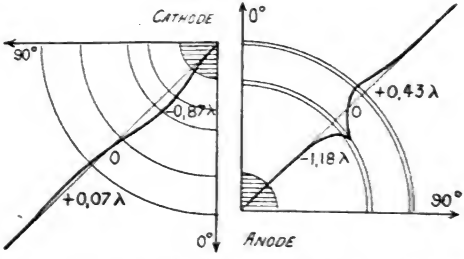
Fig. 3.



opposite faces). At the anode came a hole full of liquid; round about the jelly had a deeper tint, as also at the outer boundary of the swelling at the cathode; these places were harder than the surrounding jelly, and had suffered a loss of water or gain of gelatine. Sometimes the swelling at the cathode was radially striped or showed a network of fine cracks; with the microscope could be seen drops of liquid oozing out at the crossing of two cracks. On standing these lines disappeared, and the rings round the anode lost their sharpness; displaced water had been restored by diffusion.

The displacement of the central line in the compensator is shown in fig. 4. At the anode it increases, from the centre

Fig. 4.



out, to a maximum at the first hemisphere or inner edge of the furrow, then through zero to a maximum in the opposite direction at the second hemisphere or outer edge of the furrow; at the inner edge the dilatation is negative, at the outer edge positive, parallel to the radius. At the cathode the displacement indicates a negative dilatation parallel to the radius

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separated by a neutral zone from a positive outer zone. The figure was drawn after the current had been passing for an hour between electrodes at a potential-difference of 16 volts.

In Table III. is given the mean diameter of the furrow round the anode at various times, for the case of a cube of gelatine; also the square of the diameter divided by the time. The means of the values in the third column divided by the voltage give approximately the same number: thus the places of greatest double refraction form spherical surfaces, the square of the diameter of which is proportional to the time and to the voltage.

TABLE III.

T = Time from completion of electric circuit.
$2r$ = Diameter in mm. of the ring round the anode.

5·6 Volts.				15 Volts.				24 Volts.				34 Volts.			
T.		$2r$.	$(2r)^2/T$.	T.		$2r$.	$(2r)^2/T$.	T.		$2r$.	$(2r)^2/T$.	T.		$2r$.	$(2r)^2/T$.
0′	0″	0		0′	0″	0		0′	0″	0		0′	0″	0	
3	5	2·9	2·7	3	0	4·7	7·4	3	30	6·0	10·3	1	50	4·8	12·5
5	30	3·4	3·3	7	20	7·3	7·3	9	0	9·5	10·0	2	30	6·7	18·0
8	30	4·2	2·1	13	10	10·4	8·2	15	0	13·0	11·3	5	30	10·1	19·1
11	15	4·8	2·0	17	20	11·7	7·9	21	0	15·0	10·2	11	15	14·0	17·4
18	30	6·5	2·3	21	45	13·0	7·8	28	30	18·5	12·0	14	0	15·7	17·6
25	0	7·2	2·1	27	50	14·6	7·7	34	0	19·5	11·2	18	15	17·5	16·8
31	0	7·9	2·1	34	15	16·0	7·5	46	30	22·0	10·4	22	15	19·1	16·8
37	30	8·5	1·9	41	55	17·4	7·2					30	0	21·5	15·4
42	0	9·3	2·1												
48	0	10·7	2·3												
$((2r)^2/T) = 2·3$				$((2r)^2/T) = 7·6$				$((2r)^2/T) = 10·8$				$((2r)^2/T) = 17·2$			
$(2r)^2/TV = 0·41$				$(2r)^2/TV = 0·51$				$(2r)^2/TV = 0·45$				$(2r)^2/TV = 0·51$			

Gelatine Solution.

One per cent. gelatine solution covered the bottom of a shallow glass dish to a depth of 1 mm. Two vertical platinum wires 5 cms. apart were connected to a battery of 40 volts. At the anode a white cloud spread outwards; behind the solution had jellied through loss of water or gain in gelatine; on its surface lines running perpendicular to the direction of the current were to be seen; these moved forward from time to time with a sudden, undulatory movement. If the solution through evaporation had grown thick crystals could be watched forming as before round the cathode; if the solution were still liquid, they were driven

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away in all directions; in course of time the crystals collected on the bottom of the dish.

Silicic Acid.

A thin film of colloidal silicic acid ($\sigma = 1{\cdot}0189$, one month dialysed) was spread out on a glass slide. Liquid was driven away from the anode so that the surface of the slide dried up. Although the wires were clean the transference did not take place equally in all directions; radial furrows ran out from the electrode. At the cathode the acid was heaped up. As in the case of gelatine small (0·01 mm.) rhombohedral crystals were found round the cathode. With 440 volts feeble double refraction was to be seen in places round the anode giving a displacement of $0{\cdot}04\,\lambda$. With loss of water the acid fell into single pieces between cracks cutting everywhere at right angles. When a second slide was pressed on the top of the first after the acid had dried up, the distance apart was greater at the anode; only water had moved to the cathode.

Iron Hydrate.

Colloidal ferric hydrate was spread out on a glass slide. Liquid was driven away from the cathode; the jelly cracked and showed with the compensator a displacement of $0{\cdot}29\,\lambda$. Water collected at the anode; here crystals could be seen. The dried-up layer was thicker at the cathode than at the anode; water is moved to the anode and simultaneously colloid to the cathode, *i.e.*, in the opposite direction to gelatine, silicic acid and albumen.

Albumen.

Filtered albumen on a glass slide showed round the anode several concentric rings, the outer a zigzag or waved line. Before the cathode a crescent of small rhombohedral crystals could be seen. At the cathode the albumen was heaped up. Next day a second slide pressed on top was nearly the same distance apart at both ends. At the cathode the dry substance was folded up and loose from the slide.

Albumen coagulated in capillary tubes (prepared as above described) became transparent and liquid at the cathode; at the anode, on the other hand, it shrank and the water ran back along the sides of the tube.

My sincere thanks are due to Professor Quincke for ever-ready kindness and help during the course of the foregoing investigations.

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LXIII. *On the Theory of Surface Forces.* By G. Bakker *.

§ 1. *THE potential function.*—One can study capillarity either by imagining a liquid to be a system of molecules in movement, or by supposing some homogenic agent, which produces the same external effect as the liquid; we will follow the second method.

The force exercised between the parts of the liquid acting only through a very small range †, the potential V at some internal point depends only on the elements embraced in the sphere of action of which it is the centre, and should be proportionate to the density ρ; thus q^2 and f being constants

$$q^2 V = -4\pi f\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As to the interior of the liquid, the derivatives

$$\frac{dV}{dx}, \quad \frac{dV}{dy}, \quad \frac{dV}{dz},$$

may be regarded as *nul*. Let us refer the liquid to any set of rectangular axes.

Suppose on the left of the plane of the y- and z-axes an infinite mass of liquid limited to this plane; this plane is then a level plane and the lines of the capillary force adjacent to this plane are parallel to the x-axis; thus we should have:—

$$\frac{dV}{dy} = 0, \quad \frac{dV}{dz} = 0.$$

The character of the capillary forces demands that the force parallel to the x-axis is annulled very rapidly on leaving the yz-plane; this condition may be stated by supposing:

$$V = Ae^{-qx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which A and q are constants, where q may be very large.

Further,

$$\frac{\partial^2 V}{\partial y^2} = 0, \quad \frac{\partial^2 V}{\partial z^2} = 0, \quad \rho = 0.$$

Differentiating the equation (2) twice, one finds

$$\frac{\partial^2 V}{\partial x^2} = q^2 V.$$

By reversing the coordinate-axes, one arrives at two other relations of the same form. These relations and the conditions

* Communicated by the Author.

† Young, "On the Cohesion of Fluids," Phil. Trans. 1805. See Rayleigh, "On the Theory of Surface Forces," Phil. Mag. Oct.-Dec. 1890.

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which they should yield lead to the following differential equation :—

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = q^2 V + 4\pi f \rho. \quad . \quad . \quad . \quad (3)$$

Now I have demonstrated the following theorem *:—

"If V and ρ are functions of x, y, and z, V satisfying the three following conditions :—

1st. V and its first derivatives with respect to x, y, and z are continuous everywhere:

2nd. Some isolated points, lines, and surfaces excepted, V fulfils in an acyclic region the equation

$$\nabla^2 V = q^2 V + 4\pi f \rho ;$$

3rd. The products xV, yV, zV, $x^2 \frac{\partial V}{\partial x}$, $y^2 \frac{\partial V}{\partial y}$, and $z^2 \frac{\partial V}{\partial z}$ become nowhere infinite : then the potential of an agent, the density of which is ρ, is for that region V, meanwhile the potential function is

$$\phi(r) = -f \frac{e^{-qr}}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is interesting to remark that van der Waals in his beautiful thermodynamic theory of capillarity has demonstrated that the potential function (4) indicates the continuity between the liquid and gaseous states, no matter what the temperature. I therefore adopt as the potential function of the forces between the parts of the liqnid (the cohesion-forces) :—

$$-f \frac{e^{-qr}}{r}.$$

Observation.—C. Neumann pointed out a remarkable property of that function. He found that if a coefficient depending on the radius is left out of account, in consequence of this function the potential of a homogenic sphere for an exterior point is determined in the same way as if the whole mass were concentrated in the centre.

On account of the great importance, practical as well as theoretical, of such a function for a theory of gases and liquids, I examined the question whether there are other potential functions which possess this property †. I have found *two* solutions :—

1st,
$$\phi(r) = \frac{Ae^{-qr}}{r} + \frac{Be^{qr}}{r} + C$$

* *Koninkl. Akad. v. Wetensch.* at Amsterdam. Proceedings of the Meeting of Saturday, Nov. 25th, 1899.

† *Koninkl. Akad v. Wetensch.* at Amsterdam. Proceedings of the Meeting of Saturday, Oct. 28th, 1899.

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in which q, A, and B are arbitrary positive and negative constants. C is a constant that can be omitted.

2nd, $$\phi(r) = \frac{A_1 \sin(qr+\alpha)}{r}$$

in which A_1, q, and α are constants.

If we restrict ourselves to functions which relate to forces as they occur in nature, the second potential function must be excluded. If we further restrict ourselves to the forces which cause the cohesion of Young in the capillary theory, the constant B of the first potential function must be *nul* and we get:—

$$\phi(r) = \frac{Ae^{-qr}}{r} \text{ or } -f\frac{e^{-qr}}{r}.$$

§ 2. *Potential energy per unit of volume.*—The analogy of the equations

$$\Delta^2 V = 4\pi f\rho \quad \text{and} \quad \Delta^2 V = q^2 V + 4\pi f\rho,$$

and that of the functions

$$-\frac{f}{r} \quad \text{and} \quad -f\frac{e^{-qr}}{r}$$

have led to my seeking in the case of capillary forces (of cohesion) expressions analogous to those which Maxwell discovered for the energy and tensions in the case of electrical phenomena. The potential energy is always expressed by

$$W = \tfrac{1}{2}\int V\rho\, dx\, dy\, dz\,;$$

if one substitutes for ρ the value derived from relation (3), we see

$$W = \frac{1}{4\pi f}\int V\Delta^2 V dx\, dy\, dz - \frac{q^2}{4\pi f}\int V^2 dx\, dy\, dz,$$

and as at an infinite distance V and $\frac{dV}{dx}$ &c. annul, the integration by parts shows:—

$$W = -\frac{1}{8\pi f}\int\left\{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right\} dx\, dy\, dz - \frac{q^2}{8\pi f}.V^2 dx\, dy\, dz,$$

or, if R represents the force acting on unit of mass,

$$W = -\frac{1}{8\pi f}\int R^2 dx\, dy\, dz - \frac{q^2}{8\pi f}\int V^2 dx\, dy\, dz.$$

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Putting $\lambda = \frac{1}{q}$, λ being a length, we have thus for the potential energy per unit of volume:

$$-\frac{1}{8\pi f}\left(R^2 + \frac{V^2}{\lambda^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (4\,a)$$

That is, changing the sign, the work to be done in conveying the parts of the unit of volume to an infinite distance from each other, or, which is the same thing, beyond the sphere of sensible activity.

§ 3. *Tensions in the medium.*—Let us embrace within a surface one part of the homogenic agent we have under consideration; this would be system I., the rest we call system II. The force resolved parallel to the axis of x of the forces exercised by system II. on system I. is expressed by

$$X_1 = -\int \rho \frac{\partial V}{\partial x}\, dx\, dy\, dz,$$

or, substituting for ρ the value derived from (3),

$$-4\pi f X_1 = \int \Delta^2 V \frac{\partial V}{\partial x}\, dx\, dy\, dz - q^2 \int V \frac{\partial V}{\partial x}\, dx\, dy\, dz.$$

Putting

$$\left(\frac{\partial V}{\partial x}\right)^2 - \left(\frac{\partial V}{\partial y}\right)^2 - \left(\frac{\partial V}{\partial z}\right)^2 - q^2 V^2 = -8\pi f p_{xx}$$

&c.,

and

$$\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} = -4\pi f p_{yz} = -4\pi f p_{zy}$$

we shall find &c.,

$$X_1 = \int \left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{yx}}{\partial y} + \frac{\partial p_{zx}}{\partial z}\right) dx\, dy\, dz$$

an integral which would immediately transform into a surface-integral relative to the surface which limits system I.:—

$$X_1 = \int (l p_{xx} + m p_{yx} + n p_{zx})\, ds.$$

In the same manner one would obtain the values of Y_1 and Z_1.

In proceeding according to Maxwell in his electrical agent, one would deduce from the values of X_1, Y_1, and Z_1, the existence of a cohesion S_1 in the direction of the lines of force and a tension S_2 in the perpendicular direction, these cohesions being represented by

$$S_1 = -\frac{1}{8\pi f}\left(R^2 - \frac{V^2}{\lambda^2}\right) \quad \text{and} \quad S_2 = \frac{1}{8\pi f}\left(R^2 + \frac{V^2}{\lambda^2}\right), \quad . \quad (5)$$

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which shows (see (4*a*)) that the absolute value of the potential energy per unit of volume is equal to the cohesion per unit of surface perpendicular to the lines of force.

§ 4. *The surface-pressure* *.—Suppose a liquid in equilibrium with its vapour; if the surface is a plane, the lines of force in the capillary layer and adjacent to it may be considered as straight lines. The surface pressure is nothing but the difference of the thermic pressure † in the direction of the lines of force (normal to the surface) at the limits of the capillary layer. (We always consider the capillary layer as plane and between the liquid and saturated vapour.)

The hydrostatic pressure being the vapour-pressure and thus the same in the liquid- and vapour-phase, the absolute value of the difference of the thermic pressures $\theta_1 - \theta_2$ is consequently the same as those of the cohesions. As in the interior of the liquid and of the vapour R is *nul*, the expressions (5) for the cohesions become

$$\frac{1}{8\pi f}\frac{V^2}{\lambda^2}.$$

If V_1 and V_2 are the respective potentials for the interior of the liquid- and vapour-phase, we have therefore for the surface-pressure

$$\frac{1}{8\pi f\lambda^2}(V_1^2 - V_2^2). \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For every homogenic phase, $\Delta^2 V = 0$, and the equation (3) shows

$$V = -\frac{4\pi f}{q^2}\rho = -4\pi f\lambda^2\rho,$$

whence, calling ρ_1 and ρ_2 the densities of the liquid and of the vapour respectively, and substituting in (6), we obtain the expression

$$K = 2\pi f\lambda^2(\rho_1^2 - \rho_2^2). \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For $\rho_2 = 0$ we get

$$K_1 = 2\pi f\lambda^2\rho_1^2.$$

$2\pi f\lambda^2$ is therefore the coefficient a in the expression for the intrinsic pressure of Rayleigh or for the molecular pressure

* Molecular pressure of Laplace or intrinsic pressure of Rayleigh, when the surface of the liquid can be considered as free, *neglecting the influence of the vapour.*

† Young conceives the hydrostatic pressure as the difference between the power of repulsion and the cohesion. I have called that power of repulsion, "the thermic pressure."

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of Laplace, as we can show also immediately with the aid of the formula of Laplace:

$$K_1 = 2\pi\rho_1^2 \int_0^\infty \psi(z)dz,$$

where

$$\psi(z) = \int_0^\infty \Pi(u)u\,du \quad \text{and} \quad \Pi(u) = -f\frac{e^{-\frac{u}{\lambda}}}{u}\,*.$$

§ 5. *Superficial tension.*—This may be regarded as the force in respect to the unit of length, exercised normally to a line traced on the surface of the liquid; this is numerically the coefficient H of the term $\frac{2H}{R}$ in the formula which expresses the surface-pressure of some spherical mass of liquid enveloped in its vapour and supposed not to be weighty. In this case the fundamental equation (3) becomes

$$\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = q^2V + 4\pi f\rho, \quad . \quad . \quad . \quad . \quad (8)$$

r being the distance from the point considered as the centre of the sphere adopted as the origin of the coordinates.

Let R be the radius of the spherical mass of liquid limited by the capillary layer; for all the points of this layer one may write $r = R + h$, h varying from zero to the thickness of this layer; in this case equation (8) becomes, neglecting h with respect to R:

$$\frac{d^2V}{dh^2} + \frac{2}{R}\frac{dV}{dh} = \frac{V}{\lambda^2} + 4\pi f\rho, \qquad \left(\lambda = \frac{1}{q}\right).$$

If we multiply these two members by $2\frac{dV}{dh}$ and integrate, it becomes:

$$\frac{4}{R}\int_1^2\left(\frac{dV}{dh}\right)^2 dh = \frac{V_2^2}{\lambda^2} - \frac{V_1^2}{\lambda^2} + 8\pi f\int_1^2 \rho\frac{dV}{dh}dh,$$

in which $\int_1^2 \rho\frac{dV}{dh}dh$ is exactly the surface-pressure K.

For $R = \infty$ the last term vanishes; the difference in the values of K for $R = \infty$ and for a definite value R is therefore

$$\frac{1}{R}\cdot\frac{1}{2\pi f}\int_1^2\left(\frac{dV}{dh}\right)^2 dh = \frac{2H}{R},$$

whence

$$H = \frac{1}{4\pi f}\int_1^2\left(\frac{dV}{dh}\right)^2 dh, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

* See Rayleigh, Phil. Mag. Oct. and Dec. 1890, p. 290.

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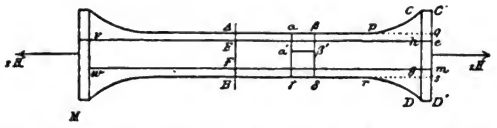
Now I will demonstrate that the capillary layer of the liquid is a true elastic membrane of which the tension is H.

Let us imagine (fig. 1) a liquid membrane, the breadth being equal to the unit, lying between two solid drops supported on the right and left by cords stretched within a space holding the vapour only of the liquid. Fig. 1 represents a

Fig. 1.



section of the membrane normally at its surface. This membrane is held in equilibrium by the action of the vapour-pressure and the tensions of the cords which must be equal to 2H, H denoting the superficial tension. Intersect the membrane by the plane AB, normal to the surface and the plane of the figure, and let us consider the system ApCC'D'DrBA. The external pressures of the vapour opposed to ApCC'q and BrDD's' are in equilibrium; it only remains for us to consider the pressure on qs and the influence exercised by the liquid situated to the left of AB. Construct the planes eh and mq at distances from the free surfaces respectively equal to the thickness of the capillary layers of the two surfaces of the membrane. *Between these two planes there is no departure from the law of Pascal*, and we have in every direction the hydrostatic pressure p_1, equivalent to the external pressure or the vapour-pressure. The pressure on EF, created by the liquid on the left, is therefore annulled by the external pressure p_1, opposed to em. It remains thus for us to consider the two masses AEeq and BFms. Allow p_2 to represent the hydrostatic pressure per unit of surface *in a direction parallel to the surface*; the force exercised by the mass to the left of AE is $\int_1^2 p_2 dh$, where h indicates the direction normal to the surface, the indices 1 and 2 corresponding respectively to the interior liquid- and vapour-phase. It is the same with the mass to the left of BF. The external pressures opposed to qe and ms are $p_1\int_1^2 dh$. The equilibrium of the system AEFBrDD'C'CpA therefore demands

$$2\int_1^2 (p_1-p_2)dh = 2\mathrm{H} \quad \text{or} \quad \mathrm{H} = \int_1^2 (p_1-p_2)dh.$$

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If our reasoning is correct the value of H must be identical with that obtained in equation (9). Indeed, if θ represents the thermic pressure and S the cohesion, we should have in every direction $p=\theta-S$.

Hence

$$p_1-p_2=\theta-S_1-(\theta-S_2)=S_2-S_1,$$

whence

$$H=\int_1^2(p_1-p_2)dh=\int_1^2(S_2-S_1)dh. \quad . \quad . \quad (10)$$

For the cohesion S_1 *in* the direction of the lines of force (that is to say in the direction normal to the surface of the membrane) we have

$$S_1=-\frac{1}{8\pi f}\left(R^2-\frac{V^2}{\lambda^2}\right).$$

For the cohesion S_2 *perpendicular* to the lines of force (or parallel to the membrane) we have found

$$S_2=\frac{1}{8\pi f}\left(R^2+\frac{V^2}{\lambda^2}\right),$$

whence

$$H=\frac{1}{4\pi f}\int_1^2 R^2dh,$$

which is exactly the formula (9) for H.

The reasoning based on fig. 1 is absolutely general and the formula for H,

$$H=\int_1^2(S_2-S_1)dh, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

is independent of the function of force adopted, *if θ is a quantity independent of the direction.* I have succeeded in demonstrating that formula (10) gives always the expression found by Rayleigh *.

§ 6. *Calculation of the surface-tension by means of the thermic pressure.*—I have found for the surface-tension H:

$$H=\frac{1}{4\pi f}\int_1^2\left(\frac{dV}{dh}\right)^2dh, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which becomes on putting $2\pi f\lambda^2=a$ (p. 561)

$$H=\frac{\lambda^2}{2a}\int_1^2\left(\frac{dV}{dh}\right)^2dh=\frac{\lambda^2}{2a}\int_1^2\frac{dV}{dh}\frac{dV}{d\rho}d\rho. \quad . \quad . \quad (9\,a)$$

* Rayleigh, "On the Theory of Surface Forces, II." Phil. Mag. Feb. 1892, p. 218, formula (22). See *Zeitschr. für phys. Chemie*, xxxiii. 4. p. 499 (1900).

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For the capillary layer equation (3) becomes:

$$\frac{d^2V}{dh^2} = q^2V + 4\pi f\rho. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If we multiply this equation by $2\frac{dV}{dh}$, after replacing q by $\frac{1}{\lambda}$ and replacing f by $\frac{a}{2\pi\lambda^2}$ and integrate, we find:

$$\left(\frac{dV}{dh}\right)^2 = \frac{V^2 - V_1^2}{\lambda^2} + \frac{4a}{\lambda^2}\int_1 \rho dV. \quad . \quad . \quad . \quad (12)$$

The equilibrium between the thermic pressure θ and the forces of cohesion demands (for every direction)

$$d\theta = -\rho dV, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

whence

$$\theta - \theta_1 = -\int_1 \rho dV,$$

where θ_1 is the thermic pressure in the interior of the liquid.

We have further for the liquid phase:

$$V_1 = -2a\rho_1 \quad . \quad . \quad . \quad (14) \text{ (Gauss and van der Waals)}$$

and

$$p_1 = \theta_1 - a\rho_1^2.$$

Hence equation (12) gives:

$$\left(\frac{dV}{dh}\right)^2 = \frac{V^2}{\lambda^2} - \frac{4a^2\rho_1^2}{\lambda^2} - \frac{4a}{\lambda^2}(\theta - \theta_1) = \frac{V^2}{\lambda^2} - \frac{4a}{\lambda^2}\theta$$
$$+ \frac{4a}{\lambda^2}(\theta_1 - a\rho_1^2) = \frac{V^2}{\lambda^2} - \frac{4a}{\lambda^2}\theta + \frac{4a}{\lambda^2}p_1,$$

or

$$\frac{dV}{dh} = \frac{2\sqrt{a}}{\lambda}\sqrt{p_1 - \theta + \frac{V^2}{4a}}. \quad . \quad . \quad . \quad (15)$$

Now, $d\theta = -\rho dV$ (equation 13) yields, noting that

$$V_1 = -2a\rho_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$V = V_1 - \int_1 \frac{d\theta}{\rho} = -2a\rho_1 - \int_1 \frac{d\theta}{\rho},$$

whence, replacing in (15) V by its value, we see:

$$\frac{dV}{dh} = \frac{2\sqrt{a}}{\lambda}\sqrt{p_1 - \theta + \frac{1}{4a}\left\{4a^2\rho_1^2 + 4a\rho_1\int_1 \frac{d\theta}{\rho} + \left[\int_1 \frac{d\theta}{\rho}\right]^2\right\}}.$$

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Let us substitute this value in (9*a*), and replace $\frac{dV}{d\rho}$ by $-\frac{1}{\rho}\frac{d\theta}{d\rho}$; (9*a*) finally becomes:

$$H=-\frac{\lambda}{\sqrt{a}}\int_{\rho_1}^{\rho_2}\left\{\sqrt{\theta_1-\theta+\rho_1\int_{\rho_1}^{\rho}\frac{d\theta}{\rho}+\frac{1}{4a}\left[\int_{\rho_1}^{\rho}\frac{d\theta}{\rho}\right]^2}\cdot\frac{1}{\rho}\frac{d\theta}{d\rho}\right\}d\rho. \quad (16)$$

Thus the constant of Laplace or the surface-tension H depends only on the thermic pressure θ, and consequently on its connexion with the density ρ. If we accept for θ the formula

$$\theta=\frac{RT}{v-b}=\frac{RT\rho}{1-b\rho}$$

given by van der Waals, the formula of H is complicated enough.

Observation.—It is interesting to observe that the surface-pressure (molecular pressure) K can be expressed, on the contrary, very simply by means of the thermic pressure θ. K, being the force by which the unit of surface of the capillary layer is attracted towards the liquid, must be in absolute value also the difference between the thermic pressures of the liquid and of the vapour:

$$K=\theta_2-\theta_1;$$

whence, observing that

$$p_1=\theta_1-a\rho_1^2, \quad \text{and } p_2=p_1=\theta_2-a\rho_2^2,$$

we see

$$K=a(\rho_1^2-\rho_2^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

§ 7. *A property of the surface-tension.*—If we assume $f(r)$ to represent the force acting between two liquid particles, and put

$$\int_r^\infty f(r)\,dr=\phi(r), \quad \text{and } -\int_r^\infty r^2\phi(r)\,dr=\psi(r),$$

we shall have as the potential energy of a volume v:

$$W=2\pi\psi(0)\rho^2 v \quad \text{(Gauss)},$$

ρ being the density.

Now I have demonstrated that for a homogenic agent the virial of the forces of cohesion B is equivalent to the potential energy W multiplied by $-\frac{3}{2}$*;

then one has: $$2B+3W=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

* G. Bakker, "Theorie der Vloeistoffen en dampen" (1888), and "Ueber die potentieile Energie und das Virial der Molekularkräfte," *Zeitschr. für phys. Chem.* (1896): L. Boltzmann, "Vorlesungen über Gastheorie, II." (1898), p. 152.

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I purpose demonstrating that the expression $-(2B+3W)$, *which is null for the liquid or vapour, is nothing but the superficial tension in the case of the capillary layer.*

Admitting the function of force to be $-f\frac{e^{-qr}}{r}$ at which I arrived above, the force which is exercised between two elements of mass becomes :

$$f\frac{mm'}{r^2}e^{-qr}+fq\frac{mm'}{r}e^{-qr}.$$

Now the virial B of the forces of cohesion is expressed by

$$B=\tfrac{1}{4}f\Sigma\frac{mm'e^{-qr}}{r}+\tfrac{1}{4}fq\Sigma mm'e^{-qr};$$

further,

$$W=-\tfrac{1}{2}f\Sigma\frac{mm'e^{-qr}}{r},$$

whence

$$2B=-W+\tfrac{1}{2}fq\Sigma mm'e^{-qr}. \quad . \quad . \quad . \quad . \quad (18)$$

The last term of (18) may be considered as the potential energy of a homogenic agent of which the function of force is

$$fqe^{-qr}.$$

Let us calculate, on this hypothesis, the potential energy of a system of plane parallel layers, such as the capillary layer is. According to Rayleigh *, one has for the potential V at a point of the capillary layer :

$$V=-2a\rho-\frac{c_2}{1\,.\,2}\frac{d^2\rho}{dh^2}-\frac{c_4}{1\,.\,2\,.\,3\,.\,4}\frac{d^4\rho}{dh^4}-\ldots, \quad . \quad (19)$$

where

$$c_n=\int_0^\infty u^n\psi(u)du, \quad \text{and } \psi(u)=2\pi\int_u^\infty u\phi(u)du,$$

$\phi(u)$ being the potential function.

The potential energy of our system of layers *per unit of surface* is thus :

$$W=\frac{1}{2}\int_1^2 V\rho d\tau=-a\int_1^2\rho^2dh+\frac{c_2}{1\,.\,2}\int_1^2\left(\frac{d\rho}{dh}\right)^2dh$$
$$-\frac{c_4}{1\,.\,2\,.\,3\,.\,4}\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2dh, \quad . \quad (20)$$

dh being the element normal to the surfaces of the system of layers, the indices 1 and 2 relating respectively to the liquid and to the saturated vapour.

* Phil. Mag. Feb. 1892, p. 210.

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The potential function $\phi(u)=fqe^{-qu}$ gives:

$$\psi(u)=2\pi fq\int_u^\infty ue^{-qu}du=-2\pi f\int_u^\infty ud(e^{-qu})=2\pi fue^{-qu}+\frac{2\pi f}{q}e^{-qu}$$

and

$$c_n=\int_0^\infty u^n\psi(u)du=2\pi f\int_0^\infty u^{n+1}e^{-qu}du+\frac{2\pi f}{q}\int_0^\infty u^ne^{-qu}du.$$

Now the function Γ of Euler is such that one has

$$\Gamma_{n+2}=\int_0^\infty u^{n+1}e^{-u}du=1\,.\,2\,.\,3\ldots(n-1)n(n+1),$$

whence

$$\int_0^\infty u^{n+1}e^{-qu}du=\frac{\Gamma_{n+2}}{q^{n+2}},\quad \int_0^\infty u^ne^{-qu}du=\frac{\Gamma_{n+1}}{q^{n+1}},$$

and

$$c_n=2\pi f\left\{\frac{\Gamma_{n+2}}{q^{n+2}}+\frac{\Gamma_{n+1}}{q^{n+2}}\right\}=2\pi f\frac{(n+2)\Gamma_{n+1}}{q^{n+2}}$$

$$\frac{c_n}{1\,.\,2\,.\,3\ldots n}=\frac{2\pi f(n+2)}{q^{n+2}}.$$

Putting $\lambda=\frac{1}{q}$, we find for the system of plane layers, according to (18):

$$\tfrac{1}{2}fq\Sigma mm'e^{-qr}=4\pi f\lambda^2\int_1^2\rho^2dh-8\pi f\lambda^4\int_1^2\left(\frac{d\rho}{dh}\right)^2dh$$
$$+12\pi f\lambda^6\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2dh+\ldots.$$

Further, the W of the equation (18) *relating to the potential function* $-f\frac{e^{-qr}}{r}$, becomes, according to equation (20):

$$W=-2\pi f\lambda^2\int_1^2\rho^2dh+2\pi f\lambda^4\int_1^2\left(\frac{d\rho}{dh}\right)^2dh-2\pi f\lambda^6\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2dh+\ldots$$

From this equation and equation (18) we arrive at

$$2B+3W=-4\pi f\lambda^4\int_1^2\left(\frac{d\rho}{dh}\right)^2dh+8\pi f\lambda^6\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2dh-\ldots. \quad (21)$$

Now I have demonstrated above that the superficial tension H is expressed by

$$H=\frac{1}{4\pi f}\int_1^2\left(\frac{dV}{dh}\right)^2dh. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

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V designating the potential for the point considered and being given by (19). From this formula (19) we derive:

$$\frac{dV}{dh} = -2a\frac{d\rho}{dh} - \frac{c_2}{1.2}\frac{d^3\rho}{dh^3} - \frac{c_4}{1.2.3.4}\frac{d^5\rho}{dh^5}\cdots$$

Moreover for the potential function $-f\frac{e^{-qr}}{r}$, we have:

$$\frac{\frac{c_2}{1.2}}{a} = \frac{\frac{c_4}{1.2.3.4}}{\frac{c_2}{1.2}} = \frac{\frac{c_6}{1.2.3.4.5.6}}{\frac{c_4}{1.2.3.4}} = \ldots = \frac{1}{q^2} = \lambda^2,$$

whence

$$-\frac{dV}{dh} = 2a\frac{d\rho}{dh} + 2a\lambda^2\frac{d^3\rho}{dh^3} + 2a\lambda^4\frac{d^5\rho}{dh^5} + \ldots$$

Hence integration with respect to h gives:

$$\frac{1}{4\pi f}\int_1^2\left(\frac{dV}{dh}\right)^2 dh = \frac{\lambda^2}{2a}\int_1^2\left(\frac{dV}{dh}\right)^2 dh = 2a\lambda^2\int_1^2\left(\frac{d\rho}{dh}\right)^2 dh$$
$$-4a\lambda^4\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2 dh + 6a\lambda^6\int_1^2\left(\frac{d^3\rho}{dh^3}\right)^3 \ldots *$$

or

$$H = 4\pi f\lambda^4\int_1^2\left(\frac{d\rho}{dh}\right)^2 dh - 8\pi f\lambda^6\int_1^2\left(\frac{d^2\rho}{dh^2}\right)^2 dh + 12\pi f\lambda^8\int_1^2\left(\frac{d^3\rho}{dh^3}\right)^2 dh -$$

The value of H, on changing the sign, is nothing but the second member of the formula (21). Finally one sees that

$$H = -3W - 2B. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This relation may be established without necessarily having recourse to the square of the expression for $\frac{dV}{dh}$ †.

Hence:

The value of the surface-tension is three times the work to be done in conveying the parts of the capillary layer (per unit of surface) *to an infinite distance from each other*, or, which is practically the same thing, beyond the sphere of sensible activity, *decreased by twice the virial of the forces of cohesion.*

$$* \ \frac{1}{4}\left(\frac{dV}{dh}\right)^2 = a^2\left(\frac{d\rho}{dh}\right)^2 + a^2\lambda^4\left(\frac{d^3\rho}{dh^3}\right)^2 + a^2\lambda^8\left(\frac{d^5\rho}{dh^5}\right)^2 + 2a^2\lambda^2\frac{d\rho}{dh}\frac{d^3\rho}{dh^3}$$
$$+2a^2\lambda^4\frac{d\rho}{dh}\frac{d^5\rho}{dh^5} + 2a^2\lambda^6\left\{\frac{d\rho}{dh}\frac{d^7\rho}{dh^7} + \frac{d^3\rho}{dh^3}\frac{d^5\rho}{dh^5}\right\} + \ldots$$

† G. Bakker, *Zeitschr. für phys. Chem.* xxxiii. p. 498 (1900). See above, § 5, and Rayleigh "On the Theory of Surface Forces, II." Phil. Mag. Feb. 1892, formula (22).

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LXIV. *On Certain Questions connected with Astronomical Physics.* *By* S. Tolver Preston, *Ph.D.* *

THE Philosophical Magazine for March 1847 contains a paper under the title "The Form of the Earth no Proof of Original Fluidity," by Herbert Spencer. A remote date to any publication may tend to its being overlooked, and some of the conclusions in this article appear to be worthy of notice.

The implication of course is that in the case of our earth †, gravitational pressure by itself alone would demonstrably suffice to have effected the compacted "flow" of solid substance into globular form, without the necessity for any preceding state of fluidity or of high temperature.

In the "flow of solids" (metals particularly) by the mere action of pressure, without any application of heat, subsequently demonstrated experimentally by Tresca, we may see a parallel effect actually produced and illustrative of the above theoretical view.

As a fact of general application, we may also notice that, provided the accumulation of aggregated material were assumed sufficiently gradual, or spread over an epoch of adequate duration, there might be time for any heat to dissipate; and so it appears there need be no real accumulation of heat from this cause, but rather a gradual yielding of materials under gravitational stress, somewhat comparable [say] to metals "flowing" under a very gradually augmented pressure. The sphere is then regarded as the natural ‡ form assumed under gravitation, even if there were no fluidity produced by heat.

But it would seem as if one fact had been overlooked by Herbert Spencer. For while, *at* the surface of the globe,

* Communicated by the Author.

† Since our earth in the remote past is generally held to have rotated more quickly about its axis than now, any quantitative inference as to its shape, if grounded on the present rate of axial rotation, would appear to be open to question.

‡ The pressure at the centre of our globe is calculated by Prof. T. J. J. See as equivalent to that of a column of mercury 2383·152 kilometres high—a pressure "so enormous that Prof. See attempts to render it more comprehensible by suggesting that it is 7838 times as great as that of a mercurial column equal in height to the Eiffel Tower" ('Nature,' March 2, 1905, p. 424). Curiously enough, this Tower is reputed to contain a mercurial manometer equal to its height [300 metres].

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gravitational stress is insufficient to entail any such "welding" together of cold ingredients; it is only at some depth below the surface [relatively insignificant, no doubt, compared with the diameter of the earth] that such crushing effect could take action. Some minor irregularities of surface would thus be inferable from this condition (on our globe).

It may be worth noticing that recently in 'Nature' it was remarked that earthquake-waves, the velocity of which is now instrumentally measured, appear to indicate a rather abrupt change* of quality or density in the globe's interior, at a point about 30–40 miles from the surface. This consideration becomes of practical value in relation to possible additions to knowledge in connexion with earthquakes. Thirty miles thickness of crust, compared with the globe's diameter, is evidently but a thin shell.

Some of the observed internal heat of the earth's crust might be attributable to the chemical action of material of diverse nature, without necessarily appealing to an original nebulous condition. We must take all evidence and cooperative causes into consideration,—every possible evolutionary process, slow, as well as catastrophic. The observed tide-resisting quality of our globe, whereby it has been inferred to resist distortion like glass or steel,—has, at any rate, to be harmoniously† accounted for. In 'Nature,' August 9, 1906, the following expression of view by Sir George Darwin is quoted:—

"On the whole, we may fairly conclude, whilst there is some evidence of a tidal yielding of the earth's mass, that yielding is certainly small, and the effective rigidity is at least as great as steel" (p. 358).

Demonstrably, there is much iron present in our universe, while this metal is known to be a chief constituent of meteorites (from actual analyses). Looking to a purpose to be fulfilled

* In an address at the recent meeting of the British Association by Principal E. H. Griffiths, attention is directed to a conclusion of Prof. Milne's, namely:—

"The difference in the rate of propagation of earthquake-waves through the earth's interior and through the crust, has led him [Prof. Milne] to the conclusion that the material below a depth approximating to thirty miles is of uniform nature, and that the change in physical constitution is abrupt at some such depth as that indicated" ('Nature,' August 9, 1906, p. 358).

† On the other hand, it is a merely speculative idea, that pressure (extremely great) might at the temperature of fluidity solidify steel. It may be worth trying a much greater pressure than is applied in the instance of the "fluid compressed steel" of Whitworth, as a research experiment.

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by meteoritic bodies known to be scattered everywhere in stellar space (through which our system is moving as a whole at a speed estimated at four * miles per second) ; also in view of the meteoritic theory of cosmical evolution, coupled with the observed powerful magnetic qualities of our globe,—its more central portions may well be constituted of the metal iron. And certainly the fact that gravitation by itself alone would suffice to produce (without the aid of heat) sphericity and the flow even of metals, would appear too interesting a point to neglect † altogether.

A metallic constitution for the chief bulk of our globe [the metal iron] would be quite consistent with the measured specific gravity of 5·5: this being considerably greater than that of ordinary rocks. The keeping of alternative possibilities in view, so long as they be not inconsistent with recognized facts, may be an aid to progress and eventual demonstration.

In an address before the British Association at South Africa, chiefly on astronomical science, by Sir George Darwin, where the subject is popularly dealt with, is the following passage, which may be quoted in connexion with the foregoing :—

"The problems of cosmical evolution are so complicated that it is well to conduct the attack in various ways at the same time. Although the several theories may seem to some extent discordant with one another, yet, as I have already said, we ought not to scruple to carry each to its logical conclusion. We may be confident that in time the false will be eliminated from each theory; and when the true alone

* Few probably realize that for the sun (or solar system), with its estimated proper motion of four miles per second, to move a distance equal to that of α Centauri [parallax = 0″·93], would demand but a small fraction of the life-history of the solar system, namely, 161,000 years (only). In the mean life of a system, average stellar distances in our universe would, it appears, be traversed some thousands of times. A diversity of physical conditions at diverse epochs may thus be presented to the speculative view, including possible gradual variation of the "temperature of space."

† At the *sun's* centre the pressure (according to the calculation of Prof. See) would be that of a mercury manometer projecting beyond the sun's radius, with the quicksilver supposed subject everywhere to the value g for acceleration at the earth's surface.

With such pressures as this, may not molecules (now recognized to consist of corpuscles) be, so to speak, "crushed out of shape" to some extent at least in the vicinity of the centre of the sun or of our globe, and so escape as "radioactive material"?

Is it likely that pressure without assigned limit can be sustained by minute configurations moving in orbits, without entailing a radical disturbance of dynamical equilibrium?

remains the reconciliation of apparent disagreements will have become obvious." ('Nature,' August 31, 1905, p. 439.)

"To conduct the attack in various ways at the same time," being tantamount to 'keeping of all alternative possibilities in view," may accordingly be a motive for inviting attention to some other aspects of the inquiry; while some extracts from Sir George Darwin's address appear to lend additional support to the view that planets may be formed by the accretion of meteoritic matter. First, by way of preface, he says :—

"The German astronomer Bode long ago propounded a simple empirical law concerning the distances at which the several planets move about the sun. It is true that the planet Neptune, discovered subsequently, was found to be considerably out of the place which would be assigned to it by Bode's law, yet his formula embraces so large a number of cases with accuracy that we are compelled to believe that it arises in some manner from the primitive conditions of the planetary system.

"The explanation of the causes which have led to this simple law as to the planetary distances presents an interesting problem, and, although it is still unsolved, we may obtain some insight into its meaning by considering what I have called a working model of ideal simplicity.

"Imagine then a sun round which there moves in a circle a single large planet. I will call this planet Jove, because it may be taken as a representative of our largest planet, Jupiter" ('Nature,' August 31, 1905, p. 439).

Next, a number of meteorites (some of which may be even comparable in size to asteroids) are supposed to be moving in all conceivable directions, so that certain of them fortuitously may enter the system composed of the sun and Jove. By mathematical analysis, it is found that there are certain "zones surrounding the sun and Jove in which stable orbits are possible, and others in which they are impossible. There is hardly room for doubt [it is added] that if a complete solution for our solar system were attainable, we should find that the orbits of the existing planets and satellites are numbered amongst the stable perpetual orbits, and should thus obtain a rigorous mechanical explanation of Bode's law concerning the planetary distances" (p. 439).

Previously it is remarked that while some of these meteoritic bodies "will be absorbed by the sun," a "minority will collide with Jove."

The process by which the meteorites,—which (it seems) must gradually *accumulate* more and more within the limits

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of the breadth of the zones—become eventual planets, is not entered upon. But the significance of the zones "in which stable orbits are possible" appears to be that only under this condition could meteoritic bodies coming from space *accumulate* [be virtually "selected"], constituting presumably material rings of discrete fragments of matter, somewhat reminding of the rings of Saturn *.

There is the difficulty, however, that a ring of material can only concentrate (aggregate) about its centre of gravity, which would imply its ultimately falling into the sun or central body.

But it might possibly be proposed that the mere fortuitous entry of meteorites (possessing a suitable "direction and speed") into these zones, would, by the laws of probability, condition [round a given zone] an *irregular* distribution of material, rather than the reverse; thus virtually constituting an ill-balanced or broken ring; which might then concentrate into a planet †. And we have seen that, without the necessity for a preceding nebulous state, gravitation by itself can produce the compacted flow of even solid substance into a natural globular (or rather oblate) form; while internal heat for the inhabitability of a planet is of no use whatever.

Here seems a fit opportunity for inviting attention to the importance of according due weight in cosmology to Hutton and Lyell's principle or (rather) factor of *slow* progressive changes, under a proper extension of *time*, coupled with natural causes at present observed to be in action,—which is acknowledged to have contributed so much to enlightenment in the allied science of geology. This slow natural process

* In reference to Saturn, Sir George Darwin remarks:—"The astonishing discovery has just been made by William Pickering of a ninth retrograde satellite of Saturn, while the rotations of the eight other satellites, of the ring, and of the planet itself, are direct." (Address, p. 440, in 'Nature').

But in that same Journal (only 27th of April previously—same year), is a letter by Prof. William J. Pickering (p. 608), where he expresses rather the reverse of surprise at this retrograde motion of the satellite discovered by him, and proposes to harmonize its motion with a fundamental property of gyrostatic motion, which, since his letter bears the title—"A little known Property of the Gyroscope," Prof. Pickering would seem to consider has escaped an adequate appreciation.

† It may be just worth a passing notice that in a so-called "meteoritic plenum," the components of any "double" (forming after the manner described in the moon's evolution) would differentiate in mass, *after* separation or severance, on account of more meteorites falling continually into the greater or central body. Thus it might seem that the mass (say of a "large planet") which was judged too small relatively to the primary for this particular evolutionary process to be applicable, might have been more equal to the primary at the very remote epoch when this "double" originated (or could have originated).

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may well cooperate or even rival in fundamental import the more catastrophic processes exemplified by the nebular theory of Kant and Laplace, which [contrasted with the *slow* procedure] involve an intense development of heat, with liquefaction or vaporization of materials.

If we leave provisionally out of view the primordial constituents of matter, namely the "corpuscles" recognized to belong to the domain of "Radioactivity": then, for the building of systems, the only available substance appears to be the substance of meteoritic bodies, which we can actually handle and analyse in the laboratory. If it be conceded that, by a natural process, systems cannot evolve out of nothing, the visualizing of a purpose to be fulfilled by this everywhere present meteoritic matter becomes of logical fitness; and it appears that no evolutionary theory of cosmical systems becomes satisfactory, unless it recognizes this universal substance as a fundamental constituent (or basis), whether the procedure concerned in any given case be regarded as slow, spread over an extensive time-epoch, with unimportant accumulation of heat; or as catastrophic, as in the instance of the nebular theory, where it appears that in the first stage the nebula itself "may be regarded as a quasi gas, the molecules of which are meteorites."

In our system, consider the numerous asteroids*. It seems to be often tacitly supposed or taken for granted that these bodies in their movement of circulation about the sun were once fluid or luminous, that they represent minor planets which have cooled down. But they might never have been incandescent or even have possessed a temperature worth mentioning; but may have been formed by the slow and gradual accretion † of meteoritic bodies into that spherical

* It appears a remarkable fact that the meteorites composing the ring of Saturn circulate all substantially in *one* plane; whereas this is *not* the case with the asteroids, the orbits of which deviate considerably from a plane. This would appear to indicate a mode of evolution not quite the same in the two cases. Any suggestion, based on recognized causes, as to the mode of origin of the asteroidal ring, seems preferable to the theory of an "exploded planet," the fragments of which are to represent the asteroids.

† There may be no objection to my quoting here a passage from a letter received on April 9, 1893, from Sir George Darwin, as it bears on the present subject:—

"I cannot help feeling that Newcomb is wrong to throw over Bode's law. It is wrong perhaps to call it a law, but for my part I believe that the sequence of mean distances observed in the planets is due to physical relationship amongst their perturbing powers on one another. I think that sometime it will be discovered what that relationship is, and then that it will appear that some term which remains negligible in the interior planets has risen to importance when we get to Neptune so as to disturb his times."

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shape which gravity by itself can condition; but the intensity of which being less in consideration of the smaller bulk of the asteroids, might entail [if this were the sole cause of accretion] an internal structure less closely packed than in the instance of our earth*. It appears, however, that whatever the existing structure of the asteroids, this structure must at least be able to hold together against the existent value of tidal action on these bodies.

September 2, 1906.

LXV. *Simultaneous Vibration-Curves of String and Air photographically obtained from a Monochord.* By Prof. E. H. Barton, *D.Sc., F.R.S.E., and* J. Penzer †.

[Plate XIX.]

SOME time ago it appeared to one of us desirable to investigate the relations between the vibrations of a string and those which consequently occur in the bridge, sound-box, and contained air of the instrument on which the string is mounted. In 1905 the first instalment ‡ of this work appeared and dealt with the motion of the belly and the string of a monochord when bowed. The present paper forms the second contribution to the entire problem, and is concerned with the string and air in the sound-box of the same monochord, both bowing and plucking being used in contrast.

Experimental Arrangements.

The string's motion is here recorded by the same method as in the former paper, and is shown in the positive prints or their reproductions by a black line on a white ground, the focussed shadow of the string rising and falling periodically on the lower portion of the photographic plate, as the plate itself shoots uniformly along the horizontal.

The motion of the air in and out of the sound-box was

* In 'Science Abstracts' (August 25, 1906, p. 394) is a communication headed "Flow of Concrete under Pressure," abstracted from 'Eng. News,' Nov. 2, 1905, in which an experiment by I. H. Woolson is described, where a tube of steel containing set concrete (four inches in diameter) widened to a diameter of five inches under pressure applied to the concrete. It is added that, "On sawing the tube, the concrete is found to have 'flowed' like a plastic material and to have taken the exact shape of the distorted tube. Reference is made to F. D. Adams, who has shown that marble will flow in this way." (Amer. Geological Soc. 1901.)

† Communicated by the Authors.

‡ Barton & Garrett, Phil. Mag. July 1905, pp. 149–157.

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recorded by placing a thin animal membrane over one of the centre holes of the instrument, which had three on each side. In the centre of this membrane was attached by shellac and a strip of membrane one end of an aluminium stalk, whose other end was in like manner attached to a small plane mirror by Hilger. This mirror rocked upon a horizontal axis, being mounted on a sewing-needle clipped to small brass brackets fastened on the vertical face of the sound-box. Now the mirror received, through a pin-hole in a sheet of tinfoil, the light from an electric arc-lamp, which, after reflexion and passing through a lens, was focussed upon the upper part of the sensitive plate about 6 feet distant.

Thus as the membrane moved, owing to the aerial motions responsive to the string's vibrations, the mirror rocked on its axis and so caused the spot of light to rise and fall on the photographic plate. The composition of this vertical periodic motion of the spot with the uniform horizontal motion of the plate gave automatically the displacement and time-curve sought, which in the positive prints and reproductions appears as a white line on a black ground.

The attachment of the aluminium stalk to the mirror was at less than 1 mm. from its axis, the absolute magnification of the diaphragm's motion working out to about 4200 for the actual negatives ($4\frac{1}{4}$ in. × $3\frac{1}{4}$ in.). The magnification of the string's motion is about 3 times only. Hence on any reproduction the relative magnification of diaphragm's and string's motions remains at the same constant value of 1400 times nearly.

The "string" used is of steel 0·92 mm. diameter, and the distance between the bridges is 100 cm.

On the positive prints the beginning of the time is at the left, the string's motion is represented inverted by its shadow owing to the presence of the lens which focussed the shadow on the plate. In the white wavy line which records the motion of the diaphragm a crest indicates a motion of the air outwards from the interior of the sound-box and *vice versa*.

Results.

The results obtained are illustrated by the 39 figures on Plate XIX., each figure containing two curves showing the simultaneous motion of string and air.

All through, the string was illuminated at points between 47 and 50 cm. from one end, usually at or near 49 cm. The frequency of the string for the figures 1–10 was 128 per sec.

Figure 1 is from the first exposure made under the present conditions, the string was bowed at one-tenth its length.

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Figures 2–5 show the effects when bowing at 1/5, 1/14, 1/14, and 1/5 respectively, thus affording a test of the uniform working of the apparatus. In fig. 2 the bowing was bad as seen by the crinkles in the string's record.

Figures 6–9 form a series in which the string was plucked throughout, but at 1/14, 1/10, 1/5, and 1/7 respectively. These allow of comparisons with the former cases of bowing and with fig. 10 of the previous paper *, which showed the motion of the sound-box when the string was plucked at 1/7.

Figures 10–13 form a set all bowed at 1/14, but with pitches ascending by about 1/3 of a tone, the frequencies being 128, 134, 139, and 145 per second respectively. The curves for the air exhibit an instructive and gradual change through this series.

The above 13 figures constitute the first column on the Plate.

Columns 2 and 3 should be viewed side by side as the pitches then correspond, but the figure in any line in col. 2 shows the effects of bowing at 1/10 for a particular pitch, while the figure on the same line in col. 3 shows the effect of plucking at 1/10 obtained without altering the pitch. Of course the bowing and plucking were made near opposite ends of the string so that the resin would not be greased by the thumb and finger.

In each of the cols. 2 and 3 the pitches vary throughout an octave and constitute the tempered chromatic scale. The actual frequencies are given between the columns on the Plate, and are the same as those used in the former paper.

Thus these two columns of the present paper show the variation due to pitch changing, and allow the effects of bowing and plucking to be contrasted side by side. Further, by comparing these with the corresponding figures of the former paper, the distinction between the motions of the belly and those of the diaphragm can be discerned.

It is perhaps too early yet to make generalizations on this distinction, but so far the tendency seems to be towards greater simplicity (fewer Fourier components) in the motion of the sound-box than in that of the air, the latter being apparently able to follow more closely the highly complicated motion of the string which, when well bowed, may be regarded as a Fourier series to infinity.

University College, Nottingham,
July 26, 1906.

* Phil. Mag. July 1905.

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LXVI. *A Kinematical Explanation of Groups of Spectrum Lines with Constant Frequency-Difference.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,

IN the November number of your Journal, Prof. R. W. Wood has published an investigation on the fluorescent and magnetic-rotation spectra of sodium, in the course of which he has demonstrated the existence of five or more groups of nearly equidistant lines in the blue-green region of those spectra. During an investigation on the radiation from systems of electrons I have obtained a result, which I think has sufficient bearing on Prof. Wood's discovery to be of interest to the readers of his paper.

Consider a circle of n equidistant electrons rotating with uniform angular velocity ω in a controlling field of such a kind as to ensure stability. The electrons can execute vibrations about their mean positions in steady motion, in which the displacement of the ith electron is of the type $A \exp. \iota\left(qt - k\frac{2\pi i}{n}\right)$, damping being neglected. The kinematics of these vibrations is given by Maxwell, 'Saturn's Rings,' Part ii. §§ 10–13.

It may be shown that the electromagnetic forces due to the vibration at a point (r, θ, ϕ) are the sums of terms of the type

$$BA \exp. \iota\{[q+(k+sn)\omega]t-(k+sn)\phi\},$$

where s is an integer. At a great distance B is very small for large values of $k+sn$; near the ring it is much larger.

Consider a second ring (n', ω') coaxal with the first; for the ith electron of this ring we have $\phi=\omega' t+\frac{2\pi i}{n'}$. Hence the electromagnetic forces relative to the second ring, and therefore also the disturbances in it, are of the type

$$CBA \exp. \iota\left\{[q+(k+sn)(\omega-\omega')]t-(k+sn)\frac{2\pi i}{n'}\right\}.$$

Just as in the case of the first ring, these disturbances of the second ring give rise to electromagnetic forces of the type

$$B'CBA \exp. \iota\{[q+(k+sn)(\omega-\omega')+(k+sn+s'n')\omega']t \\ -(k+sn+s'n')\phi\},$$

where s' is an integer; at a great distance from the ring B′ is very small for large values of $k+sn+s'n'$.

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In a paper shortly to be published I have shown that the only vibrations of this type which can give rise to spectrum-lines of sufficient intensity to be observed, are those for which $k+sn+s'n'$ has one of the values $0, \pm 1$. Of these, the strongest are given by small values of $k+sn$. We must therefore find integral values of s, s' which shall satisfy the equations

$$k+sn+s'n'=0, \quad \pm 1.$$

Convert $\frac{n'}{n}$ into a continued fraction, and let the last convergent be $\frac{P}{Q}$, so that $Pn-Qn'=\pm 1$. We find

$$s=\mp kP+jkn', \qquad s'=\pm kQ-jkn \quad (j \text{ an integer})$$

for the first case, and similar expressions for the other two.

The corresponding frequency is for the first case

$$q\mp kQn'(\omega-\omega')+jknn'(\omega-\omega').$$

In other words, the vibrations emitted in consequence of the interaction of the two rings form groups with the constant frequency-difference $knn'(\omega-\omega')$. The strongest of these, corresponding to $k=0$, $k=\pm 1$, ... have the differences 0, $\pm nn'(\omega-\omega')$, ... The application is obvious to Prof. Wood's result, as well as to the groups of lines of constant frequency-difference found by Kayser and Runge in the spectra of Sn, Pb, As, Sb, Bi. At the same time it does not lead to an explanation of these results; for in the first place it has still to be shown that the factors C, B, A, which enter into the amplitude of the vibration, can, by a suitable choice of the structures and relative positions of the rings, be given values which will account for the observed intensities of the vibrations; and secondly, the lines observed by Prof. Wood show frequency-differences which vary somewhat with the frequency. The investigation of the values of C, B, A will prove a very difficult one, and it is impossible to foresee the result. The deviations from constant frequency-difference may possibly be due to the fact that the controlling field of the rings is altered by the neighbourhood of molecules of sodium other than that to which the rings belong.

Yours faithfully,

G. A. SCHOTT.

University College of Wales,
Aberystwyth,
Nov. 6, 1906.

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LXVII. *On the "Swan" Spectrum.* By W. Marshall Watts, *D.Sc., and* H. M. Wilkinson, *M.A.**

PROBABLY no spectra have been so often observed and so much discussed as those obtainable from carbon and carbon compounds; and, in particular, the chemical origin of the Swan spectrum, or that observed at the blue base of the flame of a Bunsen burner, has been a subject of controversy for the last fifty years. This spectrum, first noticed by Wollaston in 1802, was examined by Swan in 1857, and by him attributed to a hydrocarbon. Attfield in 1862 came to the conclusion that the spectrum was that of the *element* carbon, since it was common to compounds of carbon with hydrogen, nitrogen, oxygen, and sulphur.

This conclusion was combated by several writers. Morren, in 1865, undertook a series of experiments to prove that Attfield was wrong, but in the end became convinced that he was right. Morren particularly describes the brilliant spectrum produced by burning cyanogen with oxygen. Dibbits arrived at the same conclusion as to the origin of the spectrum. He reasons that carbon exists in combination with hydrogen in the gas which is burnt, and after combustion that carbon is in combination with oxygen; it is possible, therefore, that in some intermediate stage it existed as uncombined carbon.

Plücker and Hittorf, Van der Willigen and Wüllner all arrived at the same conclusion.

Thus, from 1862 till 1875 this view of the origin of the Swan spectrum was all but universally held, but in the latter year Thalén published a paper in which it was contended that the spectrum was due to acetylene, and this view was supported by Profs. Liveing and Dewar. They dwelt much upon the difficulty of getting rid of all hydrogen from gases like cyanogen, carbonic oxide, and carbon tetrachloride, from all of which the Swan spectrum can be obtained; or from the carbon electrodes used in the arc, in the spectrum of which the Swan bands were always seen, whatever the atmosphere in which the arc was taken.

This view of the origin of the Swan spectrum which Profs. Liveing and Dewar held so strongly, they abandoned later on as the result of their own further experiments.

It seems, then, that the theory which ascribes the Swan spectrum to elementary carbon was finally adopted by nearly every experimenter—whether chemist or physicist—who

* Communicated by the Authors.

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entered upon a study of the subject, and the question was supposed to have been set at rest.

But in 1900, the late Sir G. Stokes suggested that the Swan spectrum belonged to carbonic oxide, and this view was adopted by Prof. Smithells, who in a paper in the Phil. Mag. for April 1901 has sought to establish it by a critical study of the spectra of carbon compounds generally and of flame spectra in particular.

He sets aside the evidence of many observers that the Swan spectrum is obtainable by the electric discharge in hydrocarbons, carbon chloride, and carbon sulphide, by the supposition that all these different materials contained oxygen as an impurity. He points to the difficulty of removing films of air or moisture from glass, the occlusion of gases by electrodes, the oxidizing character of glass itself; and there is indeed no doubt that the evidence obtained from exhausted tubes must be received with much caution; but the case would seem to be different when the spark is taken in gases at ordinary pressure, and the results will be still less open to objection on account of possible contamination by impurities when the spark is taken in the *liquid* carbon compound itself. We beg to offer the results of some experiments we have made on the spectra of the spark in various liquids containing carbon and not oxygen, as far as it lay in our power to secure the absence of this element, as a contribution towards the determination of the chemical origin of the "Swan" spectrum.

By the "Swan" spectrum we mean the band-spectrum as obtained at the base of a Bunsen flame, and more brightly by the combustion of hydrocarbons such as olefiant gas with oxygen in a "mixed" blowpipe jet. In the visible portion of the spectrum we have five groups of bands each made up of several "edges" sharply defined on the red side and fading away on the more refrangible side. The wave-lengths of these edges are as follows:—

Group α	6188·1
	6120·0
	6052·0
	5999·0
	5955·0
Group γ	5635·4
	5585·5
	5540·9
	5500·0
	5470·0

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Group δ	5165·2 5129·4 5101·0 5081·9
Group ϵ	4737·2 4715·3 4697·6 4684·0 4677·0

and a group f of somewhat different structure, viz., a band fading both ways, a sharply defined detached line, and a band fading away towards the blue :—

Group f	4381·9 to 4356·9	band.
	4334·4	"detached line."
	4315·0 to 4234·0	band.

The same bands are obtained by taking the spark in certain gases at atmospheric pressure, viz., carbonic oxide, olefiant gas, cyanogen. In this last gas (and in the others if air is present) we observe besides the groups given above, certain bands no doubt due to cyanogen (or at least dependent upon the presence of nitrogen). These are :—

Group ζ	4600 4574 4550 4532 4515 4505 4500
Group θ	4216·1 4197·2 4181·5 4177·7 4165·4 4158·2

We have experimented with the following liquids, in each case of as great purity as we could procure :—Chloroform, benzene, naphthalene, aniline, ethylene dibromide, ethyl bromide. In most cases the liquid proved highly insulating and only a minute spark could be obtained, only a fraction of a millimetre in length. In all these liquids carbon is

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rapidly deposited, so that there is only a very short time in which observation of the spectrum is possible. For this reason, it was arranged in some cases to have a continuous flow of fresh liquid so as to carry the separated carbon away as rapidly as possible.

We observe the "Swan" spectrum also in liquids containing oxygen, such as ethyl and methyl alcohols, and glacial acetic acid; and in these liquids the decomposition takes place without separation of carbon (or only slight blackening) but with evolution of a combustible gas. In the liquids not containing oxygen there is rapid separation of carbon, so rapid in the case of naphthalene, benzol, &c., that the observation of the spectrum is only possible for a few seconds.

We describe more at length our observations with chloroform.

The chloroform employed was the purest we could obtain: it was allowed to stand for some weeks in contact with freshly-fused zinc chloride and then redistilled. The distillate was allowed to stand in contact with bright sodium and again distilled, and the process repeated several times. Pure chloroform does not appear to act upon bright sodium.

When subjected to the action of the spark in the liquid, chloroform is rapidly decomposed with separation of much carbon and evolution of torrents of hydrochloric acid. It gives the "Swan" spectrum superposed upon a continuous spectrum, due, no doubt, to the solid carbon deposited. A rough quantitative determination was made in order to obtain some light upon the nature of the decomposition of chloroform by the spark. For this purpose the chloroform was sparked in a bent piece of combustion-tube, and the gas evolved passed through water in absorption-bulbs. The deposited carbon was filtered off, in the same tube, through asbestos, and burnt in a current of oxygen: the weight of carbon dioxide obtained was 0·2196. The hydrochloric acid—precipitated and weighed as silver chloride—amounted to 0·5472. The chloroform filtered from the carbon gave white crystals on standing which seemed to be carbon sesquichloride, and the decomposition thus appears to take place according to the equation $3CHCl_3 = C_2Cl_6 + C + 3HCl$, which would require 0·5449 of HCl for 0·2196 carbon dioxide.

Carbon tetrachloride also gives the Swan spectrum and evolves chlorine with very rapid separation of carbon. The liquid employed had been repeatedly redistilled from bright sodium.

Ethyl bromide gave the Swan spectrum brightly, the groups γ, δ, and ϵ were observed.

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Ethylene dibromide gives a brilliant Swan spectrum in which the groups α, γ, δ, and ϵ were observed. Ethylene dibromide melts at 9° C. The specimen employed was frozen and allowed to melt slowly and drain, the last half to melt being taken for the experiment, and this process repeated several times. After standing in contact with zinc chloride the liquid was distilled over into a discharge-tube connected with an air-pump, and boiled in the tube before observing the spectrum. The lines of the spectrum were quite sharp when the pressure in the tube was small, but became less sharp when the air was readmitted.

Aniline and pyridine give the Swan spectrum brilliantly.

In liquid naphthalene the spark gives the Swan spectrum but the blackening is extremely rapid. The same result is obtained with benzol. We have repeated an experiment described by one of us (Phil. Mag. [4] xlviii. p. 456). Benzol was distilled over sodium into a vacuum-tube containing bright sodium at one end, and at the other a fragment of potassium chlorate wrapped up in platinum-foil. This was connected to the pump, the benzol made to boil, and the tube sealed off when exhausted. This tube gave the spectrum formerly described as C (2), consisting of the following "edges":—

6079·0
5610·3
5198·3
4835·4
4511·0
4393·2
4123·4

On heating the sodium this spectrum disappeared giving place to the Swan spectrum, but on heating the chlorate so as to evolve a little oxygen the original spectrum reappeared.

LXVIII. *The Intensification of Glass Diffraction Gratings and the Diffraction Process of Colour Photography.* By R. W. Wood *.

IN the course of some experiments made last year by my assistant Mr. H. E. Ives on the diffraction process of colour photography, much difficulty was found in ruling on glass large gratings of sufficient brilliancy. Diamond points could be found without trouble which gave very brilliant rulings, but they almost invariably broke down after ruling an inch or so of the surface. Gratings which are to be

* Communicated by the Author.

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used for the purpose of reproducing objects in their natural colours must be of uniform intensity over their whole extent; that is, they must appear uniformly illuminated when the pupil of the eye is brought into coincidence with the focussed spectrum. Very often a point would rule a millimetre or two of great brilliancy and then break down, ruling the remainder of the surface (10 cms.) with perfect uniformity, but with lines so fine that the spectra were scarcely visible when it was held up to a gas-flame. Cross-sections of these gratings, examined under the microscope, showed that the lines were usually deep enough, but that they lacked sufficient breadth.

Remembering Lord Rayleigh's interesting work on the development of latent scratches on glass by etching with hydrofluoric acid, it occurred to me that treatment with dilute acid might improve the gratings. On referring to Lord Rayleigh's paper I found that he had thrown out this same suggestion, but without trying it apparently. An old grating, which, when placed on the viewing-stand for the diffraction colour photographs, showed scarcely any luminosity at all, was partially submerged in the acid (about 1 part to 100 of water) for half a minute, washed, dried, and put back on the viewing-stand. The portion treated fairly blazed with red, green, or blue light, according to the position of the eye. Further experimenting showed that nearly all of the bad gratings could be made first class by a bath of a minute or two in the acid. The back surface was painted with paraffine to prevent the development of latent scratches by the acid.

Lord Rayleigh has shown that the action of the acid is remarkably uniform, it being an easy matter to remove a layer of glass one or more wave-lengths in thickness. Furthermore the surface remains optical, even after long immersion, which makes it appear probable that the performance of a glass grating would be quite as good after etching as before. In some cases I have succeeded, by starting with a fairly brilliant grating, in producing a grating which almost completely suppressed the central image, did so in fact for a limited region of the spectrum. When held up to a distant window, backed by the sky, the ruled surface appears like heavily smoked glass. The gratings are "blacker" in fact than any that I have ever been able to produce on bichromated gelatine, or by converting "silver on glass" gratings into silver iodide, the method used by Quincke in his study of gratings giving coloured central images. It is important to determine exactly how much

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etching is good for a given grating. The intensity appears to rise to a maximum, and then remain constant for some time, further etching having but little effect; if carried on for half an hour or so the grating is usually ruined.

The following scheme has proved very reliable. One corner of the ruled surface is dipped into the acid to a depth of a few millimetres, for perhaps 15 seconds, washed, dried, and examined by holding it at arm's length up to a distant gas-flame. On bringing any given spectrum into the etched corner, it is easy to see at once whether matters have been improved or not. If there is a decided improvement, the whole grating is immersed for 15 seconds and then examined again. Usually the corner first etched will be found still further improved. This process is to be repeated until the rest of the grating catches up to the corner in intensity—in other words, as long as the corner is brighter than the rest of the grating we can be sure that a further etching will improve things.

I believe that all glass gratings can be vastly improved in this way. By testing the grating with a spectroscope after each treatment, the experimenter can satisfy himself that he is not injuring its optical qualities. Personally I have never made a test of this kind, as the gratings did not need to be very perfect for the work for which they were intended. We found no indication, however, that they had lost in flatness of surface, as they copied as well after the etching as before, and it is well known that a grating cannot be uniformly copied by contact printing unless both the grating and plate are plane. The greater part of the gratings experimented with had rulings varying from 3000 to 5000 lines to the inch, but it was found that 14,000 line gratings could be greatly intensified in the same way.

A few experiments were made on speculum gratings with various acids, without much hope of success. These were complete failures. Probably the crystalline structure would etch too unevenly, even if a non-tarnishing etching fluid could be found. I shall make a few trials with electrolysis, but am very doubtful of the results. It is much easier, however, to get brilliant gratings on speculum metal, and in many cases it is doubtful if etching would improve them, as they show very dark central images.

The effect of the etching is to widen the groove, as is clearly shown by microscopic examination of sections cut before and after etching.

Inasmuch as the diffraction process of colour photography was first published in this Magazine, a few words about the

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recent improvements which have been made may not be out of place. Thanks to the skill and ingenuity of Mr. F. E. Ives, and his son Mr. H. E. Ives, the finished pictures are now quite as good as the pictures shown by the Ives Kromoscope: in fact it seems to me that they are perhaps a little better, if that be possible. Three gratings are used as before, but in the finished picture the mixed colours are made by juxtaposed gratings instead of by superposed gratings, as originally. This is a great advantage, for, as I showed some years ago, superposed gratings show spectra of spectra which may be very troublesome. A full description of the details of the process, which has been most successfully worked out by Mr. Ives, will be found elsewhere *. The pictures are as "structureless" as those shown in the kromoscope, and can be made much brighter if desired. The number of linear elements has now been increased to 400 to the inch, which makes them absolutely invisible. Mr. F. E. Ives has designed a simple and compact viewing instrument, which is placed before a well-lighted window, instead of being pointed at a lamp in a dark room. By the very ingenious device of employing four parallel slits suitably disposed, he utilizes the light of the first two orders of spectra on each side. Thus finished pictures are duplicated *ad infinitum* by a purely mechanical process.

LXIX. *On the Electric Radiation from Bent Antennæ.* By J. A. Fleming, *M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London* †.

[Plates XVII. & XVIII.]

IN March 1906, Mr. Marconi described in a Paper presented to the Royal Society his inventions and interesting investigations with wireless telegraph antennæ consisting partly of vertical and partly of horizontal wires for the purposes of directive electric-wave telegraphy. At the conclusion he says ‡:—

"I have found the results (*i. e.*, the unsymmetrical radiation and reception) to be well marked for wave-lengths of 150 metres and over, but have not been able to obtain as well defined results when employing much shorter waves, the effect following some law I have not had time to investigate."

At the same meeting the present writer gave a brief

* H. E. Ives, Phys. Rev. July 1906.

† Communicated by the Physical Society: read November 23rd, 1906.

‡ See G. Marconi, "On Methods whereby the Radiation of Electric Waves may be mainly confined to certain Directions, and whereby the receptivity of a Receiver may be restricted to Electric Waves emanating from certain Directions," Proc. Roy. Soc. Lond. A. vol. lxxvii. p. 413, 1906.

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explanation of the nature of the electric radiation from such a bent antenna, and subsequently expanded it into a more detailed mathematical theory *.

In this last Paper it was analytically proved (as previously discovered experimentally by Mr. Marconi) that a bent antenna radiates unsymmetrically, that is, not equally in all directions but most energetically in its own plane and in a direction opposite to that in which the free ends point. The theory showed that this inequality in fore and aft radiation depends upon the ratio of the lengths of the vertical and horizontal portions of the antenna and upon the ratio of the wave-length employed to the distance between the transmitter and receiver. Hence it appeared clear that with a suitable bent antenna the effect should be obtained just as well at comparatively short distances when using relatively small wave-lengths as at long distances with long wave-lengths.

During the past summer numerous experiments and measurements have been made for me by Mr. G. B. Dyke in the quadrangle of University College, London, employing short bent antennæ 10 or 20 feet in length receiving at distances varying from a few yards up to 140 feet or so, with the object of confirming the general theory of the action given by the writer.

It may be convenient in the first place to explain generally and briefly the reasons for the unsymmetrical radiation of such a bent oscillator.

Suppose a pair of rods terminating in spark-balls to be placed with the balls in apposition and the rods in one straight line. It is obvious by reason of symmetry that this radiator must radiate equally in all directions which have the line of the rods for an axis. If, however, the rods each have a bend made in them so as to make an oscillator resembling

Fig. 1.

in shape three sides of a rectangle (see fig. 1) then the symmetry of radiation is destroyed.

* See J. A. Fleming, "A Note on the Theory of Directive Antennæ or Unsymmetrical Hertzian Oscillators," Proc. Roy. Soc. A. vol. lxxviii. p. 1 (1906).

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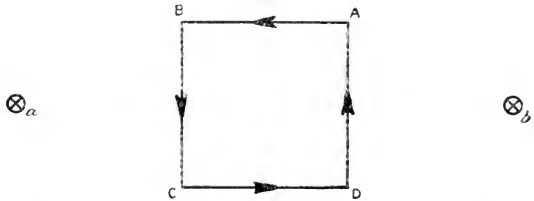
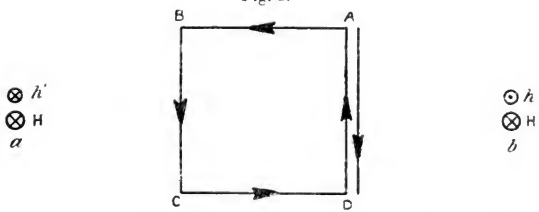
We may consider the oscillator as constructed in the following manner. Imagine a rectangular circuit ABCD of wire (see fig. 2) placed perpendicularly to the earth's surface,

Fig. 2.



and let it be traversed by a high frequency oscillation. Then if a horizontal line is drawn through the centre of the rectangle and two points *b* and *a* chosen at equal distances on the right and left side, the magnetic force at those points will be equal and normal to the plane of the rectangle. If at any instant the current is flowing counter clockwise round the circuit, the magnetic forces H at these selected points will be directed away from the spectator *. Next suppose a wire to be placed in contiguity to one side of the rectangle and to be traversed by an equal current the same at all points of its

Fig. 3.



length and always in opposite phase to that in the side of the rectangle adjacent to it (see fig. 3).

* It is convenient sometimes to represent an end-on view of a tube or line of electric or magnetic force. We then must indicate in some way the direction of the force whether to or from the spectator. This may be done by representing the section of the tube by a small circle and putting a dot in the circle if the force is towards the reader, and a cross on it if it is away from the reader, as is done to denote current directions in similar diagrams in other cases.

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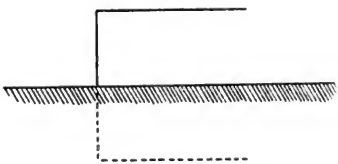
The field due to this open circuit current at the two selected points a and b will be towards the spectator at the right-hand point b and away from it at the left-hand point a. Hence if h and h' are the magnetic forces due to the open circuit current at the points in question, the resultant fields are $H-h$ at the right-hand point and $H+h'$ at the left. But since the open circuit and the adjacent side of the rectangle are traversed at any instant by equal and opposite currents, we may consider them both annihilated, and we have as a consequence that the magnetic field due to a bent oscillator similar to that in fig. 1 at two symmetrical and equidistant points on its axis is greater at that side away from which the free ends point than it is at the other. If we suppose a doubly bent oscillator of the above kind to be half buried in the earth, then we have a singly bent earthed oscillator of the kind used by Mr. Marconi (see fig. 4) *. The antennæ used in

Fig. 4.



the experiments here described consisted of a couple of No. 16 bare copper wires loosely twisted together and from 10 to 20 feet in length. On the grass of the Courtyard some large sheets of zinc were laid down to form earth-plates. At one of these posts used as a transmitting station, a bent radiating antenna was constructed by connecting one of a pair of spark-balls to the earth-plate and the other to a length of the above said copper wire which was so arranged and upheld by insulators that any fraction of its length could be placed vertical and the remainder horizontal as in fig. 4. The free insulated farther end of the horizontal portion generally carried a plate of zinc 18 inches square which served as an additional capacity. The receiving antenna consisted of a vertical copper wire upheld by insulators from a light bamboo rod.

* In the discussion on Mr. Marconi's paper at the Royal Society on March 22nd, 1906, Prof. J. Larmor, Sec.R.S., pointed out that a bent oscillator as in fig. 4 is equivalent to a magnetic oscillator *plus* an electric oscillator.

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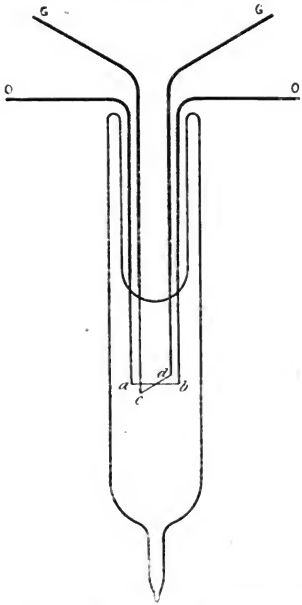
The wave detector used was of the thermoelectric type, and made as follows:—

A glass test-tube had four platinum wires sealed through the bottom (see fig. 5). These were soldered to four copper strips spaced apart on the inside of the test-tube. On the external side at the base one pair of platinum wires were connected by a fine constantan wire *a b* about 1 cm. long and

Fig. 5.



Tellurium-Bismuth vacuum thermal detector for electric oscillations. *ab*. Constantan wire. *cd*. Thermojunction. GG. Galvanometer terminals. OO. Antenna and earth terminals.

0·02 mm. in diameter. The other pair of platinum wires were connected with small pieces of bismuth wire and tellurium which were soldered to each other and to the centre of the constantan wire, thus making a thermoelectric junction *cd* with one junction on the constantan wire. This test-tube was then sealed into another vessel which was highly exhausted,

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thus providing a tellurium-bismuth thermojunction in vacuum, one junction in contact with a fine resistance wire which could be heated by electric oscillations sent through it (see fig. 5).

The thermojunction was connected by long wires with a sensitive movable-coil galvanometer placed in a room about 40 feet from the receiving antenna. The fine wire circuit was included in the receiving antenna between the base and the earth-plate. This thermal receiver could be calibrated by passing a continuous current through the heating wire, and in the case of the instrument mostly used in these experiments the sensitiveness was such that 1 milliampere passed through the heating wire produced a deflexion of 1 cm. of the spot of light on the galvanometer scale.

Observation showed that the deflexion of the spot of light on the scale was almost exactly proportional to the square of the current through the heating wire. Hence an oscillatory current, the root-mean-square value of which was as small as 300 microamperes, could be read by the spot deflexion. This sensitiveness cannot be called great, but it was sufficient for the purpose, and such resistance-wire combined with a vacuum tellurium-bismuth couple is not only more easily calibrated but less troublesome to use than a bolometer bridge arrangement.

No attempt was made to tune the receiving antenna to the transmitting antenna with any great exactness. The actual experiment consisted in placing the bent transmitting antenna at a certain distance from the fixed vertical receiver antenna and then swivelling round the horizontal part of the transmitter into various directions and taking readings of the current in the receiving antenna in each case. To avoid errors due to variation in the sending antenna current, a comparison measurement was always made in each case between the current in the receiver with the free end of the bent transmitting antenna pointing directly away from the receiver and the current when the transmitting antenna had any other assigned azimuth.

In the first set of experiments the sending antenna consisted of a double copper wire 20 feet in length having at the free end a terminal capacity plate, the wire being bent over at various heights from the ground or earthed end, so as to make a bent antenna with 1 foot vertical and 19 feet horizontal, 2 feet vertical and 18 feet horizontal, 3 feet vertical and 17 feet horizontal, 4 feet vertical 16 feet horizontal, 5 feet vertical 15 feet horizontal, &c. &c.; and in each case observations were taken of the current in a receiving antenna at the same distance but at various angular positions round the transmitter. The currents produced in the receiving antenna, as calculated from the galvanometer deflexion, were plotted as

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radii vectores of a polar curve; and these radii were therefore proportional to the magnetic and electric fields of the radiator taken at equal distances round its centre, but in various azimuths. The reduced observations are given in Table I., and from them the polar curves (see Plate XVII. figs. 1 to 5) have been plotted. These show the results for five such experiments when the transmitting antenna had its bend made at various heights as indicated in the diagrams.

TABLE I.

Radiation from Bent Earthed Transmitting Antenna 20 feet in total length.
Receiving antenna vertical and 20 feet high.
Distance between receiver and transmitter, 138 feet.

Length in feet of Vertical part of Transmitter ...	5	4	3	2	1
Length in feet of Horizontal part of Transmitter.	15	16	17	18	19
Radiated wave-length in feet...	100	92 (?)	105	106	110
Azimuth of horizontal part of transmitter.	Current in the Receiving Antenna in arbitrary units.				
°					
0	100	100	100	100	100
15	98	97	94	92	93
30	92	85	96	83	75
45	82	79	79	77	67
60	78	74	70	71	58
75	77	67	59	56	45
90	72	66	57	52	48
105	71	65	57	46	41
120	70	66	62	53	49
135	72	64	60	54	48
150	73	80	58	67	59
165	70	74	56	69	60
180	82	69	64	63	68
Plate XVII....	Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.	Fig. 5.

The azimuths are reckoned in degrees counting from the position in which the free end of the bent transmitting antenna points directly away from the receiving wire. It will be seen from the curves that the greatest fore and aft inequality is obtained with this particular antenna bent so that 2 feet of it is vertical and 18 feet horizontal, and that at the distance

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chosen, viz., 138 feet, the field in the direction in which the free end points is then about 60 per cent. of that in the opposite direction (see fig. 4, Plate XVII.). The polar diagrams moreover confirm Mr. Marconi's observation and also my mathematical theory that the minimum radiation is not in the direction 90° from the maximum but more nearly 105°, to 110°. It will be seen that the effect of making the horizontal part of the antenna longer is not so much to increase the fore and aft inequality as to squeeze in the sides of the polar diagram and make it narrower.

The wave-lengths emitted in each case were measured with a special form of cymometer consisting of a condenser of known capacity and a variable inductance of calculable magnitude. This cymometer circuit was placed near to the horizontal portion of the transmitting antenna and tuned to it until the current in the cymometer circuit was found to be a maximum, as determined by a thermoelectric detector. The frequency in the antenna thus being known from the constants of the cymometer circuit, the radiated wave-length could be calculated. On account of the capacity plate at the end of the antenna, the emitted wave had a length rather greater than 5 times the total length of the antenna, the exception being the 4 : 16 case in which the wave-length found by several measurements was 92 feet.

It is clear then that when using electric waves of about 100 feet in length and receiving at a distance of 138 feet, or not much greater than one and one-third of a wave-length, the inequality in radiation in different azimuths is well marked, and is greatest for a certain ratio of horizontal to vertical portions of the antenna. This fore and aft inequality moreover, as well as the form of the polar curve, varies with the distance of the receiving antenna as shown by the following experiment. A bent transmitting antenna 10 feet long was used of which 3 feet of the length was vertical and 7 feet horizontal. The receiver was placed successively at 138 feet and 78 feet distance and the polar curves taken. The values of the current in the receiving antenna in different azimuths were as shown in Table II. (p. 596). It will be seen that at the shorter distance the inequality in fore and aft radiation is less evident. These values are plotted in the polar curves in figs. 7 and 8 (Plate XVII.).

In comparing these results with the theory given we must notice that the theory presupposes that the current in the sending antenna or circuit is everywhere the same. The mathematical investigation of the problem becomes immensely difficult unless we make this assumption.

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TABLE II.

Bent transmitting antenna 3 feet vertical, 7 feet horizontal. Straight receiving antenna (vertical) 10 feet high.

Azimuth of horizontal part of Antenna.	Current in Receiving Antenna in arbitrary units.	
	At 78 feet distance.	At 138 feet distance.
°		
0	100	100
15	91	94
30	96	87
45	91	76
60	81	77
75	83	67
90	81	65
105	74	60
120	72	52
135	70	56
150	71	57
165	71	43
180	64	50

On the other hand, in the actual antennæ the current varies from a maximum at the earthed end up to zero at the free end. The effect of this is to make the actual magnetic moment of the bent oscillator less than would be the case on the assumption of a uniform current in it.

Suppose, however, we consider a small closed rectangular circuit to be traversed by an oscillatory current of maximum value I. Through the centre of the rectangle draw a line parallel to one side and produce it both ways. Take points a and b at equal distances r from the centre both on the right and left-hand sides of the rectangle. Let H be the maximum magnetic force at a and b perpendicular to the plane of the rectangle, and E the electric force in the plane of the rectangle and perpendicular to the bisecting line. Then it can be shown that

$$H = \frac{I\delta y\,\delta z}{r^3}\sqrt{m^4r^4 - m^2r^2 + 1},$$

$$E = \frac{I\delta y\,\delta z}{r^3}\sqrt{m^4r^4 + m^2r^2},$$

where δy, δz are the sides of the rectangle and $m = 2\pi/\lambda$, where λ is the wave-length of the radiation.

These formulæ can be arrived at on the principles explained

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in the writer's Paper to the Royal Society ("A Note on the Theory of Directive Antennæ or Unsymmetrical Hertzian Oscillators," Proc. Roy. Soc. Lond., ser. A, vol. lxxviii. p. 1).

If δz be the length of the side perpendicular to the bisecting line on which r is measured, and if we consider the oscillation in this one side only, then on the same principles it can be shown that the maximum magnetic (h) and electric (e) forces at distances r from the oscillation taken in the same directions as H and E are given by

$$h = \frac{I\delta z}{r^2}\sqrt{m^2r^2+1},$$

$$e = \frac{I\delta z}{r^2}\sqrt{m^2r^2-1+\frac{1}{m^2r^2}}.$$

At distances such that mr is large compared with unity, all these forces vary inversely as the distance, and $E=H$ and $e=h$.

Also $$H/h = m\delta y = 2\pi\delta y/\lambda.$$

Hence when using the bent earthed oscillator as in the experiments here described, it is clear that the field at a distance r from the centre, not small compared with the wave-length, taken in the direction in which the free end of the oscillator points, is to the field in the opposite direction at an equal distance in the ratio of $h-H$ to $h+H$, that is in the ratio of $\lambda-2\pi\delta y$ to $\lambda+2\pi\delta y$.

Under the assumption here made as to the equality of the current at all points of the oscillator, the quantity δy is the length of the horizontal part of the oscillator. In comparing the theory with the practical results, we must notice that the value to be assigned to δy will always be very much less than the actual length of the horizontal part, because the actual magnetic moment of the bent oscillator is always much less than the product of the lengths of its vertical and horizontal portions and the maximum current at the earthed end. A consideration of all the circumstances shows that the δy in the formula may, as a first approximation, be taken to be half the length of the horizontal portion of the antenna, whilst the wave-length in the present case was about five times the total length. Accordingly, in the case of the antenna 10 feet long bent over so that 7 feet were horizontal, the theory would predict that the ratio of the two magnetic forces H and h should be in the ratio of $2\pi\times3\frac{1}{2}$ to 5×10 or 22 to 50, and the ratio of the fore and aft fields or radiation

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in the ratio of 40 to 100. Experiment shows this to be approximately the case.

In a further set of experiments a number of measurements were made by erecting a vertical antenna having a lateral wire terminated in a capacity plate, attached to some point on its height (see figs. 9 to 13, Pl. XVII.). In all cases such a transmitting antenna was found to radiate more equally in all azimuths than if the vertical part above the lateral projection was removed. Hence the polar curves are made more nearly circular by adding this vertical part. This showed that strengthening the electric moment of such a bent antenna makes it a less unsymmetrical radiator, and that what is required to improve the simple bent antenna and make it radiate more unequally in a fore and aft direction is an increase in the magnetic moment.

To investigate this point a little more, the whole of the observations taken with the 20-foot antenna bent over at various heights were plotted to the same scale in a family of polar diagrams together with the results obtained from the same antenna in a vertical position (see Plate XVIII.). Thus if the 20-foot antenna is placed in a vertical position it radiates equally in all azimuths, and the polar curve obtained by plotting its field at a constant distance is a circle. If the antenna is bent over, say, 5 feet from the top so that 15 feet are vertical and 5 feet horizontal, the radiation is still nearly equal in all directions, but the field is not so great as when the antenna is wholly vertical. Hence the polar curve is smaller than the above circle and not quite circular. If it is bent over at the height of 10 feet, then the polar curve is smaller still, also decidedly non-circular and has little depressions in it. If bent over at 15 feet so that 5 feet are vertical, the polar curve is still less circular. Hence as the antenna is bent over so that less and less of it is vertical and more and more horizontal, the radiation in various azimuths becomes more and more unequal, and has in an increased degree a decidedly minimum value at about 105° reckoning 180° in the direction in which the free top end points. The family of polar curves so obtained and depicted in Plate XVIII. show very well the gradual diminution in the current in the 20-foot receiving antenna as the 20-foot bent-over transmitting antenna is swivelled round, and they show the gradual increase in the asymmetry of radiation in various azimuths, for constant distance between radiator and receiver.

The curves here given, however, never reach the well defined figure-of-8 form of those given by Mr. Marconi, since he employed a transmitting antenna the vertical part

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of which was 1·5 metres and the horizontal part 60 metres long. In other words, his ratio of horizontal to vertical was as much as 40 : 1, and the greatest ratio in my experiments was only 19 : 1. This entirely confirms Mr. Marconi's remark, *loc. cit.* that "in order that the effects should be well marked it is necessary that the length of the horizontal conductors should be great in proportion to their height above the ground." This is only another way of saying that the magnetic moment of the oscillator must not be small compared with the electric moment. This is achieved by making the length of the horizontal portion of the antenna large compared with that of the vertical portion.

One point of great interest in the family of polar curves above mentioned is that the minimum radius always lies near to 105°, reckoned from a zero opposite to the direction in which the free end of the antenna points. In the mathematical theory given by the writer (Proc. Roy. Soc. vol. lxxviii. p. 7, 1906) it is shown that for a doubly bent insulated antenna azimuthal angle (θ) of minimum force is given by the expression

$$\cos\theta = \frac{2\phi v}{\mathrm{M}} \cdot \frac{1}{mr},$$

where ϕ is the electric moment of the oscillator, M the magnetic moment, v the velocity of radiation, $m = 2\pi/\lambda$, and r is the distance between radiator and receiver.

Now $\qquad \phi = \mathrm{Q}\delta z \qquad$ and $\qquad \mathrm{M} = \mathrm{I}\delta y\,\delta z = \mathrm{Q}n\,\delta y\,\delta z,$

where Q is the maximum end charge of the oscillator and $n = 2\pi/\mathrm{T}$.

But $v = n/m$. Hence we have

$$2\,\frac{\phi v}{\mathrm{M}} = \frac{2}{m\delta y},$$

and

$$\cos\theta = \frac{2}{m^2 r\,\delta y} = \frac{\lambda^2}{2\pi^2 r\,\delta y} = \frac{1}{20}\,\frac{\lambda}{r}\cdot\frac{\lambda}{\delta y}.$$

Mr. Macdonald has shown (see 'Electric Waves') that in the case of a linear oscillator the emitted wave-length is 2·5 times the total length. Hence for our case

$$\lambda = 2{\cdot}5(2\delta y + \delta z) = 7{\cdot}5\delta y.$$

Therefore we have for the bent oscillator with three equal branches

$$\cos\theta = \frac{3}{8}\,\frac{\lambda}{r} = 0{\cdot}375\,\lambda/r.$$

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The experiments here detailed were made at distances equal to about one and one-third of a wave-length between the radiator and receiver. Hence for this case $\lambda/r = 0{\cdot}72$, and therefore $\cos\theta = 0{\cdot}27$ or $\theta = 74^\circ\ 20'$ or $\theta = 105^\circ\ 40'$, according to the way of reckoning the angle. It is not a little remarkable that in the case both of Mr. Marconi's measurements and mine, made with bent oscillators having the ratio of the horizontal to the vertical parts varying from 40 : 1 to 3, the angle of minimum radiation always lies near to 110°, as shown by fig. 2 in Mr. Marconi's paper (*loc. cit.*) and the diagrams in Plate XVIII. in this present paper.

It is also very interesting that the actual results obtained from bent antennæ of such different proportions as those mentioned should agree so well with the theory evolved for a doubly bent antenna with three equal branches.

One other interesting point remains to be considered. The experiments above mentioned were all made with antennæ bent at a right angle. The question arises, what would be the effect of inclining the upper part of the bent antennæ at various angles? A series of experiments was accordingly made with a radiating antenna 20 feet in total length, the vertical part of which was kept always at 5 feet and the remainder tilted at various angles from 20° above the horizontal to 20° below the horizontal. The receiving wire was 21 feet vertical and at a distance of 138 feet. The current in the receiving wire as read by the thermoelectric receiver was taken for various inclinations of the upper part of the sending antenna, both with the free end pointing away from the receiving wire and with the free end pointing towards it. The ratio of these two currents is given in the following Table III. and plotted out as ordinates in terms of the inclination angle as abscissæ in fig. 6.

The figures of Table III. when plotted out into a curve (see fig. 6) show a gradual increase in ordinate as the free end of the antenna is elevated above the ground, but no decided minimum value of the ratio of the fore and aft radiation. One very striking difference was, however, found between a transmitting antenna with the upper part horizontal and one with the upper part placed in a down-sloping position when the complete polar diagram was taken in the two cases. The following experiment shows this difference. An antenna was formed with the 20-foot wire and capacity-plate above described, and was fixed to a wooden frame so that 5 feet of it was vertical and contained the spark-gap, and 15 feet was bent in a down-slanting direction so that the free end was only 2 feet from the ground. This was then swivelled round

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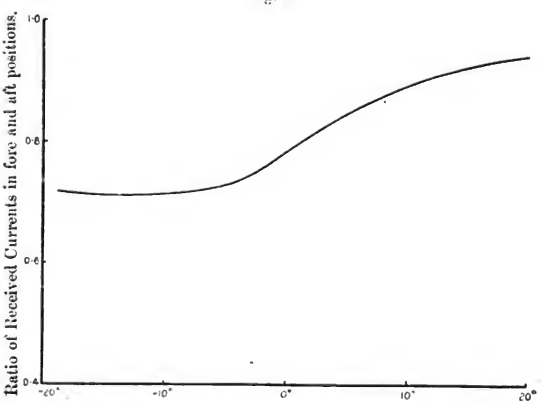
TABLE III.

Observations made with bent Transmitting Antenna 20 feet in total length, 5 feet vertical, and 15 feet inclined at various angles to the horizontal line.

Receiving wire vertical and 21 feet high at 138 feet distance.

Height of free end of sending wire above ground in feet.	Angle of inclination of upper part of sending wire to horizontal.	Ratio of currents in receiving wire with free end of sending wire towards and away from receiver.
	°	
0·5	−17·5	0·72
1	−15·5	0·71
2	−11·55	0·72
3	− 7·65	0·71
4	− 3·85	0·73
5	0	0·77
6	+ 3·85	0·84
7	+ 7·65	0·87
8	+11·55	0·94
9	+15·5	0·92
10	+19·45	0·91

Fig. 6.



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into various azimuths, and the current in a vertical receiving wire 21 feet high and at 138 feet distance was measured and plotted as a polar diagram (see fig. 14, Plate XVII.). When this polar diagram is compared with the polar diagram (fig. 1, Plate XVII.) for the antenna of same length but having its upper part 15 feet long kept in a horizontal position whilst being swivelled round, a very considerable difference is seen in their form.

The polar diagram (fig. 14, Plate XVII.) for the antenna with down-sloping upper part has two deep indentations in it. The ratio of the radiation in the fore and aft directions in the two cases is not very different, that in the direction of the free end being 72 to 76 per cent. of that in the opposite, but whereas the diagram for the horizontal antenna is a smooth oval curve, the other is of a distorted figure-of-8 form.

Moreover, the absolute value of the current in the receiving wire in the two cases for the maximum readings at 0° are in the ratio of about 100 : 70, so that the maximum value of the current in the receiver is reduced by bending down the free end so that it lies below the horizontal.

The explanation of the distortion is not difficult. By bending down the free end of the antenna we increase the magnetic moment and reduce the electric moment of the antenna. The polar curve for an antenna with magnetic but no electric moment, viz., for a closed circuit, is a figure-of-8-shaped curve. The polar diagram for an antenna with electric moment but no magnetic moment, viz., for a vertical wire, is a circle.

In proportion as we lower the free end of the antenna by bending down the upper part into a down-sloping position, we make the magnetic moment of that antenna more pronounced in its effect on the polar curve.

We do not alter much the ratio of the fore and aft radiation in the plane of the oscillator, but we cut it down immensely in two regions lying between 30° and 85° reckoned from the direction in which the free end points.

These differences in the polar curves are perfectly accounted for on the theory given by the writer (see Proc. Roy. Soc. Lond. vol. lxxviii. A. p. 1, 1906). It has there been shown that for the bent oscillator, as used in the above experiments, the magnetic force H parallel to the earth's surface, which cuts across the vertical receiving wire, at the distance r, is given by the expression

$$H = \frac{1}{r^3}\sqrt{(\phi v m^2 r^2)^2 + \left(\phi v m r - \frac{M}{2} m^2 r^2 \cos\theta\right)^2},$$

where ϕ is the electric moment and M the magnetic moment

of the transmitting antenna. It is obvious that if $\phi=0$, then H varies as $\cos\theta$, assuming a constant value for the distance r, and since the current in the receiving antenna varies as H, it follows that the plotting of the current for various azimuths of the sending antenna gives us a polar cosine curve consisting of two circles with perimeters in contact at the origin. If, however, $M=0$, then H is constant for constant values of r and the polar curve is a circle with centre at the origin. For various intermediate ratios of M to ϕ the polar curve takes some irregular figure-of-8-shape as depicted.

The similarly shaped polar curves which Mr. Marconi has given (*loc. cit.*) for the current in the receiving antenna when a *bent receiving antenna* is swivelled round its earthed end are, however, to be explained in a slightly different manner. If a receiving antenna is employed which is partly vertical and partly horizontal, then, when acted upon by a similar or even a vertical transmitting antenna, there are three sources of electromotive force in the receiving wire :—

1st. That due to the action on the vertical part of the receiving wire of the electric force of the incident wave which is perpendicular to the earth's surface.

2nd. That due to the cutting of the vertical part of the receiving wire by the magnetic force of the incident wave which is parallel to the earth's surface.

3rd. That due to the magnetic-force lines passing under the horizontal part of the receiving wire being alternately reversed in direction, aided by the wave-length being roughly equal to 4 times the length of that wire.

The total E.M.F. is the vector sum of these three separate E.M.F.s. The numerical values of (1) and (2) are equal, and (1) is proportional to the minimum radius of the polar curve in the direction 105° to 110°.

It is not difficult to show that when the receiving antenna has its free end pointing anywhere in the semicircle which lies nearest to the sending station, the E.M.F. (3) is opposed in direction to E.M.F. (2), and for a certain azimuth these nullify each other. Hence the minimum radius of the polar curve is proportional to E.M.F. (1). Again, since (1) and (2) differ in phase by 90°, it follows that the receiver current, when the free end of the receiving wires bears 90° from the shortest line connecting the stations, is proportional to the vector sum of E.M.F. (1) and E.M.F. (2).

On the other hand, when the free end of the receiving wire points away from the sending station, the current in it is proportional to the vector sum of E.M.F. (1) and the scalar sums of (2) and (3).

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An analysis of Mr. Marconi's numerous polar curves in the light of this explanation shows that we have E.M.F. (1) = E.M.F. (2), as it should be, and that in the case of the various bent receiving antennæ used by him E.M.F. (3) is from one to four times E.M.F. (2).

In conclusion I wish to mention my obligations to my Assistant, Mr. G. B. Dyke, B.Sc., for his willing and energetic as well as intelligent assistance in the above described work. The measurements not only required accurate observations and care to avoid various sources of error, but necessitated a large amount of active exercise in going to and fro between the stations mostly taken in very hot weather.

LXX. *Electrical Resistance of Alloys.* *By* R. S. Willows, *M.A.*, *D.Sc.**

THE experiments of Fleming and Dewar † and others have shown clearly the wide differences between the electrical properties of alloys and of the pure metals composing them. The well-known investigations of Matthiessen show that the resistance of alloys containing two of the metals lead, tin, cadmium, or zinc, and no other substance, can be calculated from the resistances of their constituents, when the proportions present of each metal are known. The specific resistance of all other alloys is greater than would be given by a calculation based on the assumption that the components conduct proportionally to the volume of each present. This gives rise to the curious fact that the addition of pure silver (a good conductor) to gold (a worse conductor) decreases the conductivity of the latter. Fleming and Dewar have also shown that the resistance of pure metals decreases very greatly as the absolute zero is approached, while alloys still retain a great part of their resistance.

Lord Rayleigh ‡ has advanced a theory, intended to account for these differences, which is based on the thermo-electric properties of a mixture of two metals. Liebenow §, in various publications in Germany, but at a later date, has advanced a theory which, physically, is identical with that of Rayleigh. (I might mention that in German publications the theory is always referred to as Liebenow's; Lord Rayleigh's contribution seems to have escaped notice.)

* Communicated by the Physical Society: read November 23rd, 1906.

† Phil. Mag. [5] vol. xxxvi. p. 271 (1893).

‡ Scientific Papers, vol. iv. p. 232.

§ *Encyklopædie der Elektrochemie*, Band 10.

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The theory is as follows:—When electricity flows from one metal to another there is an absorption or development of heat at the junction—the Peltier effect. The temperature disturbance thus created increases until the conduction of heat through the metals balances the Peltier effect at the junctions, and it sets up a back electromotive force. The difference of temperature at the alternate junctions is proportional to the current, so is also the back E.M.F. called into play. But a reverse E.M.F. proportional to current is indistinguishable experimentally from a resistance, so that an alloy should on these grounds possess a spurious resistance, differing in nature from that of a pure metal. Rayleigh's calculation shows that the false resistance, R, per unit length is given by

$$R = 273e^2/(\kappa/p + \kappa'/p');$$

where e=thermo-electric force of a couple for 1° difference of temperature between the junctions; κ and κ' are the heat conductivities of the metals in ergs; p and p' are the proportions by volume in which the two metals are taken. The temperature is supposed to be near 0° C.

It will be noticed that n, the number of couples per unit length, does not enter into the above expression. The number co-operating is indeed increased by finer subdivision, but the efficiency of each is decreased owing to the readier conduction of heat between the junctions. An alloy of equal volumes of copper and iron should have a false resistance amounting to 1·5 per cent. of that of copper, as is readily shown by substitution of the appropriate numbers in the above formula.

It may be noted in passing that this expression shows that it is possible always to choose the proportions of the metals present so that the resulting alloy shall have a maximum resistance. For $p + p' = 1$,

$$\therefore \qquad R = \frac{273e^2}{\frac{\kappa}{p} + \frac{\kappa'}{1-p}},$$

and for a maximum or minimum $dR/dp = 0$;

whence $$\kappa/p^2 - \kappa'/(1-p)^2 = 0;$$

or $$p = \frac{1}{1 + \sqrt{\frac{\kappa'}{\kappa}}}.$$

The positive sign is to be taken with the square root, since

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the other sign would make p come outside the limits 0–1, and this value it is readily seen corresponds to a maximum R.

It appeared possible to put this false resistance in evidence in two ways, one direct, the other not so. It was only after working for a considerable time that I became acquainted with Liebenow's work, which is entirely concerned with the indirect method.

This paper is a description of the attempts I have made to separate the true and false resistances directly. Suppose a current runs through an alloy and that it sets up a back E.M.F.: if it is now quickly reversed, this inverse E.M.F. will at first assist its passage, and more will flow in the second direction than in the first, or, what comes to the same thing, the resistance will appear to be less for the quickly reversed current than for the steady direct one. As the temperature of the junctions will be equalized rapidly on account of their small distance apart, the current reversals must be rapid. I therefore used an alternating current.

The alloy to be tested formed one of the arms of a Wheatstone's bridge, the adjacent arm being a simple metal such as copper or lead, generally the latter on account of its greater resistivity. The resistances were first balanced for alternating and then for direct current. The pure metal possesses no spurious resistance, and hence the apparent resistance of the alloy should decrease when the alternating current is used.

The figure (p. 607) shows the arrangement of apparatus.

A is the resistance-coil of alloy; B the copper or lead resistance made from specially purified material obtained from Johnson and Matthey; C, D, are two coils of manganin or eureka wire wound on the same bobbin; G is the secondary of a small induction-coil; H, a Leclanché cell; P, a galvanometer; Q, a telephone specially wound for bridge work, or a vibration galvanometer; S_1, S_2, mercury-paraffin switches by means of which the source of current or the detector could be changed.

There is, in addition to the false resistance of the alloy, the skin effect, arising from the concentration of current in the outer layers, to be taken into account. If R is the true resistance of a wire of length l to steady currents, then the resistance to currents of moderate frequency and of simple periodic form is

$$R' = R\left(1 + \frac{1}{12}\frac{\mu^2 p^2 l^2}{R^2}\right),$$

where μ is the permeability and $p = 2\pi$ (frequency). The

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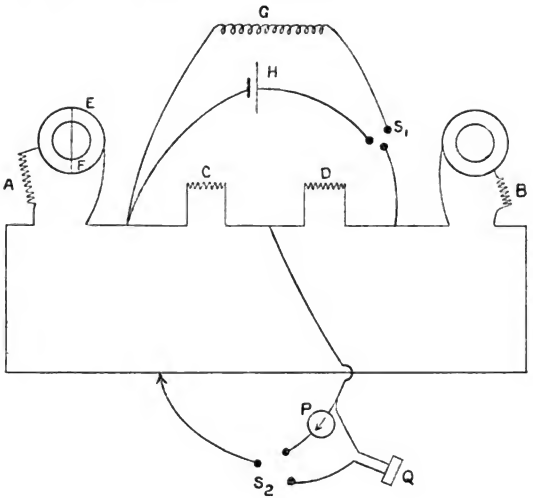
wave-form of the current used was not known, but the second term in the bracket was made small by using wires of small diameter. Were any skin effect present, only the difference between that in lead and in the alloy would be given by the experiment.



For the same frequency the value of this term depends on l/R, and this is greatest for the pure metal, and hence if $\mu=1$, the skin effect, when it first becomes appreciable, would make the alloy appear to decrease in resistance when referred to copper or lead as standard.

All the coils in the bridge were made of wire 0·33 mm. in diameter, formed after the manner given by Chaperon, by first winding a definite number of turns in the right-handed direction and then an equal number the opposite way. The resistance of each was 3 ohms approximately; the induction in the arms by this method was rendered very small, but was still large enough, with these low resistances, to give a flat minimum in the telephone. With an ordinary set of coils out of a resistance-box it was impossible to obtain a balance. In order to render the minimum sharper the method of

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Rayleigh was adopted *. E is a wooden ring of 4 inches diameter. F one of 2 inches; the smaller can be rotated round a common diameter of the two. Each ring carries three turns of copper wire, 0·33 mm. diameter, joined in series, and the whole arrangement is in series with A. By rotating the coil F, the induction in the arm could be varied within narrow limits. A similar pair of coils is in series with B. By this means the balance point on the wire of the bridge with alternating current could be found readily to 1 mm., generally much nearer, and as the resistance of 1 mm. of the bridge wire was 0·0006 ohm, this corresponds to a minimum accuracy of about 0·02 per cent.

Various current interrupters were used. In the preliminary experiments a secolumneter and the galvanometer were tried, but soon abandoned, because thermo-currents were troublesome, and also because I desired to get a greater range of frequency. Finally a vibrating wire with mercury contact was used in the low-frequency experiments. This broke the primary circuit of a small induction-coil, the secondary of which was joined to the bridge, and it possessed the advantage that it could readily be tuned to unison with the vibration-galvanometer used as detector. For most of the other observations a wheel interrupter was used. This was made by letting into the circumference of a disk of beech-wood, 1 foot in diameter, about 120 pieces of brass whose width along the circumference was about $\frac{1}{10}$ inch, the whole being carefully turned. Pressing on the circumference were two springs which completed the circuit through the brass pieces as the wheel revolved when driven by a motor. A small condenser was connected to the springs to prevent sparking.

The bridge was balanced first with alternating current by shifting the movable contact and adjusting the coils E, F until the inductances as well as the resistances of the arms were equal; then with direct current. The coils A and B were then generally interchanged and the observations repeated. Mr. F. G. Bratt helped me considerably by taking readings alternately with me.

The alloys used were eureka, brass, platinoid, German silver, platinum-iridium, and platinum-silver. The frequency of the current varied between 10 and 980 per second.

No certain differences could be detected between the resistance of an alloy to direct and alternating current at temperatures 20° and 100° C.

As already mentioned, Fleming and Dewar found the resistance of a pure metal very small at low temperatures,

* Phil. Mag. Dec. 1886.

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while that of alloys was not greatly changed. It seemed possible, therefore, as the temperature was reduced, that the true resistance of an alloy might decrease and the false resistance become relatively great. Experiments similar to those described above were therefore made with alloys at the temperature of solid CO_2 and of liquid air, but in no case could any false resistance be found.

Other experiments with a 20-ohm coil of German silver balanced against one of lead gave similarly negative results. Whether this is due to the frequency being too small to overtake the equalisation of temperature of the junctions, future experiments, which I hope to carry out, will perhaps determine.

The experiments of Hagen and Rubens * on the reflecting and emissive powers of metals for infra-red rays do not help us. They show that the electrical conductivity can be found by measuring the reflecting or emissive powers to within a few per cent. Alloys do not occupy an exceptional position. The experiments do not put in evidence the magnet properties of iron and nickel.

In any case indirect evidence in favour of Rayleigh's theory is not wanting. Liebenow gives the results of his calculation in a form involving two constants, which he calls the inner molecular heat conductivity of the metals. By assigning proper values to these the formula expresses well, in some cases such as gold and silver excellently, the relation between resistance and composition when applied to the measurements of Matthiessen, Feussner, Haas, and others. They are not, however, altogether convincing. The heat conductivities to be ascribed to the different molecules seem to bear no relation to the conductivities as usually measured in the mass. Thus, taking the heat-conduction of copper as unity, silver requires to be taken as 2·3 and gold as 2, while copper-zinc has the value 3 ascribed to it.

I might also point out that a frequent constituent of the high resistance alloys in common use is nickel, a metal for which the Peltier effect and other thermo-electric properties are strongly pronounced.

In conclusion I have to thank the Government Grant Committee of the Royal Society for help in defraying the cost of this research.

Cass Institute,
London, E.C.

* *Annal. d. Physik.* vol. ii. p. 873 (1903).

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LXXI. *Notices respecting New Books.*

A Practical Manual of Tides and Waves. *By* W. H. Wheeler, M.Inst.C.E. With illustrations. London: Longmans, Green & Co. 1906. Pp. x + 202.

THE aim of the author of this book has been "to give as practical an account as possible, and free from all mathematical demonstration, of the action of the sun and moon in producing the tides; and also of the physical causes by which the tides are affected after their generation, and of their propagation throughout the tidal waters of the earth." In this object we think the author has succeeded admirably; and although here and there there is a looseness of diction which one might like to see removed, the book as a whole is admirably fitted for the practical and non-mathematical class of reader for whom it is primarily intended. After an introduction containing a sketch of the history of tidal science, the author considers the effects of the sun and moon as tide-producing agents, the propagation of the tidal wave, tidal currents, the mean level of the sea and the range of the tides, the effects of wind and atmospheric pressure on the tides, river tides, wind waves, seismic and cyclonic storm-waves, and tidal bores in rivers. The concluding chapters deal with tide-tables and tide-gauges, secondary undulations and storm warnings, and the tides regarded as a source of power. The numerous appendices contain, besides various data, a comprehensive bibliographical list.

Petrogenesis. *Von* Dr. C. Doelter, O. Professor der Mineralogie und Petrographie an der Universität Graz. Mit einer Lichtdrucktafel und 5 eingedruckten Abbildungen. Braunschweig: F. Vieweg und Sohn. 1906. Pp. xii + 262.

In this little treatise, which forms No. 13 of the series bearing the general title of *Die Wissenschaft*, the author gives a clear and interesting account of the present state of our knowledge regarding the origin of rocks. In the opening chapter, the various rival theories regarding the state of the earth's interior and the nature of volcanic eruptions are passed in critical review. Then follow accounts of the structure, chemical and physical characteristics and relative ages of rocks of volcanic origin. Of particular interest is the chapter dealing with the artificial production of rocks, a subject to whose development the author himself has largely contributed by his experimental researches. The remaining portion of the book treats of slates, chalks, flints, sandstones, saline deposits, &c.

The work is based on the modern methods of physico-chemical research as well as on geological observation. It should form an excellent guide for the geologist, mineralogist and chemist; and will be found by no means devoid of interest to the general reader.

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INDEX TO VOL. XII.

Actinium, on the origin of the β rays emitted by, 177; on the ionization ranges of the α rays of, 244; on the mass and velocity of the α particles expelled from, 348.
Albumen, on double refraction in, produced by electric endosmose, 556.
Alloys, on the electrical resistance of, 604.
Alpha particle, from radium, on the retardation of the, in passing through matter, 134; on the mass and velocity of the, from radium and actinum, 348; on the mass of the, from thorium, 371.
—— rays, of radiothorium, on some properties of the, 82; of actinium, on the ionization ranges of the, 244; on the recombination of ions made by, 273.
Antennæ, on the electric radiation from bent, 588.
Aqueous solutions, on the molecular constitution of, 1.
Argon, on the evolution of gas from the cathode in, 481.
Astronomical physics, on questions connected with, 570.
Atmosphere, on the radioactive matter in the, 189.
Atomic conductivities of the ions, on the, 150, 532.
Bakker (Dr. G.) on the theory of surface-forces, 557.
Baldwin (J. M.) on the propagation of longitudinal waves of magnetic flux along iron wires and rods, 433.
Baldwin-Wiseman (W. R.) on the influence of pressure and porosity on the motion of subsurface water, 531.
Bars, on the lateral vibration of, 233.
Barton (Prof. E. H.) on simultaneous vibration-curves of string and air obtained from a monochord, 576.
Beta rays, on the origin of the, emitted by thorium and actinium, 177; on the recombination of ions made by, 273; on the coefficient of absorption of the, from uranium, 379.
Blackman (P.) on the atomic conductivities of the ions, 150, 532.
Bonney (Prof. T. G.) on the relations of the chalk and boulder-clay near Royston, 169.
Books, new:—Jäger's Die Fortschritte der kinetischen Gastheorie, 93; Dennstedt and Voigtländer's Baumert's Lehrbuch der Gerichtlichen Chemie, 94; Schmid's Philosophisches Lesebuch, 94; Zahikjanz's Die Theorie, Berechnung und Konstruktion der Dampfturbinen, 95; Wood's Physical Optics, 162; Barus's Nucleation of the Uncontaminated Atmosphere, 164; Guarini's Derniers Progrès du Telphérage Electrique, 164; Muir's Theory of Determinants, 431; Strong's Lectures on the Method of Science, 431; Niethammer's Moteurs à Collecteur à Courants Alternatifs, 432; Fleming's The Principles of Electric Wave Telegraphy, 525; v. Neumayer's Anleitung zu Wissenschaftlichen Beobachtungen auf Reisen, 526; Vogl's Die Physik Roger Bacos, 527; Jamin's Cours de Physique de l'Ecole Polytechnique, 527;

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Millikan and Gale's First Course in Physics, 528; de Metz's La Double Réfraction accidentelle dans les Liquides, 528; Wheeler's Practical Manual of Tides and Waves, 610; Doelter's Petrogenesis, 610.
Bronson (Dr. H. L.) on the periods of transformation of radium A, B, and C, 73.
Brown (J. A.) on the potential required to maintain a current between parallel plates in a gas at low pressures, 210.
Burton (E. F.) on the action of electrolytes on colloidal solutions, 472.
Campbell (A.) on the use of chilled cast iron for permanent magnets, 468.
Cathode, on the evolution of gas from the, in helium and argon, 481.
Chemical changes induced by ultra-violet light, on, 397.
Cockin (G. M.) on the occurrence of limestone in Cannock Chase, 168.
Colloidal solutions, on the action of electrolytes on, 472.
Colloids, on double refraction in, produced by electric endosmose, 548.
Colour of light scattered by small absorbing particles, on the, 147.
—— phenomena in photometry, on 120.
—— photography, on the diffraction process of, 585.
Contour integration, on the solution of problems in diffraction by the aid of, 63.
Crowther (J. A.) on the coefficient of absorption of the β rays from uranium, 379.
Crystal, on the determination of the optical constants of a, by means of refraction through a prism, 99.
Davies (H.) on the solution of problems in diffraction by the aid of contour integration, 63.
Dead points of a galvanometer-needle for transient currents, on the, 202.
Dielectric strain along the lines of force, on the, 268.
Diffraction, on the solution of problems in, by the aid of contour integration, 63.
—— bands, on unsymmetrical, due to a rectangular aperture, 494.
—— gratings, on the intensification of glass, 585.
Disruptive voltage of thin liquid films, on the, 317.
Double refraction in colloids produced by electric endosmose, on, 548.
Dow (J. S.) on colour phenomena in photometry, 120.
Ductile materials, on the strength and behaviour of, under combined stress, 533.
Dynamical theory of gases, on the, 60.
Earth, on the constitution of the interior of the, as revealed by earthquakes, 165; on the radio-active matter in the, 189.
Eccles (Dr. W. H.) on the effect of electrical oscillations on iron in a magnetic field, 100.
Elastic properties of iron and steel, on the effect of combined stresses on the, 418.
Electric currents, on the potential required to maintain, between parallel plates in a gas at low pressures, 210; on leakage, in the moisture condensed on glass surfaces, 336.
—— discharge, on the effect of radium in facilitating the, in vacuo, 70.
—— endosmose, on double refraction in colloids produced by, 548.
—— oscillations, on the effect of, on iron in a magnetic field, 100.
—— radiation from bent antennæ, on the, 588.
—— resistance of alloys, on the, 604.
—— units, on the experimental determination of the ratio of the, 97.
—— waves, on the effect of a long straight wire on, 46.
Electrical changes induced by ultra-violet light, on, 397.
Electrolytes, on the action of, on colloidal solutions, 472.
Electron theory of matter, on the, 21.
Ellipsoidal harmonics, on, 36.
Evans (D. C.) on the ordovician rocks of W. Caermarthenshire, 174.

Digitized by Google

Eve (A. S.) on the radioactive matter in the earth and the atmosphere, 189.
Ewing (Dr. J. A.) on the molecular structure of metals, 254.
Ferric hydrate, on double refraction in, produced by electric endosmose, 556.
Films, on the disruptive voltage of thin liquid, 317.
Fish-eye views and vision under water, on, 159.
Fleming (Prof. J. A.) on the electric radiation from bent antennæ, 588.
Fluorescence spectrum, on the, of iodine vapour, 329; of sodium vapour, 499.
Galvanometer-needle, on the dead points of a, for transient currents, 202.
Galvanometers, on the theory of moving-coil, 269.
Gamma rays, on the recombination of ions made by, 273.
Gases, on the H theorem and the dynamical theory of, 60; on the potential required to maintain a current between parallel plates in, at low pressures, 210; on the evolution of, from the cathode in helium and argon, 481.
Gelatine, on double refraction in, produced by electric endosmose, 548.
Geological Society, proceedings of the, 95, 165, 529.
Glass surfaces, on leakage currents in the moisture condensed on, 336.
Gordon (Dr. M. M. Ogilvie) on interference-phenomena in the Alps, 529.
Gratings, on the intensification of glass diffraction, 585.
Gravitation, on the explanation of, 21.
Gravity, on the acceleration of, at Johannesburg, 479.
H theorem, on the, 60.
Hahn (Dr. O.) on properties of the α rays of radiothorium, 82; on the ionization ranges of the α rays of actinium, 244; on the mass of the α particles from thorium, 371.
Haidinger's interference-rings, on, 489.
Hancock (Prof. E. L.) on the effect of combined stresses on the elastic properties of iron and steel, 418; on tests of metals in reverse torsion, 426.
Hargreaves (R.) on ellipsoidal harmonics, æolotropic and isotropic, 36.
Harmonics, on ellipsoidal, 36.
Hawthorne (W.) on the deflexions caused by a break in an overhead wire carried on poles, 200.
Heat waves, on a new method of isolating, 67.
Helium, on the evolution of gas from the cathode in, 481.
Hill (Rev. E.) on the chalk and drift in Möen, 168.
Interference colours of chlorate of potash crystals, on the, 67.
—— rings described by Haidinger, on the, 489.
Iodine vapour, on the fluorescence, magnetic rotation, and temperature emission spectra of, 329.
Ionization in the atmosphere, on, 198.
—— ranges of the α rays of actinium, on the, 244.
Ions, on the atomic conductivities of the, 150, 532; on the recombination of, made by α, β, γ, and X rays, 273.
Iron, on the effect of electrical oscillations on iron in a magnetic field, 109; on the effect of combined stresses on the elastic properties of, 418; on the use of chilled cast, for permanent magnets, 468.
Iron wires and rods, on the propagation of longitudinal waves of magnetic flux along, 433.
Jeans (Prof. J. H.) on the thermodynamical theory of radiation, 57; on the H-theorem and the dynamical theory of gases, 60.
Johannesburg, on the acceleration of gravity at, 479.
Kelvin (Lord) on some remarkable results of a discharge of lightning upon an oak-tree, 62.
Kleeman (R. D.) on the recombination of ions made by α, β, γ, and X rays, 273.
Lateral vibration of bars, on the, 233.
Leakage currents in the moisture condensed on glass surfaces, on, 336.

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Leathem (Dr. J. G.) on the effect of a long straight wire on electric waves, 46.
Lebour (Prof. G. A. L.) on a case of unconformity and thrust in the coal-measures of Northumberland, 170.
Lehfeldt (Prof. R. A.) on the acceleration of gravity at Johannesburg, 479.
Levin (Dr. M.) on the origin of the β rays emitted by thorium and actinium, 177.
Light, on abnormal polarization and colour of, scattered by small absorbing particles, 147; on chemical and electrical changes induced by ultraviolet, 397.
Lightning, on some remarkable results of a discharge of, upon an oak-tree, 62.
Liquid films, on the disruptive voltage of thin, 317.
Lorenzo (Prof. G. de) on the eruption of Vesuvius in April 1906, 173.
Lyle (Prof. T. R.) on the propagation of longitudinal waves of magnetic flux along iron wires and rods, 433.
Magnetic field, on the effect of electrical oscillations of iron in a, 109.
—— flux, on the propagation of longitudinal waves of, along iron wires and rods, 433.
—— rotation spectrum, on the, of iodine vapour, 329; of sodium vapour, 499.
Magnets, on the use of chilled cast iron for permanent, 468.
Malcolm (Dr. H. W.) on double refraction in colloids produced by electric endosmose, 548.
Marr (Dr. J. E.) on the influence of the geological structure of English lakeland upon its present features, 95.
Materials, on the strength and behaviour of ductile, under combined stress, 533.
Metals, on the molecular structure of, 254; on tests of, in reverse torsion, 426.
Milner (S. R.) on the use of the secohmmeter for the measurement of combined resistances and capacities, 297.
Moisture condensed on glass surfaces, on leakage currents in the, 336.
Molecular constitution of aqueous solutions, on the, 1.
—— structure of metals, on the, 254.
Monochord, on simultaneous vibration-curves of string and air obtained from a, 576.
Moore (Prof. R. B.) on new methods for separating uranium X from uranium, 393.
More (Prof. L. T.) on the dielectric strain along the lines of force, 268.
Morrow (Dr. J.) on the lateral vibration of bars, 233.
Morton (Prof. W. B.) on the deflexions caused by a break in an overhead wire carried on poles, 200.
Moving-coil galvanometers, on the theory of, 269.
Oldham (R. D.) on the constitution of the interior of the earth as revealed by earthquakes, 165.
Penzer (J.) on simultaneous vibration-curves of string and air obtained from a monochord, 576.
Photometry, on colour phenomena in, 120.
Pjetursson (Dr. H.) on the crag of Iceland, 176.
Polarization, on abnormal, and colour of light scattered by small absorbing particles, 147.
Potassium chlorate crystals, on the interference colours of, 67.
Potential, on the, required to maintain a current between parallel plates in a gas at low pressures, 210.
Preston (Dr. S. T.) on questions connected with astronomical physics, 570.
Prism, on the determination of the optical constants of a crystal by means of refraction through a, 29.
Radiation, on the thermodynamical theory of, 57, 498; on the distribution of the intensity of the, from radioactive sources, 152.
Radioactive matter in the earth and the atmosphere, on the, 189.
—— sources, on the distribution of the intensity of the radiation from, 152.

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Radiothorium, on some properties of the α rays of, 82.
Radium, on the effect of, in facilitating the electric discharge in vacuo, 70; on the retardation of the α particle from, in passing through matter, 134; on the amount of, in the earth, 194; on the mass and velocity of the α particles expelled from, 348.
—— A, B, and C, on the periods of transformation of, 73.
Raman (C. V.) on unsymmetrical diffraction-bands due to a rectangular aperture, 494.
Ramsay (Sir W.) on chemical and electrical changes induced by ultra-violet light, 397.
Rayleigh (Lord) on the experimental determination of the ratio of the electrical units, 97; on the interference-rings, described by Haidinger, observable by means of plates whose surfaces are absolutely parallel, 489.
Russell (A.) on the dead points of a galvanometer-needle for transient currents, 202.
Rutherford (Prof. E.) on the retardation of the α particle from radium in passing through matter, 134; on the distribution of the intensity of the radiation from radioactive sources, 152; on the mass and velocity of the α particles expelled from radium and actinium, 348; on the mass of the α particles from thorium, 371.
Schott (Dr. G. A.) on the electron theory of matter and the explanation of fine spectrum-lines and of gravitation, 21; on a kinematical explanation of groups of spectrum-lines with constant frequency-difference, 579.
Scoble (W. A.) on the strength and behaviour of ductile materials under combined stress, 533.
Searle (C.) on leakage currents in the moisture condensed on glass surfaces, 336.
Secohmmeter, on the use of the, for the measurement of combined resistances and capacities, 297.
Shaw (Dr. P. E.) on the disruptive voltage of thin liquid films, 317.
Silicic acid, on double refraction in, produced by electric endosmose, 556.
Skinner (Prof. C. A.) on the evolution of gas from the cathode in helium and argon, 481.
Smith (G. F. H.) on the determination of the optical constants of a crystal by means of refraction through a prism, 29.
Smythe (Dr. J. A.) on a case of unconformity and thrust in the coal-measures of Northumberland, 170.
Sodium vapour, on the fluorescence and magnetic rotation spectra of, 499.
Sollas (Prof. W. J.) on recumbent folds produced as a result of flow, 175.
Solutions, on the molecular constitution of aqueous, 1; on the action of electrolytes on colloidal, 472.
Spectra, on the fluorescence, magnetic rotation, and temperature emission, of iodine vapour, 329; on the fluorescence and magnetic rotation, of sodium vapour, 499.
Spectrum, on the Swan, 581.
Spectrum-lines, on the explanation of fine, 21; on a kinematical explanation of groups of, with constant frequency-difference, 579.
Spencer (Dr. J. F.) on chemical and electrical changes induced by ultra-violet light, 397.
Starkey (W. I. S. B.) on some remarkable results of a discharge of lightning upon an oak-tree, 62.
Steel, on the effect of combined stresses on the elastic properties of, 418.
Stresses, on the effect of combined, on the elastic properties of iron and steel, 418.
Surface-forces, on the theory of, 557.
Sutherland (W.) on the molecular constitution of aqueous solutions, 1.
Swan spectrum, on the, 581.
Swinton (A. A. C.) on the effect of radium in facilitating the visible electric discharge in vacuo, 70.
Temperature emission spectrum of iodine vapour, on the, 329.
Thermodynamical theory of radiation, on the, 57, 498.

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Thorium, on the origin of the β rays emitted by, 177; on the mass of the α particles from, 371.
Torsion, on tests of metals in reverse, 426.
Treacher (Ll.) on the phosphatic chalks of Winterbourne and Boxford, 171.
Trouton (Prof. F. T.) on leakage currents in the moisture condensed on glass surfaces, 336.
Tuckerman (L. B., jr.) on the thermodynamical theory of radiation, 498.
Ultraviolet light, on chemical and electrical changes induced by, 397.
Uranium, on the coefficient of absorption of the β rays from, 379; on new methods for separating uranium X from, 393.
Vibration-curves of string and air obtained from a monochord, on simultaneous, 576.
Vision under water, on, 159.
Watts (Dr. W. M.) on the Swan spectrum, 581.
Waves of magnetic flux, on the propagation of longitudinal, along iron wires and rods, 433.
——, electric, on the effect of a long straight wire on, 46.
——, heat, on a new method of isolating, 67.
White (H. J. O.) on the phosphatic chalks of Winterbourne and Boxford, 171.
Wilkinson (H. M.) on the Swan spectrum, 581.
Willows (Dr. R. S.) on the electrical resistance of alloys, 604.
Wilson (Prof. H. A.) on the theory of "moving-coil" galvanometers, 269.
Wire, on the effect of a long straight, on electric waves, 46; on the deflexions caused by a break in an overhead, carried on poles, 200.
Wireless telegraphy, on the radiation from bent antennæ in, 588.
Wood (Miss E. M. R.) on the Tarannon series of Tarannon, 166.
Wood (Prof. R. W.) on interference-colours of chlorate of potash crystals and a new method of isolating heat-waves, 67; on abnormal polarization and colour of light scattered by small absorbing particles, 147; on fish-eye views and vision under water, 159; on the fluorescence, magnetic rotation, and temperature emission spectra of iodine vapour, 329; on the fluorescence and magnetic rotation spectra of sodium vapour, 499; on the intensification of glass diffraction-gratings and the diffraction process of colour photography, 585.
X rays, on the recombination of ions made by, 273.

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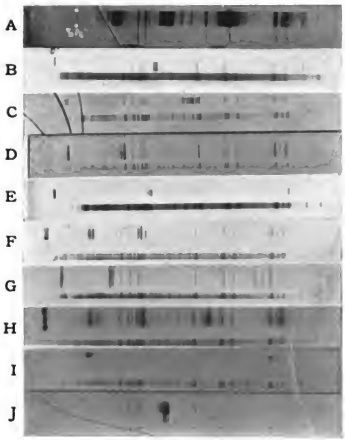

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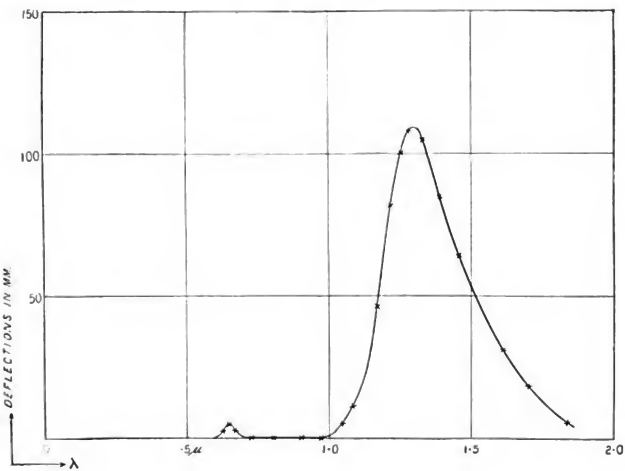

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Fig. 2.

Fig. 4.

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Fig. 3.

Fig. 5.

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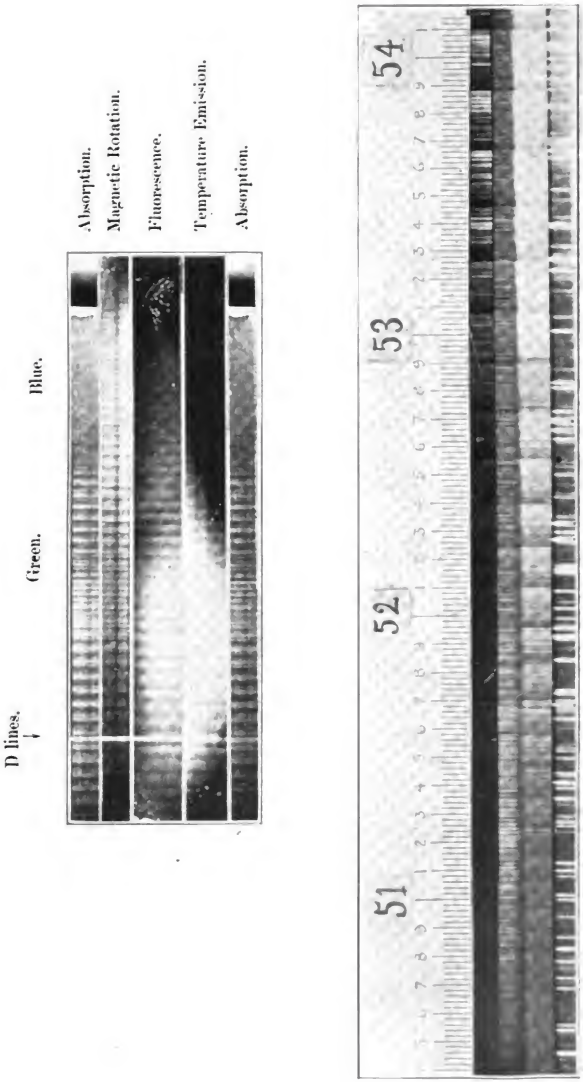

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RUTHERFORD.]

FIG. 2.

A B C D

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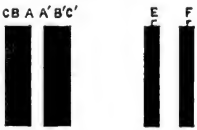

FIG. 2.

A B

FIG. 3.

A B

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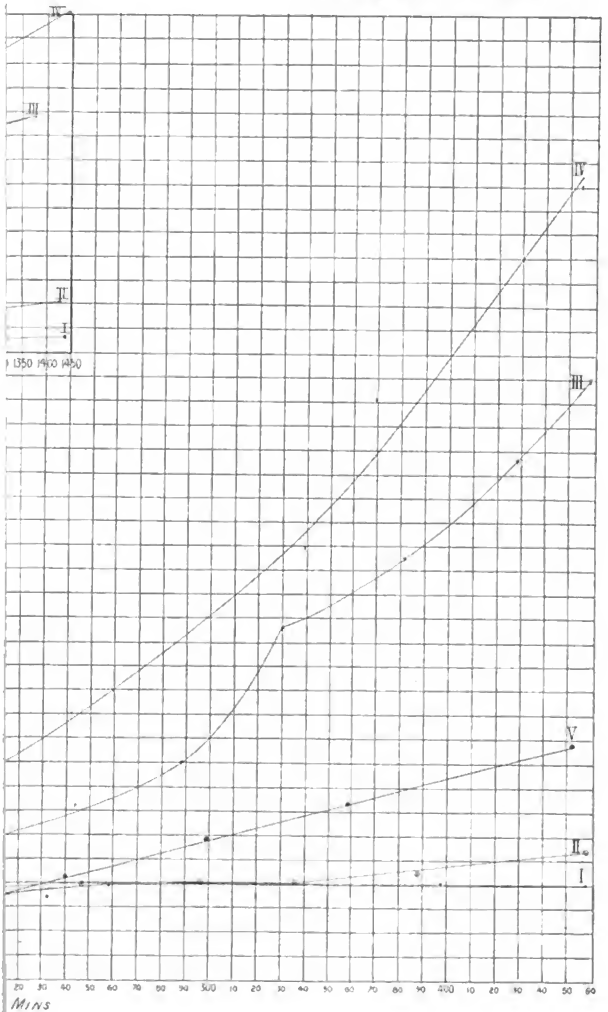

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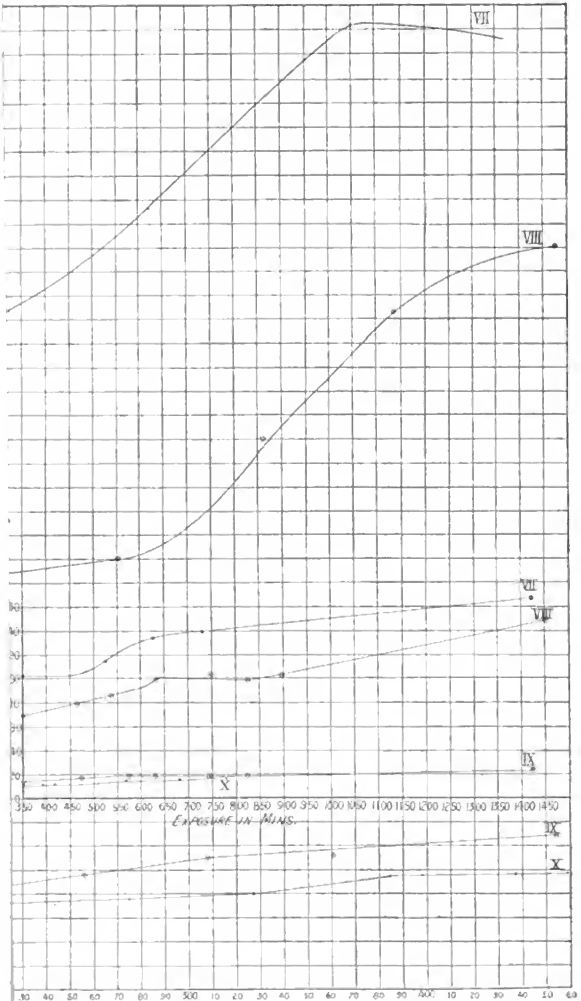

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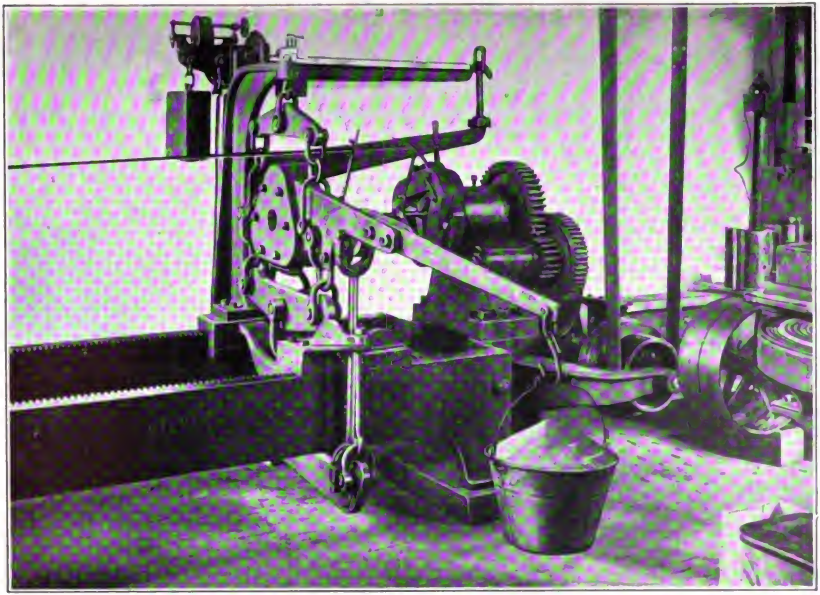

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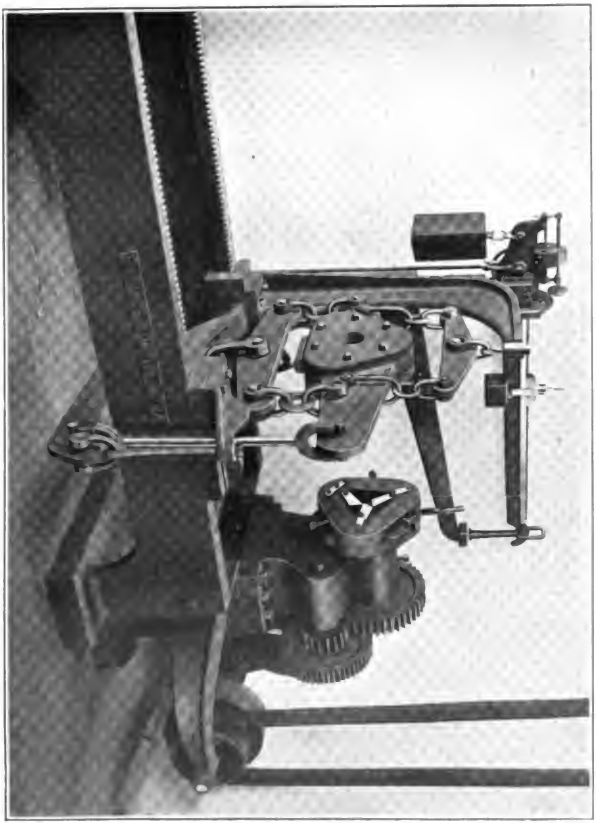

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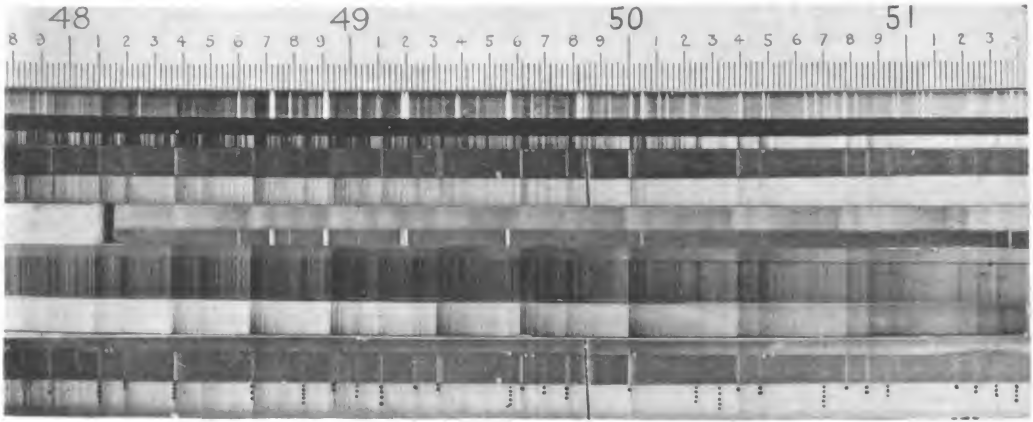

Fig 1

Fluorescence of Na Fe Comparison

Fig 2

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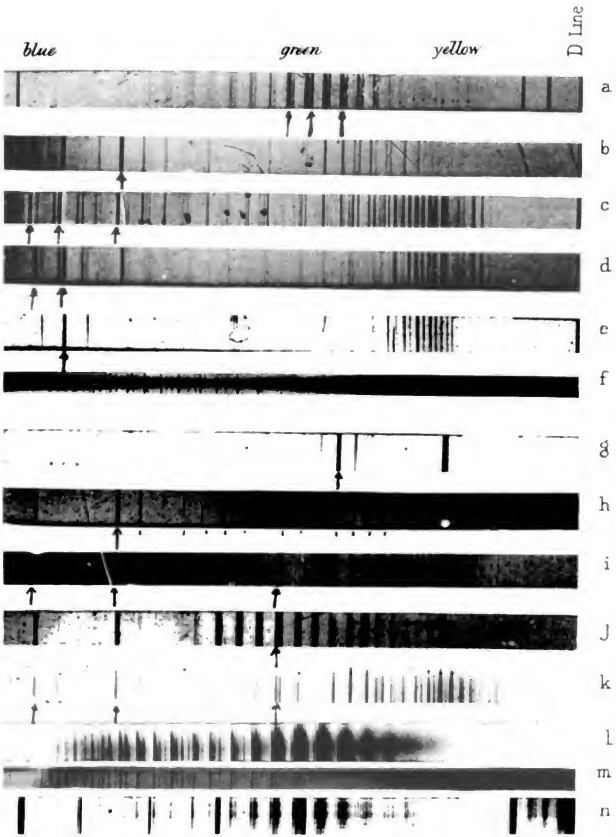

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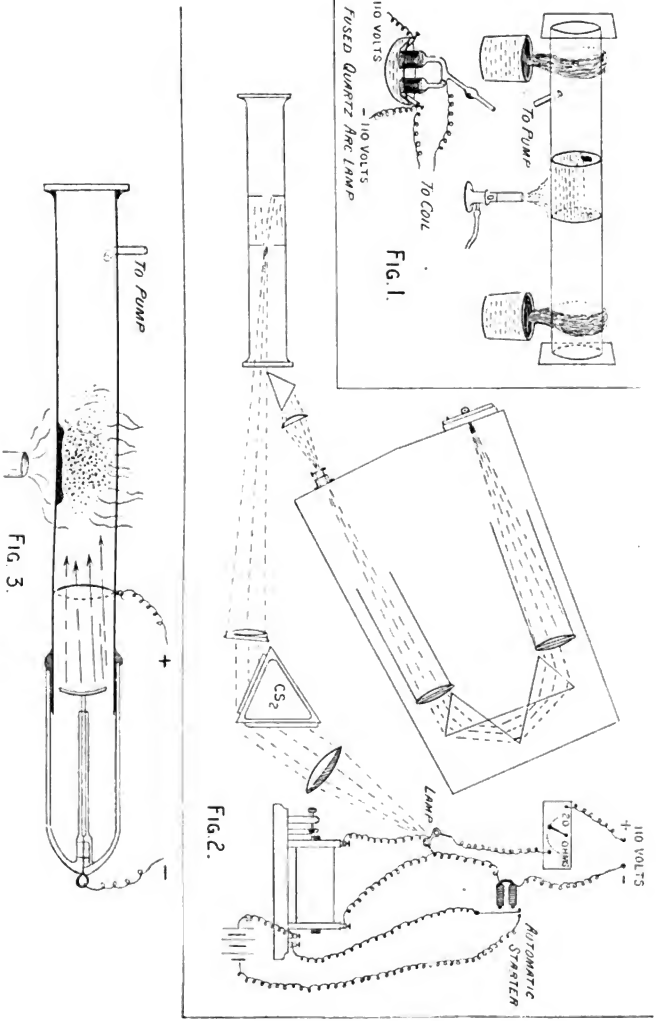

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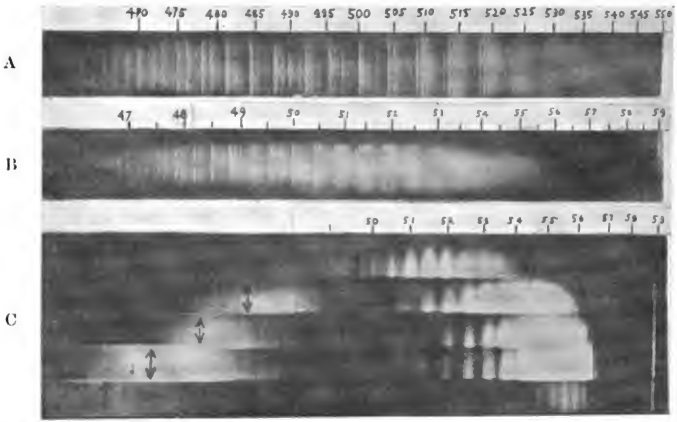

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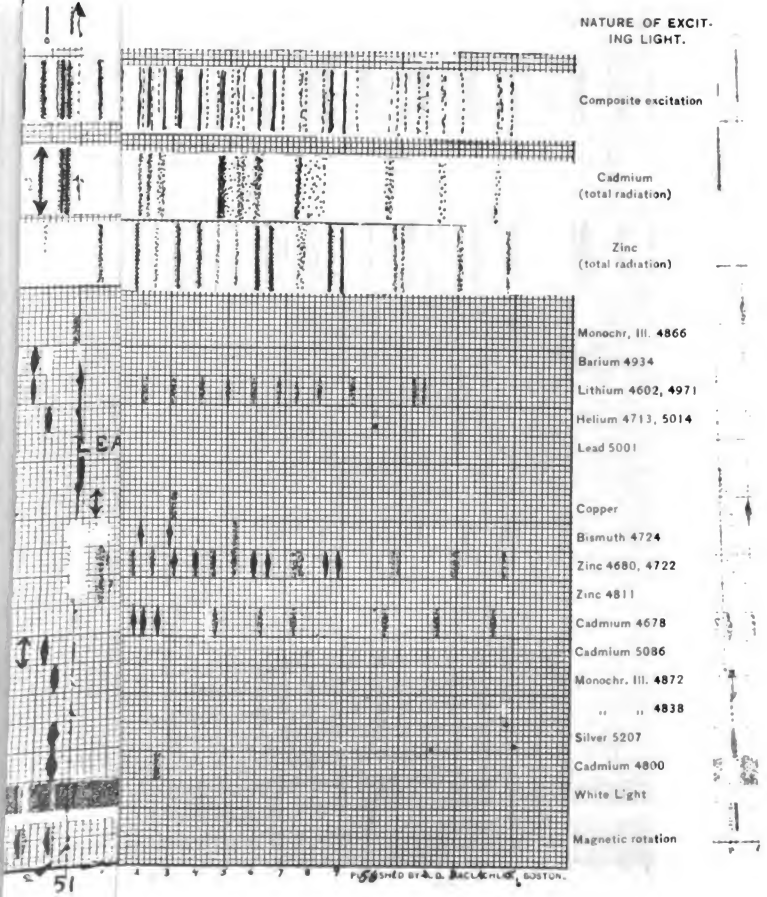

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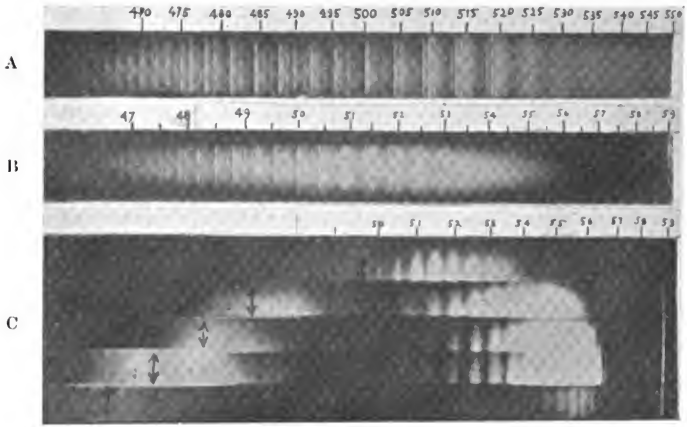

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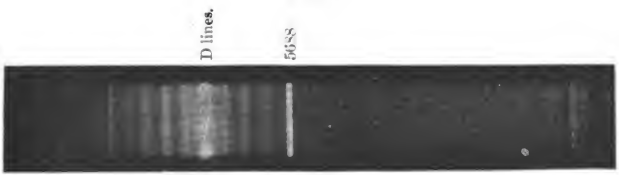

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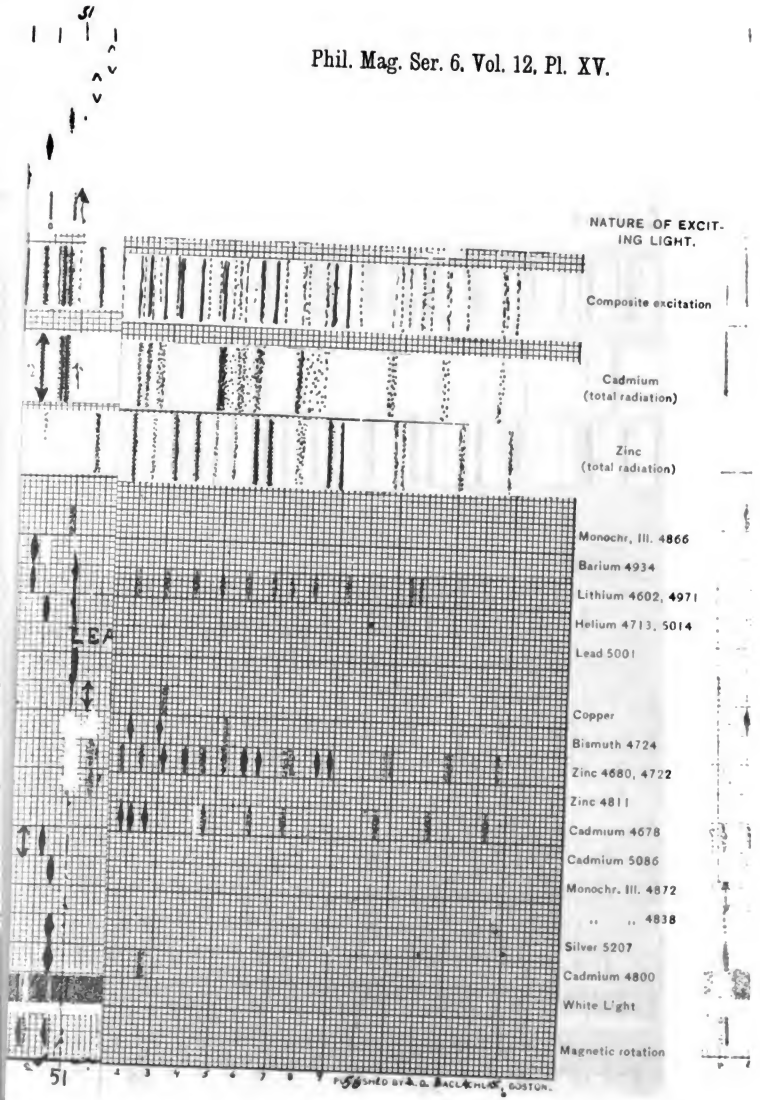

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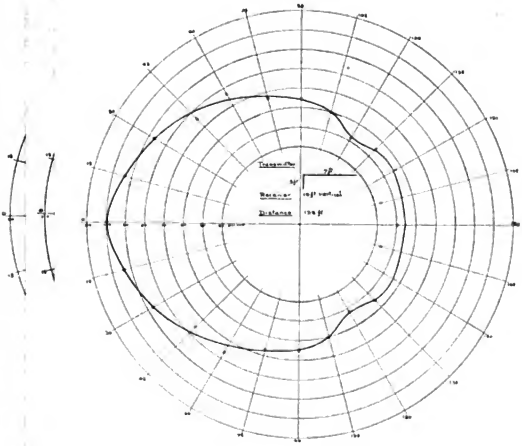

Transmitter 3 : 7.

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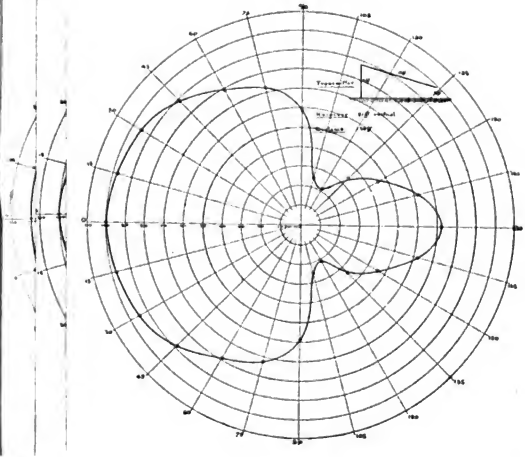

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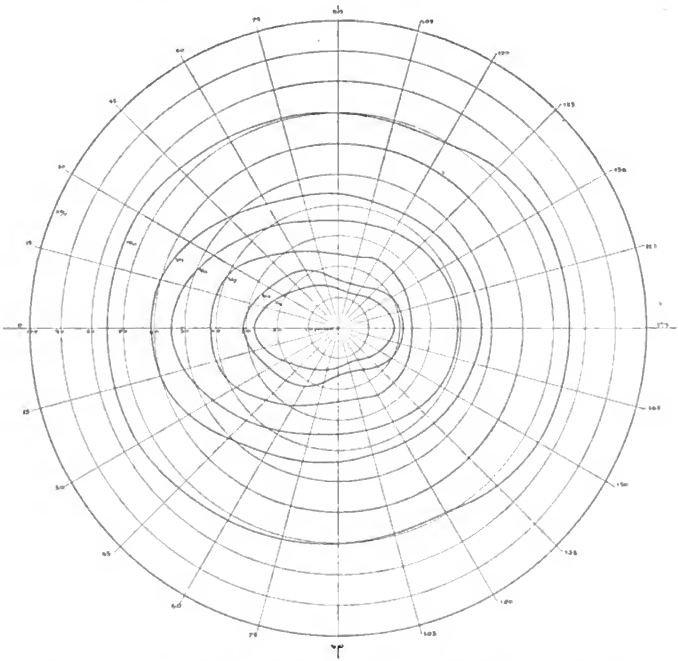
TRANSMITTING ANTENNA.	**Total length 20 ft.**	
RECEIVING	**,,**	**Height 21 ft.**
DISTANCE.		**138 ft.**



The radii of the curves are proportional to the currents in the receiving antenna when the bent transmitter wire has its vertical to horizontal parts in the ratio indicated by the fractions 10/10, 5/15, 4/16, 3/17, 2/18, 1/19 marked on the curves; the receiver current when the transmitter is vertical is taken as 100.

The zero position of the transmitter is that in which the horizontal part is directed away from the receiver.

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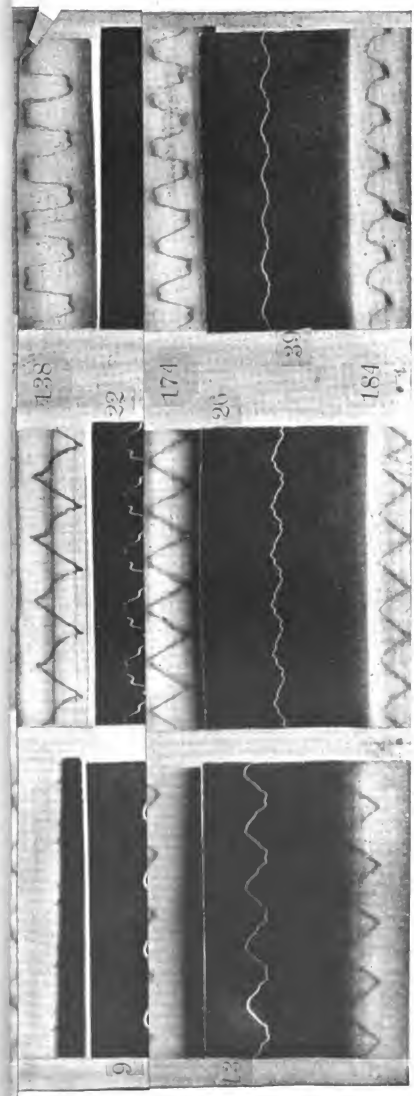

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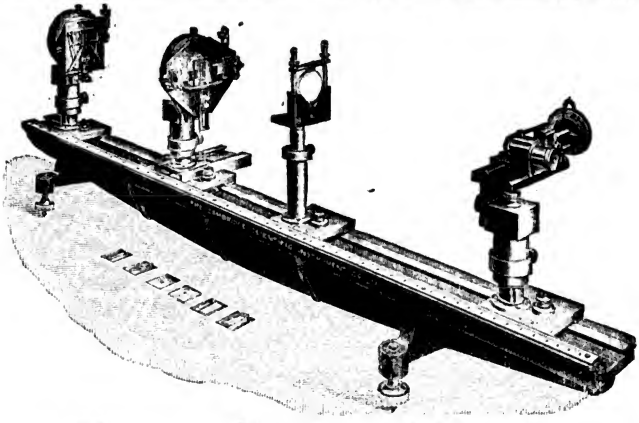

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LXI. The Strength and Behaviour of Ductile Materials under Combined Stress. By WALTER A. SCOBLE, A.R.C.Sc., B.Sc., Whitworth Scholar ..... *page* 533

LXII. Double Refraction in Colloids produced by Electric Endosmose. By H. W. MALCOLM, M.A., B.Sc., Ph.D., Carnegie Fellow of Aberdeen University 548

LXIII. On the Theory of Surface Forces. By Dr. G. BAKKER .. 557

LXIV. On Certain Questions connected with Astronomical Physics. By S. TOLVER PRESTON, Ph.D. 570

LXV. Simultaneous Vibration-Curves of String and Air photographically obtained from a Monochord. By Prof. E. H. BARTON, D.Sc., F.R.S.E., and J. PENZER. (Plate XIX.) 576

LXVI. A Kinematical Explanation of Groups of Spectrum Lines with Constant Frequency-Difference. By G. A. SCHOTT 579

LXVII. On the "Swan" Spectrum. By W. MARSHALL WATTS, D.Sc., and H. M. WILKINSON, M.A. 581

LXVIII. The Intensification of Glass Diffraction Gratings, and the Diffraction Process of Colour Photography. By Prof. R. W. WOOD 585

LXIX. On the Electric Radiation from Bent Antennæ. By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London. (Plates XVII. & XVIII.) 588

LXX. Electrical Resistance of Alloys. By R. S. WILLOWS, M.A., D.Sc. 604

LXXI. Notices respecting New Books:—W. H. WHEELER's Practical Manual of Tides and Waves; Dr. C. DOELTER's Petrogenesis 610

Index 611

With Titlepage, Contents, &c.

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